1 Modeling Concurrent Processes with LTL

- Regarding Problem 1 of problem set 7 (posted Friday):
  - Two processes \( P \) and \( Q \) running concurrently with a shared variable \( n \) storing some integer \( i \in (-2,-1,0,1,2) \). As for the code itself, I didn’t write it down, but the basic idea is that \( n \) is initialized to 0, process \( Q \) decrements \( n \) while \( P \) increments \( n \) (each by 1), and once \( n=-2 \), process \( Q \) terminates and \( n \) is re-initialized to 0.

- To model this system, think of a model \( M \) with 8 states:
  - \( S[P,Q,i] \), with \( i \in (-2,-1,0,1,2) \), representing when both \( P \) and \( Q \) are running.
  - \( S[P,i] \) with \( i \in (0,1,2) \) when only \( P \) is running.

- Such a model can be represented in terms of the following directed graph:
2 Regular Expressions and Regular Language

- Consider a directed graph with nodes representing states of some system, with each edge labeled either $a$ or $b$. Thus, any path in the system will be represented by a series of $a$'s and $b$'s, and will look something like $aababba \cdots$.

- The expression $L(A)$ is used to denote a particular execution path through the system to one of its final states. More formally, $L(A) =$ "The language recognized by the automaton $A$".
  
  - "+" means 'or', so $(a + b)$ means that either path $a$ or path $b$ can be taken.
  - An expression like $a^*$ means that the path $a$ can be taken an arbitrary number of times, like when a node has an edge which leads immediately to itself.
  - So, an expression like $(a+b)^*b(a+b)$ means "either path $a$ or $b$ is taken (possibly both, alternating) an arbitrary number of times, followed by path $bb$, followed by either path $a$ or path $b$.”

3 $\omega$-Regular Expressions and $\omega$-Regular Language

- Extends finite automata for infinite paths.
  
  - One type of $\omega$-automata is called a "Buchi Automaton". It accepts an infinite input sequence if there exists a run of the automaton which visits at least one of the final states infinitely often.
  - Relevant for deciding validity for models in LTL.