Four Projects Proposed by Mark Reynolds

1. Factorization (Isabelle)

Use Isabelle to prove or refute the statement:
\[ \gcd(N, M) \times \text{lcm}(N, M) = N \times M \]
for all positive integers \( N \) and \( M \). If the statement is false, produce a counterexample.

2. Fibonacci Numbers (Isabelle)

Let \( F_n \) denote the Fibonacci numbers. Use Isabelle to prove the following statements:
\[ \gcd(F_{n+1}, F_n) = 1 \quad \text{and} \quad \gcd(F_{n+2}, F_n) = 1 \]

3. Fibonacci Numbers (Alloy)

Using a scope of at least 8, prove the two statements of Project 2 above in Alloy. Then find an Alloy counterexample to the statement:
\[ \gcd(F_{n+3}, F_n) = 1 \]

4. A Paradox of Set Theory (Alloy)

Consider a system where there are 31 branch libraries, 31 branch librarians, and one head librarian. At the end of each year, the branch librarians compile a catalog of all books at their library. They send these catalogs to the head librarian. This year, the head librarian has discovered an inconsistency: some catalogs list themselves, while others do not. The head librarian sorts the catalogs into two piles. Pile G contains the catalogs that list themselves, while pile B contains the catalogs that do not list themselves. Create an Alloy model of this system.

The head librarian files all the catalogs in pile G, and then decides to create a new catalog \( U \) that lists all the catalogs that do not list themselves. First, show that the assertion "\( U \) lists itself" is inconsistent with your Alloy model of the catalog system. Second, show that the assertion "\( U \) does not list itself" is also inconsistent with your Alloy model. Finally, make a simple change to your model that makes it consistent with the second assertion.