CS 512, Spring 2017, Handout 01

Syntax of Propositional Logic

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Syntax of the WWF's of Propositional Logic

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  5. if φ and ψ are WFF's, then so is $(\varphi \rightarrow \psi)$

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- More succintly, in **BNF (Backus Naur Form)**:

$$\varphi \ ::= \ p \mid (\neg\varphi) \mid (\varphi \wedge \varphi) \mid (\varphi \vee \varphi) \mid (\varphi \rightarrow \varphi)$$

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- Or, more abstractly by omitting parentheses, in **Extended BNF**:

  $$\varphi \ ::= \ p \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid \varphi \to \psi$$

  Parentheses are used only to set an order of precedence among logical connectives $\{\neg, \wedge, \vee, \to\}$.

Parse Trees of WFF's

- A fully-parenthesized WFF:

$$\Big(\big(\neg((\neg P) \lor (Q \land (\neg P)))\big)$$
$$\to \big(\neg((\neg P) \to (Q \lor (\neg R)))\big)\Big)$$

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$$\Big(\big(\neg((\neg P) \vee (Q \wedge (\neg P)))\big)$$
$$\quad \to \big(\neg((\neg P) \to (Q \vee (\neg R)))\big)\Big)$$

- Same WFF with all parentheses omitted:

$$\neg\ \neg P\ \vee\ Q \wedge\ \neg P$$
$$\quad \to\ \neg\ \neg P\ \to\ Q \vee\ \neg R$$

(an incomprehensible mess!)

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(an incomprehensible mess!)

- Same WFF minimally parenthesized:

$$\neg \; (\neg P \;\; \vee \;\; (Q \wedge \neg P)) \\ \rightarrow \;\; (\neg P \;\; \rightarrow \;\; (Q \vee \;\; \neg R))$$

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- Same WFF with all parentheses omitted:

$$\neg \neg P \;\vee\; Q \wedge \;\neg P \\ \to \;\neg\; \neg P \;\to\; Q \vee \;\neg R$$

(an incomprehensible mess!)

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$$\neg\, (\neg P \;\vee\; (Q \wedge \neg P)) \\ \to\; (\neg P \;\to\; (Q \vee \;\neg R))$$

No parentheses in the parse tree