CS 512, Spring 2017, Handout 04

Classical Propositional Logic
versus
Intuitionistic Propositional Logic

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examples of **classical tautologies** that are also **intuitionistic tautologies**

Yet to be studied: An **intuitionistic tautology** is a WFF that is true in every Heyting algebra. There are also **soundness** and **completeness** theorems for intuitionistic logic.

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1. $P \to (Q \to P)$ or, with the convention that **"$\to$" is right-associative**,

   $P \to Q \to P$

2. $P \to \big(Q \to (P \wedge Q)\big)$, *i.e.*, $P \to Q \to (P \wedge Q)$

3. $\Big(\big(P \to (Q \to R)\big) \to \big((P \to Q) \to (P \to R)\big)\Big)$

   i.e., $\big(P \to Q \to R\big) \to (P \to Q) \to (P \to R)$

4. $P \to \neg\neg P$

5. $\neg(P \wedge \neg P)$

6. $(\neg P \vee Q) \to (P \to Q)$

7. $(P \to Q) \to (\neg Q \to \neg P)$

examples of **classical tautologies** that are **not intuitionistic tautologies**

1. $P \vee \neg P$
2. $\neg\neg P \to P$
3. $(P \to Q) \to (\neg P \vee Q)$
4. $\neg(P \wedge Q) \to (\neg P \vee \neg Q)$
5. $(\neg P \to Q) \to (\neg Q \to P)$
6. $(\neg P \to \neg Q) \to (Q \to P)$
7. $((P \to Q) \to P) \to P$