CS 512 Formal Methods, Spring 2017	Instructor: Assaf Kfoury

Lecture 2: Semantics of Classical Propositional Logic

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(These lecture notes are **not** proofread and proof-checked by the instructor.)

General Information:

- Lecture notes (by scribes) are not proof-checked (by me). They are a service from the from the scribe to the rest of class.
- Information about the website:
  1. *To Go Deeper*: for interest, not responsible. Get to know SAT/SMT solver through this class (for term project). Model checker is for the following class. Interactive theorem is also important.
  2. *LaTex*: useful source, e.g. Latex templates.
- Functional Language:
  1. Scheme, LISP
  2. ML, Haskell (typed)
  3. Python (half-function)
- Curry-Howard correspondance: very powerful. Proof-as-programs, Formulas-as-types
- KB — knowledge base = premise

Review:

- Formal System:
  Formal Syntax
  Formal Proof Theory — corresponding to soundness
  Formal Semantics — corresponding to completeness
  Applications
- $\wedge i$ $1, 2$: wedge introduction from Line 1, 2 in LaTex.
  $\wedge$: and logical.
  There's elimination. e_1: left elimination. e_2: right elimination.
  $\neg\neg e$ can't be used in intuitionistic logic.
- Nature Deduction: introduce an assumption by opening a box(have to close it eventually).
- We can invent a rule that a box can have two temporary assumptions.

Classical Propositional Logic:

- The semantics (or formal semantics) of a formal logic L is sometimes called the model theory of L. The model theory of classical propositional logic is defined in terms of Boolean algebras. The standard boolean operations can be defined using truth tables.

- Logical operators and symbols:

Logical Operator/Symbol	English Name	LaTeX Command
$\neg$	negation	\neg
$\wedge$	and / conjunction	\wedge
$\vee$	or / disjunction	\vee
$\oplus$	exclusive or	\oplus
$\rightarrow$	conditional / implication	\rightarrow
$\leftrightarrow$	biconditional / double implication	\leftrightarrow
$\top$	tautology / verum	\top
$\bot$	contradiction / falsum	\bot
$\vdash$	turnstile	\vdash
$\models$	semantic model	\models

- Propositional WFF ϕ is *satisfiable* if *there is* an assignment of truth-values to the propositional atoms which makes ϕ true.
  Propositional WFF ϕ is *tautology* if *every* an assignment of truth-values to the propositional atoms which makes ϕ true.

- Truth table allows you to deduce more validities.

- $\phi_1, \phi_2, ..., \phi_n \models \psi$ say "$\phi_1, \phi_2, ..., \phi_n$ semantically entails ψ" or "every model of $\phi_1, \phi_2, ..., \phi_n$ is a model of ψ".

- Theorem:

  1. Soundness:
     $\phi_1, ...\phi_n \vdash \psi$, $\phi_1, ...\phi_n \models \psi$
     If you can prove it, then it is true.
  2. Completeness:
     $\phi_1, ...\phi_n \models \psi$, $\phi_1, ...\phi_n \vdash \psi$
     If it is true, you can prove it
  3. For understanding:
     ϕ tautology $\models \phi$
     ϕ logical valid $\vdash \phi$
     $\vdash \phi$ iff $\models \phi$

- Minimum requirement: be soundness. Everything if you can prove it must be true.

- Examples of classical tautologies that are also intuitionistic tautologies.
  However, examples of classical tautologies that are not intuitionistic tautologies. Classical tautologies have to use three forbidden rules.

- Universal WFFs includes tautologies. Tautologies include formally deducible. Soundness is the proof that doesn't go beyond the tautologies. (E.g.. NP-Completeness we can't prove.)

Reference:

1. http://www.cs.bu.edu/faculty/kfoury/UNI-Teaching/CS512-Spring17/Lecture/HD03.propositional-logic-semantics.pdf

2. http://www.cs.bu.edu/faculty/kfoury/UNI-Teaching/CS512-Spring17/Lecture/HD04.classical-vs-intuitionistic-propositional-logic.pdf

3. http://www.cs.bu.edu/faculty/kfoury/UNI-Teaching/CS512-Spring17/index.html scribe template