| CS 512 Formal Methods, Spring 2017 | Instructor: Assaf Kfoury |
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# Lecture 8(QBF)

*Feb 17, 2017* Geng Lan

(These lecture notes are **not** proofread and proof-checked by the instructor.)

Genral Notes:

- Assignment 4 due Wed, Feb 22
- Assignment 5 posted next Tuesday
- Tuesday, Feb 21 no lecture

1. QBF: Quantified Boolean Formula.
   Quantifiers: $\forall$, $\exists$

2. Scope: the scope of two binding occurrences "$Q_1x_1$" and "$Q_2x_2$" should be either disjoint or nested, but never overlap. The situation that "$Q_1x_1$" and "$Q_2x_2$" are partially joint cannot happen.

   Consider situation like:

   | | | |
   |---|---|---|
   | ... | ... | ... |
   | ... | $fun.x_1$ | ... |
   | ... | ... | ... |
   | ... | $fun.x_2$ | ... |
   | ... | ... | ... |

   $x_1$ *and* $x_2$ are not the same.

3. Closed and Open QBF:
   closed: all occurrences of propositional variables are bound.
   open: some occurrences of propositional variables are free.
   (page 9, handout 13)
   Substitution: cannot happen to the bound occurrences, like x in the example of handout 13.
   should also remove any conflict about the name. In this example we cannot substitute $\neg x$ for y, because x has been name-captured.

4. Problems we do not know:
   $P \subseteq NP \subseteq PSPACE \subseteq EXPTIME \subseteq NEXPTIME$
   . =? =? =? =?
   Problems we do know:
   PSPACE=NPSACE
   P$\subsetneq$EXPTIME
   NP$\subsetneq$NEXPTIME

SAT belongs to NP, QBF belongs to PSPACE.

5. QBF-solvers are just like SAT-solvers, but the development of QBF-solvers is currently far behind that of SAT-solvers.

6. validity and satisability of closed QBFs coincide, not open QBFs, why? Take this example:
$\varphi = (p \rightarrow q) \wedge (q \rightarrow p)$, we can convert it to:
$\varphi = (\neg p \vee q) \wedge (\neg q \vee p)$
if we assign "T" to q and p, what happen? $\varphi \vdash T$
WFFs of QBF:
$\exists q\ \varphi$ - open
$\exists p \exists q\ \varphi$ - closed
$\forall p\ \varphi$ - open
$\exists q \forall p\ \varphi$ - closed
$\forall q \exists p\ \varphi$ - closed
note that $\exists q \forall p\ \varphi$ is not true, cause when we assign "T" to q, if p is assigned "F", $\varphi$ will be fasle.

7. satisfiable: one, or more, but not necessarily all of the row in truth-table is true.
tautology: all of the row in truth-table are true.

8. Prenexform: QBF is in prenex form iff $Q_1x_1Q_2x_2...Q_nx_n\ \varphi$
$Q_1,\ Q_2, ...Q_n\ \in \{\forall, \exists\}$
and $\varphi$ quantifier free.
$Q_1,\ Q_2, ...Q_n$ is quantifier prefix, $\varphi$ is matrix.

9. Sometimes quantifiers are inside matrix $\varphi$, we want to push them out:
$\neg(\exists x\ \varphi) \rightsquigarrow (transform\ to)(\forall x\ \neg\varphi)$
define $* \in \{\wedge, \vee\}$.
$(Q_1x_1\ \varphi_1) * (Q_2x_2\ \varphi_2) \rightsquigarrow Q_1x_1\ Q_2x_2(\varphi_1 * \varphi_2)$ provided $x_1$ not free in $\varphi_2$ and $x_2$ not free in $\varphi_1$.
Why should we do this? Because QBF solvers work on QBF prenex form, where the matrix is in CNF.