CS512 Formal Methods, Spring 2017

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(These lecture notes were **not** proofread and proof-checked by the instructor)

1. Binary decision diagrams. Refer to Handout 14, sections 6.1 and 6.2 to be read from LCS book. 6.3 and 6.4 for further reading.

2. λ-calculus is a formal system in mathematical logic for expressing computation based on function abstraction and application using variable binding and substitution. It sets the foundation for functional programming languages. Analogically speaking, what a turing machine is to real computers, is what λ-calculus is to programming languages. Propositional logic provides us with an idea as to how we study other logical languages for modeling.

3. Read the introduction of - Donald Knuth, 2009, Art of Programming Languages, Vol 4. Available at *http://www.cs.utsa.edu/ wagner/knuth/*.

4. Exercise problem in Slide 12. Solution below for the ordering: r q p.

$\varphi \triangleq \big((q \to p) \land r\big) \to \big(p \leftrightarrow r\big) \land q$

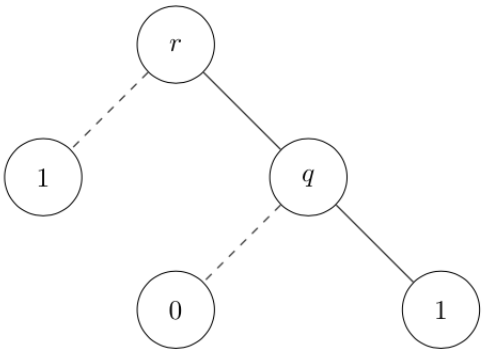
Case 1: $r := \bot$; then $\big((q \to p\big) \land \bot\big) \to \big(p \leftrightarrow \bot\big) \land q$; whose result is true.

Case 2: $r := \top$;
$\big((q \to p\big) \land \top\big) \to \big(p \leftrightarrow \top\big) \land q$;
$\big(q \to p\big) \to p \land q$; We have two sub cases now based on q.

Case 2.1: $q := \bot$;
$\big(\bot \to p\big) \to p \land \bot$; whose result is false.

Case 2.2: $q := \top$;
$\big(\top \to p\big) \to p \land \top$;
$p \to p$; whose result is true.

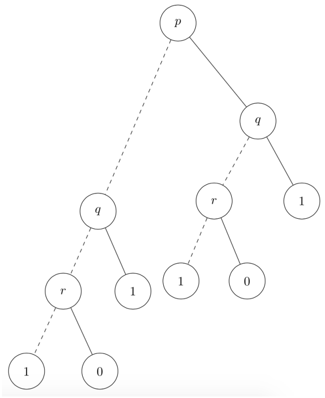
So, ultimately tree looks like,



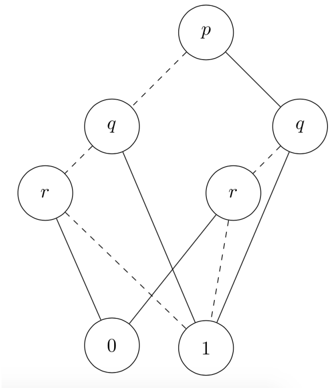
5. Write $if - then - else$ using the standard logical connectives.

if φ then ψ else $\theta \Leftrightarrow (\varphi \to \psi) \wedge (\neg\varphi \to \theta)$

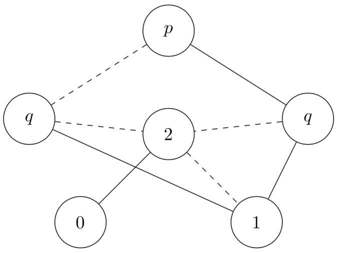
6. Consider the second tree in Slide 13.
Applying the $\{C1, C2, C3\}$ steps in Slide 18, we get the following results.



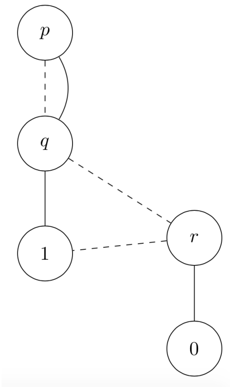
Apply rule C1,



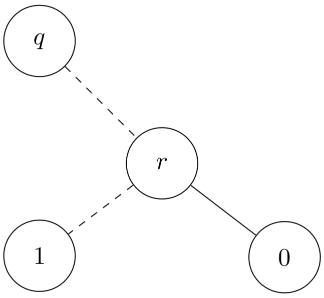
Applying rule C2,



Applying rule C3,



Applying rule C1 again,



which is the ROBDD obtained in the end.