CS 512 Formal Methods, Spring 2017	Instructor: Assaf Kfoury
Lecture Note	
Mar 23, 2017	Desheng Zhang

(These lecture notes are **not** proofread and proof-checked by the instructor.)

Genral Notes:

- Assignment 7 due Fri, Mar 31
- Assignment 8 will be posted on Mar 30

1. Last handout(handout 19) is about Prenex Normal Form and Skolemization. For a Prenex Normal Form, we can skolemize it and get its Skolem normal form.
   Prenex Normal Form: For every WFF ϕ there is an equivalent WFF ψ with the same free variables where all quantifiers appear at the beginning. ψ is called the prenex normal form of ϕ.
   Skolemization: A method for removing existential quantifiers from formal logic statements.

2. **Ground terms**: In mathematical logic, a ground term of a formal system is a term that does not contain any free variables.
   (1) elements of C are ground terms;
   (2) If fF is an n-ary function symbol and 1, 2, ..., n are ground terms, then f(1, 2, ..., n) is a ground term.
   Example: GrTerms $\supseteq \{a, b\} \cup \{f(t)|t \in GrTerms\} \cup \{g(t)|t \in GrTerms\}$

   Ground atoms: A ground predicate or ground atom or ground literal is an atomic formula all of whose argument terms are ground terms.

3. **Herbrand Base:** Roughly speaking, the Herbrand base is the set of all ground atoms, while a Herbrand interpretation assigns a truth value to each ground atom in the base.

   Herbrand Universe: The Herbrand universe of a closed formula in Skolem normal form F, is the set of all terms without variables, that can be constructed using the function symbols and constants of F. If F has no constants, then F is extended by adding an arbitrary new constant.

   Herbrand Structure: In first-order logic, a Herbrand structure S is a structure over a vocabulary , that is defined solely by the syntactical properties of . The idea is to take the symbols of terms as their values, e.g. the denotation of a constant symbol c is just 'c' (the symbol).
   Example: $H = (GrTerms, \doteq, f^H, g^H, P^H) = (GrTerms, \doteq, f, g, P^H)$

4. **Lemma 24** Let ϕ be an arbitrary first-order sentence, Then ϕ is satisfiable iff $\Theta_{P,SK}(\phi)$ has a Herbrand model.

   Given a universal sentence $\phi \equiv \forall x_1 \ldots x_n \phi_0$,
   its herbrand expression should be $\{\phi_0[x_1 = t_1][x_2 = t_2] | t_1, t_2, \ldots, t_n \in GrTerms\}$

5. **Lemma 30** Let ψ be an arbitrary first order sentence and $\Gamma =$ Groud Expression of ψ, Then ψ is satisfiable iff Γ is satisfiable.
   note that Γ is just in the sense of PL, but still not PL.

6. Some other notations
   (1) Boolean algebra & Heying algebra(See definition at handout 20): Every Boolean algebra is a Heyting algebra, which means Boolean algebra is a subset of Heyting algebra.
   (2) Suppose $\{\phi | M \in AandM \models \phi\}$,
   A is a family of first order structure
   B is a family contains A
   Then $B \supseteq A$