Lecture 1: Introduction to Propositional Logic

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Propositions:

- A proposition is a statement that can either be $true$ or $false$. Example: "Joe is a professor".
- Propositional variables refer to propositions (sometimes lower case convention is used). These are also called propositional atoms. Example: p.
- Logical operators and symbols include:

Logical Operator/Symbol	English Name	LaTeX Command
$\neg$	negation	\neg
$\wedge$	and / conjunction	\wedge
$\vee$	or / disjunction	\vee
$\oplus$	exclusive or	\oplus
$\rightarrow$	conditional / implication	\rightarrow
$\leftrightarrow$	biconditional / double implication	\leftrightarrow
$\top$	tautology / verum	\top
$\bot$	contradiction / falsum	\bot
$\vdash$	turnstile	\vdash

- Simple propositions can be combined to obtain more compound propositions using logical operators. Example: $\neg p \wedge q$. The expression "well-formed formulas" (wffs) usually refers to gramatically correct formulas. The wffs of propositional logic are obtained by applying the construction rules below, and only these, finitely many times (more succintly in BNF or eBNF):

  1. Every propositional atom (i.e., propositional variable) p is a WFF
  2. if ϕ is a wff, then so is $\neg\phi$
  3. if ϕ and ψ are wffs, then so is $\phi \wedge \psi$
  4. if ϕ and ψ are wffs, then so is $\phi \vee \psi$
  5. if ϕ and ψ are wffs, then so is $\phi \rightarrow \psi$

  A minimally parenthesized wff is one that can be represented as a parse tree, and therefore has no ambiguity.

- When all of the propositional variables in a logical expression are assigned truth values, the expression itself acquires a truth value. We can then evaluate a logical expression just as we would an arithmetic expression or a relational expression. Example: If we know that p is $false$ and q is $true$, then we can evaluate $p \vee q$ as $false \vee true$ which is $true$.
- There is a rule of introduction (i) for every logical operator. Example: $\wedge i$ means introduction of the and logical operator.

- ⋄ There is a rule of elimination (e) for every logical operator. Example: $\wedge e$ means elimination of the and logical operator. In particular, $\wedge e_1$ means elimination of the and operator by removing the right operand, and $\wedge e_2$ means elimination of the and operator by removing the left operand.
- ⋄ Example of using notation. Let:
  p be the proposition: "you study"
  q be the proposition: "you attend the exam"
  r be the proposition: "you pass"

  English proposition: IF you study AND you attend the exam THEN you pass

  Proposition in notation: $p \wedge q \to r$
- ⋄ Propositional logic is an algebra whose original purpose was to model reasoning. It was then expanded and used for higher level applications in Computer Science like theorem proving and circuit design.
- ⋄ *Natural Deduction* is a style of deduction prepared by computer scientists. Although there exist alternative ways to write a formal proof, example: Hilbert Style Proof System, Natural Deduction will be used in this course.

Informal vs formal reasoning example.

Informal Reasoning	Formal Reasoning
IF you study AND you attend the exam THEN you pass	$p \wedge q \to r$
you studied	p
you did not pass	$\neg r$
$\vdots$	$\vdots$
we can reason that	*we can reason that*
$\downarrow$	$\downarrow$
$\vdots$	$\vdots$
you did not attend the exam	$\neg q$

Question: How do we write that reasoning symbolically?
Answer: $p \wedge q \to r, p, \neg r \vdash \neg q$. This is called a sequent or a judgement. This is a valid/deducible/derivable sequent because there is a proof for it.

The general form of a *sequent* is:
premises/antecedents/hypotheses $\vdash$ *conclusion/succedent*
The proof of a sequent validity is not necessarily unique.

A formal proof is a sequence of wffs starting with the premises and ending with the conclusion using proof rules. Refer to [1] for proof rules. An example is:

$$\frac{\phi \qquad \psi}{\phi \wedge \psi} \wedge i$$

What this means is: if ϕ is true and ψ is true, then we can say that $\phi \wedge \psi$ is also true.

References:

1. http://www.cs.bu.edu/faculty/kfoury/UNI-Teaching/CS512-Spring15/Lecture/HD01.propositional-logic-example.pdf
2. http://www.cs.bu.edu/faculty/kfoury/UNI-Teaching/CS512-Spring15/Lecture/HD01A.propositional-logic-syntax.pdf
3. http://infolab.stanford.edu/ ullman/focs/ch12.pdf
4. http://www.labri.fr/perso/casteran/CoqArt/Tsinghua/C3.pdf