In October of '86, the Internet had the first of what became a series of 'congestion collapses'. During this period, the data throughput from LBL to UC Berkeley (sites separated by 400 yards and three IMP hops) dropped from 32 Kbps to 40 bps. Mike Karels and I were fascinated by this sudden factor-of-thousand drop in bandwidth and embarked on an investigation of why things had gotten so bad. We wondered, in particular, if the 4.3BSD (Berkeley UNIX) TCP was mis-behaving or if it could be tuned to work better under abysmal network conditions. The answer to both of these questions was "yes".

Since that time, we have put seven new algorithms into the 4BSD TCP:

(i) round-trip-time variance estimation
(ii) exponential retransmit timer backoff
(iii) slow-start
(iv) more aggressive receiver ack policy
(v) dynamic window sizing on congestion
(vi) Karn's clamped retransmit backoff
(vii) fast retransmit

Our measurements and the reports of beta testers suggest that the final product is fairly good at dealing with congested conditions on the Internet.

This paper is a brief description of (i) – (v) and the rationale behind them (vii) is an algorithm recently developed by Phil Karn of Bell Communications Research, described in [KP87]. (vii) is described in a soon-to-be-published RFC.

Algorithms (i) – (v) spring from one observation: The flow on a TCP connection (or ISO TP-4 or Xerox NS SPP connection) should obey a 'conservation of packets' principle. And, if this principle were obeyed, congestion collapse would become the exception rather than the rule. Thus congestion control involves finding places that violate conservation and fixing them.

By 'conservation of packets' I mean that for a connection in equilibrium, i.e., running stably with a full window of data in transit, the packet flow is what a physicist would call 'conservative': A new packet isn't put into the network until an old packet leaves. The physics of flow predicts that systems with this property should be robust in the face of congestion. Observation of the Internet suggests that it was not particularly robust. Why the discrepancy?

There are only three ways for packet conservation to fail:

1. The connection doesn't get to equilibrium, or
2. A sender injects a new packet before an old packet has exited, or
3. The equilibrium can't be reached because of resource limits along the path.

In the following sections, we treat each of these in turn.

1 Getting to Equilibrium: Slow-start

Failure (1) has to be from a connection that is either starting or restarting after a packet loss. Another way to look at the conservation property is to say that the sender uses acks as a 'clock' to strobe new packets into the network. Since the receiver can generate acks no faster than data packets can get through the network,
This is a schematic representation of a sender and receiver on high bandwidth networks connected by a slower, long-haul net. The sender is just starting and has shipped a window’s worth of packets, back-to-back. The ack for the first of those packets is about to arrive back at the sender (the vertical line at the mouth of the lower left funnel).

The vertical direction is bandwidth, the horizontal direction is time. Each of the shaded boxes is a packet. Bandwidth x Time = Bits so the area of each box is the packet size.

The number of bits doesn’t change as a packet goes through the network so a packet squeezed into the smaller long-haul bandwidth must spread out in time. The time $P_b$ represents the minimum packet spacing on the slowest link in the path (the bottleneck). As the packets leave the bottleneck for the destination net, nothing changes the inter-packet interval so on the receiver’s net packet spacing $P_r = P_b$. If the receiver processing time is the same for all packets, the spacing between acks on the receiver’s net $A_r = P_r = P_b$. If the time slot $P_b$ was big enough for a packet, it’s big enough for an ack so the ack spacing is preserved along the return path. Thus the ack spacing on the sender’s net $A_s = P_b$.

So, if packets after the first burst are sent only in response to an ack, the sender’s packet spacing will be exactly match the packet time on the slowest link in the path.

Figure 1: Window Flow Control ‘Self-clocking’

The protocol is ‘self clocking’ (fig. 1). Self clocking systems automatically adjust to bandwidth and delay variations and have a wide dynamic range (important considering that TCP spans a range from 800 Mbps Cray channels to 1200 bps packet radio links). But the same thing that makes a self-clocked system stable when it’s running makes it hard to start — to get data flowing there must be acks to clock out packets but to get acks there must be data flowing.

To start the ‘clock’, we developed a slow-start algorithm to gradually increase the amount of data in transit. Although we flatter ourselves that the design of this algorithm is rather subtle, the implementation is trivial — one new state variable and three lines of code in the sender:

- Add a congestion window, cwnd, to the per-connection state.
- When starting or restarting after a loss, set cwnd to one packet.
- On each ack for new data, increase cwnd by one packet.
- When sending, send the minimum of the receiver’s advertised window and cwnd.

$^2$Slow-start is quite similar to the CUTE algorithm described in [Jai86b]. We didn’t know about CUTE at the time we were developing slow-start but we should have — CUTE preceded our work by several months.

When describing our algorithm at the Feb., 1987, Internet Engineer-
The horizontal direction is time. The continuous time line has been chopped into one-round-trip-time pieces stacked vertically with increasing time going down the page. The grey, numbered boxes are packets. The white numbered boxes are the corresponding acks.

As each ack arrives, two packets are generated: one for the ack (the ack says a packet has left the system so a new packet is added to take its place) and one because an ack opens the congestion window by one packet. It may be clear from the figure why an add-one-packet-to-window policy opens the window exponentially in time.

If the local net is much faster than the long haul net, the ack's two packets arrive at the bottleneck at essentially the same time. These two packets are shown stacked on top of one another (indicating that one of them would have to occupy space in the gateway's outbound queue). Thus the short-term queue demand on the gateway is increasing exponentially and opening a window of size $W$ packets will require $W/2$ packets of buffer capacity at the bottleneck.

Figure 2: The Chronology of a Slow-start

Actually, the slow-start window increase isn't that slow: it takes time $R \log_2 W$ where $R$ is the round-trip-time and $W$ is the window size in packets (fig. 2). This means the window opens quickly enough to have a negligible effect on performance, even on links with a large bandwidth-delay product. And the algorithm guarantees that a connection will source data at a rate at most twice the maximum possible on the path. Without slow-start, by contrast, when 10 Mbps Ethernet hosts talk over the 56 Kbps Arpanet via IP gateways, the first-hop gateway sees a burst of eight packets delivered at 200 times the path bandwidth. This burst of packets often puts the connection into a persistant failure mode of continuous retransmissions (figures 3 and 4).

2 Conservation at equilibrium: round-trip timing

Once data is flowing reliably, problems (2) and (3) should be addressed. Assuming that the protocol implementation is correct, (2) must represent a failure of sender's retransmit timer. A good round trip time estimator, the core of the retransmit timer, is the single most important feature of any protocol implementation that expects to survive heavy load. And it is frequently botched ([Zha86] and [Jai86a] describe typical problems).

One mistake is not estimating the variation, $\sigma_R$, of
Trace data of the start of a TCP conversation between two Sun 3/50s running Sun OS 3.5 (the 4.3BSD TCP). The two Suns were on different Ethernets connected by IP gateways driving a 230.4 Kbps point-to-point link (essentially the setup shown in fig. 7).

Each dot is a 512 data-byte packet. The x-axis is the time the packet was sent. The y-axis is the sequence number in the packet header. Thus a vertical array of dots indicate back-to-back packets and two dots with the same y but different x indicate a retransmit.

'Desirable' behavior on this graph would be a relatively smooth line of dots extending diagonally from the lower left to the upper right. The slope of this line would equal the available bandwidth. Nothing in this trace resembles desirable behavior.

The dashed line shows the 20 KBps bandwidth available for this connection. Only 35% of this bandwidth was used; the rest was wasted on retransmits. Almost everything is retransmitted at least once and data from 54 to 58 KB is sent five times.

![Graph showing TCP startup behavior](image)

**Figure 3: Startup behavior of TCP without Slow-start**

the round trip time, $R$. From queuing theory we know that $R$ and the variation in $R$ increase quickly with load. If the load is $\rho$ (the ratio of average arrival rate to average departure rate), $R$ and $\sigma_R$ scale like $(1 - \rho)^{-1}$. To make this concrete, if the network is running at 75% of capacity, as the Arpanet was in last April’s collapse, one should expect round-trip-time to vary by a factor of sixteen ($\pm 2\sigma$).

The TCP protocol specification, [RFC793], suggests estimating mean round trip time via the low-pass filter

$$R = \alpha R + (1 - \alpha)M$$

where $R$ is the average RTT estimate, $M$ is a round trip time measurement from the most recently acked data packet, and $\alpha$ is a filter gain constant with a suggested value of 0.9. Once the $R$ estimate is updated, the retransmit timeout interval, $rto$, for the next packet sent is set to $\beta R$.

The parameter $\beta$ accounts for RTT variation (see [Cla82], section 5). The suggested $\beta = 2$ can adapt to loads of at most 30%. Above this point, a connection will respond to load increases by retransmitting packets that have only been delayed in transit. This forces the network to do useless work, wasting bandwidth on duplicates of packets that will be delivered, at a time when it’s known to be having trouble with useful work. I.e., this is the network equivalent of pouring gasoline on a fire.

We developed a cheap method for estimating vari-
Same conditions as the previous figure (same time of day, same Suns, same network path, same buffer and window sizes), except the machines were running the 4.3+TCP with slow-start.

No bandwidth is wasted on retransmits but two seconds is spent on the slow-start so the effective bandwidth of this part of the trace is 16 KBps — two times better than figure 3. (This is slightly misleading: Unlike the previous figure, the slope of the trace is 20 KBps and the effect of the 2 second offset decreases as the trace lengthens. E.g., if this trace had run a minute, the effective bandwidth would have been 19 KBps. The effective bandwidth without slow-start stays at 7 KBps no matter how long the trace.)

Figure 4: Startup behavior of TCP with Slow-start

Another timer mistake is in the backoff after a retransmit: If a packet has to be retransmitted more than once, how should the retransmits be spaced? Only one scheme will work, exponential backoff, but proving this is a bit involved. To finesse a proof, note that

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3 We are far from the first to recognize that transport needs to estimate both mean and variation. See, for example, [Edg83]. But we do think our estimator is simpler than most.

4 An in-progress paper attempts a proof. If an IP gateway is viewed as a ‘shared resource with fixed capacity’, it bears a remarkable resemblance to the ‘ether’ in an Ethernet. The retransmit backoff problem is essentially the same as showing that no backoff ‘slower’ than an exponential will guarantee stability on an Ethernet. Unfortunately, in theory even exponential backoff won’t guarantee stability (see [Ald87]). Fortunately, in practise we don’t have to deal with the theorist’s infinite user population and exponential is “good enough”.

5 The phrase congestion collapse (describing a positive feedback instability due to poor retransmit timers) is again the coinage of John Nagle, this time from [Nag84].

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Figure 5: Performance of an RFC793 retransmit timer

3 Adapting to the path: congestion avoidance

If the timers are in good shape, it is possible to state with some confidence that a timeout indicates a lost packet and not a broken timer. At this point, something can be done about (3). Packets get lost for two reasons: they are damaged in transit, or the network is congested and somewhere on the path there was insufficient buffer capacity. On most network paths, loss due to damage is rare ($\ll 1\%$) so it is probable that a packet loss is due to congestion in the network.

A ‘congestion avoidance’ strategy, such as the one proposed in [JRC87], will have two components: The network must be able to signal the transport endpoints that congestion is occurring (or about to occur). And the endpoints must have a policy that decreases utilization if this signal is received and increases utilization if the signal isn’t received.

If packet loss is (almost) always due to congestion and if a timeout is (almost) always due to a lost packet, we have a good candidate for the ‘network is congested’ signal. Particularly since this signal is delivered automatically by all existing networks, without special modification (as opposed to [JRC87] which requires a new bit in the packet headers and a modification to all existing gateways to set this bit).

The other part of a congestion avoidance strategy, the endnode action, is almost identical in the DEC/ISO scheme and our TCP and follows directly from a first-order time-series model of the network: Say network load is measured by average queue length over fixed intervals of some appropriate length (something near the round trip time). If $L_i$ is the load at interval $i$, an uncongested network can be modeled by saying $L_i$ changes

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6 The congestion control scheme we propose is insensitive to damage loss until the loss rate is on the order of one packet per window (e.g., 12–15% for an 8 packet window). At this high loss rate, any window flow control scheme will perform badly—an 12% loss rate degrades TCP throughput by 60%. The additional degradation from the congestion avoidance window shrinking is the least of one’s problems. A presentation in [IETF88] and an in-progress paper address this subject in more detail.

7 This is not an accident: We copied Jain’s scheme after hearing his presentation at [IETF87] and realizing that the scheme was, in a sense, universal.
Same data as above but the solid line shows a retransmit timer computed according to the algorithm in appendix A.

Figure 6: Performance of a Mean+Variance retransmit timer

slowly compared to the sampling time. I.e.,

\[ L_i = N \]

\( (N \text{ constant}) \). If the network is subject to congestion, this zeroth order model breaks down. The average queue length becomes the sum of two terms, the \( N \) above that accounts for the average arrival rate of new traffic and intrinsic delay, and a new term that accounts for the fraction of traffic left over from the last time interval and the effect of this left-over traffic (e.g., induced retransmits):

\[ L_i = N + \gamma L_{i-1} \]

(These are the first two terms in a Taylor series expansion of \( L(t) \). There is reason to believe one might eventually need a three term, second order model, but not until the Internet has grown substantially.)

When the network is congested, \( \gamma \) must be large and the queue lengths will start increasing exponentially.\(^8\) The system will stabilize only if the traffic sources throttle back at least as quickly as the queues are growing. Since a source controls load in a window-based protocol by adjusting the size of the window, \( W \), we end up with the sender policy

\[ W_i = dW_{i-1} \quad (d < 1) \]

I.e., a multiplicative decrease of the window size (which becomes an exponential decrease over time if the congestion persists).

On congestion:

\[ L_i = N + \gamma L_{i-1} \]

If there’s no congestion, \( \gamma \) must be near zero and the load approximately constant. The network announces, via a dropped packet, when demand is excessive but says nothing if a connection is using less than its fair share (since the network is stateless, it cannot know this). Thus a connection has to increase its bandwidth utilization to find out the current limit. E.g., you could have been sharing the path with someone else and converged to a window that gives you each half the available bandwidth. If she shuts down, 50% of the bandwidth will be wasted unless your window size is increased. What should the increase policy be?

The first thought is to use a symmetric, multiplicative increase, possibly with a longer time constant, \( W_i = bW_{i-1}, 1 < b \leq 1/d \). This is a mistake. The result will oscillate wildly and, on the average, deliver poor throughput. There is an analytic reason for this but it’s tedious to derive. It has to do with that fact that it is easy to drive the net into saturation but hard for the net to recover (what [Kle76], chap. 2.1, calls the rush-hour effect).\(^9\) Thus overestimating the available bandwidth

\[ L_i = N + \gamma L_{i-1} \]

which goes exponentially to infinity for any \( \gamma > 1 \)

\(^8\) I.e., the system behaves like \( L_i \approx \gamma L_{i-1} \), a difference equation with the solution \( L_n = \gamma^n L_0 \) which goes exponentially to infinity for any \( \gamma > 1 \)

\(^9\) In fig. 1, note that the ‘pipesize’ is 16 packets, 8 in each path, but
is costly. But an exponential, almost regardless of its time constant, increases so quickly that overestimates are inevitable.

Without justification, I’ll state that the best increase policy is to make small, constant changes to the window size:

\[
W_i = W_{i-1} + u \quad (u < W_{\text{max}})
\]

where \(W_{\text{max}}\) is the pipesize (the delay-bandwidth product of the path minus protocol overhead — i.e., the largest sensible window for the unloaded path). This is the additive increase / multiplicative decrease policy suggested in [JRC87] and the policy we’ve implemented in TCP. The only difference between the two implementations is the choice of constants for \(d\) and \(u\). We used 0.5 and 1 for reasons partially explained in appendix C. A more complete analysis is in yet another in-progress paper.

The preceding has probably made the congestion control algorithm sound hairy but it’s not. Like slow-start, it’s three lines of code:

- On any timeout, set \(cwnd\) to half the current window size (this is the multiplicative decrease).
- On each ack for new data, increase \(cwnd\) by \(1/cwnd\) (this is the additive increase).\(^{10}\)

the sender is using a window of 22 packets. The six excess packets will form a queue at the entry to the bottleneck and that queue cannot shrink; even though the sender carefully clocks out packets at the bottleneck link rate. This stable queue is another, unfortunate, aspect of conservation: The queue would shrink only if the gateway could move packets into the skinny pipe faster than the sender dumped packets into the fat pipe. But the system tunes itself so each time the gateway pulls a packet off the front of its queue, the sender lays a new packet on the end.

A gateway needs excess output capacity (i.e., \(\rho < 1\)) to dissipate a queue and the clearing time will scale like \((1-\rho)^{-1}\) ([Kle76], chap. 2 is an excellent discussion of this). Since at equilibrium our transport connection ‘wants’ to run the bottleneck link at 100% (\(\rho = 1\)), we have to be sure that during the non-equilibrium window adjustment, our control policy allows the gateway enough free bandwidth to dissipate queues that inevitably form due to path testing and traffic fluctuations. By an argument similar to the one used to show exponential timer backoff is necessary, it’s possible to show that an exponential (multiplicative) window increase policy will be ‘faster’ than the dissipation time for some traffic mix and, thus, leads to an unbounded growth of the bottleneck queue.

\(^{10}\)This increment rule may be less than obvious. We want to increase the window by at most one packet over a time interval of length \(R\) (the round trip time). To make the algorithm ‘self-clocked’, it’s better to increase by a small amount on each ack rather than by a large amount at the end of the interval. (Assuming, of course, that the sender has effective silly window avoidance (see [Cla82, section 3) and doesn’t attempt to send packet fragments because of the fractionally sized window.) A window of size \(cwnd\) packets will generate at most \(cwnd\) acks in one \(R\). Thus an increment of \(1/cwnd\) per ack will increase the window by at most one packet in one \(R\). In TCP, windows and packet sizes are in bytes so the increment translates to \(\text{maxseg} * \text{maxseg} / \text{cwnd}\) where \(\text{maxseg}\) is the maximum segment size and \(\text{cwnd}\) is expressed in bytes, not packets.

\(^{11}\)We have also developed a rate-based variant of the congestion avoidance algorithm to apply to connectionless traffic (e.g., domain server queries, RPC requests). Remembering that the goal of the increase and decrease policies is bandwidth adjustment, and that ‘time’ (the controlled parameter in a rate-based scheme) appears in the denominator of bandwidth, the algorithm follows immediately: The multiplicative decrease remains a multiplicative decrease (e.g., double the interval between packets). But subtracting a constant amount from interval does not result in an additive increase in bandwidth. This approach has been tried, e.g., [KR87] and [PP87], and appears to oscillate badly. To see why, note that for an inter-packet interval \(J\) and decrement \(c\), the bandwidth change of a decrease-interval-by-constant policy is

\[
\frac{1}{J} - \frac{1}{J} = c
\]

\(c\) is the desired rate of increase in packets per sec. We have simulated the above algorithm and it appears to perform well. To test the predictions of that simulation against reality, we have a cooperative project with Sun Microsystems to prototype RPC dynamic congestion control algorithms using NFS as a test-bed (since NFS is known to have congestion problems yet it would be desirable to have it work over the same range of networks as TCP).
Test setup to examine the interaction of multiple, simultaneous TCP conversations sharing a bottleneck link. 1 MByte transfers (2048 512-byte packets) were initiated 3 seconds apart from four machines at LBL to four machines at UCB, one conversation per machine pair (the dotted lines above show the pairing). All traffic went via a 230.4 Kbps link connecting IP router *csam* at LBL to IP router *cartan* at UCB.

The microwave link queue can hold up to 50 packets. Each connection was given a window of 16 KB (32 512-byte packets). Thus any two connections could overflow the available buffering and the four connections exceeded the queue capacity by 160%.

Figure 7: Multiple conversation test setup

the buffer capacity of the UCB-MIT IMP path and would lead to the behavior shown in the following figures.

4 Future work: the gateway side of congestion control

While algorithms at the transport endpoints can insure the network capacity isn't exceeded, they cannot insure fair sharing of that capacity. Only in gateways, at the convergence of flows, is there enough information to control sharing and fair allocation. Thus, we view the gateway 'congestion detection' algorithm as the next big step.

The goal of this algorithm to send a signal to the endnodes as early as possible, but not so early that the gateway becomes starved for traffic. Since we plan to continue using packet drops as a congestion signal, gateway 'self protection' from a mis-behaving host should fall-out for free: That host will simply have most of its packets dropped as the gateway tries to tell it that it's using more than its fair share. Thus, like the endnode algorithm, the gateway algorithm should reduce congestion even if no endnode is modified to do congestion avoidance. And nodes that do implement congestion avoidance will get their fair share of bandwidth and a minimum number of packet drops.

Since congestion grows exponentially, detecting it early is important — If detected early, small adjustments to the senders' windows will cure it. Otherwise massive adjustments are necessary to give the net enough spare capacity to pump out the backlog. But, given the bursty nature of traffic, reliable detection is a non-trivial problem. [JRC87] proposes a scheme based on averaging between queue regeneration points. This should yield good burst filtering but we think it might have convergence problems under high load or significant second-order dynamics in the traffic. We plan to use some of our earlier work on ARMAX models for round-trip-time/queue length prediction as the basis of detection. Preliminary results suggest that this approach works well at high load, is immune to second-order dynamics in the traffic.  

12 The time between regeneration points scales like $(1 - p)^{-1}$ and the variance of that time like $(1 - p)^{-3}$ (see [Fel71], chap. VI.9). Thus the congestion detector becomes sluggish as congestion increases and its signal-to-noise ratio decreases dramatically.

13 The time between regeneration points scales like $(1 - p)^{-1}$ and the variance of that time like $(1 - p)^{-3}$ (see [Fel71], chap. VI.9). Thus the congestion detector becomes sluggish as congestion increases and its signal-to-noise ratio decreases dramatically.
Trace data from four simultaneous TCP conversations without congestion avoidance over the paths shown in figure 7.

4,000 of 11,000 packets sent were retransmissions (i.e., half the data packets were retransmitted).

Since the link data bandwidth is 25 KBps, each of the four conversations should have received 6 KBps. Instead, one conversation got 8 KBps, two got 5 KBps, one got 0.5 KBps and 6 KBps has vanished.

Figure 8: Multiple, simultaneous TCPs with no congestion avoidance

order effects in the traffic and is computationally cheap enough to not slow down kilopacket-per-second gateways.

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A fast algorithm for rtt mean and variation

A.1 Theory

The RFC793 algorithm for estimating the mean round trip time is one of the simplest examples of a class of estimators called recursive prediction error or stochastic gradient algorithms. In the past 20 years these algorithms have revolutionized estimation and control theory\(14\) and it's probably worth looking at the RFC793 estimator in some detail.

Given a new measurement \(M\) of the RTT (round trip time), TCP updates an estimate of the average RTT \(A\) by

\[
A \leftarrow (1 - g)A + gM
\]

where \(g\) is a 'gain' (\(0 < g < 1\)) that should be related to the signal-to-noise ratio (or, equivalently, variance)

\(14\) See, for example [LS83].
Trace data from four simultaneous TCP conversations using congestion avoidance over the paths shown in figure 7.

89 of 8281 packets sent were retransmissions (i.e., 1% of the data packets had to be retransmitted).

Two of the conversations got 8 KBps and two got 4.5 KBps (i.e., all the link bandwidth is accounted for — see fig. 11). The difference between the high and low bandwidth senders was due to the receivers. The 4.5 KBps senders were talking to 4.3 BSD receivers which would delay an ack until 35% of the window was filled or 200 ms had passed (i.e., an ack was delayed for 5–7 packets on the average). This meant the sender would deliver bursts of 5–7 packets on each ack.

The 8 KBps senders were talking to 4.3 BSD receivers which would delay an ack for at most one packet (the author doesn’t believe that delayed acks are a particularly good idea). I.e., the sender would deliver bursts of at most two packets.

The probability of loss increases rapidly with burst size so senders talking to old-style receivers saw three times the loss rate (1.8% vs. 0.5%). The higher loss rate meant more time spent in retransmit wait and, because of the congestion avoidance, smaller average window sizes.

**Figure 9:** Multiple, simultaneous TCPs with congestion avoidance

of \( M \). This makes a more sense, and computes faster, if we rearrange and collect terms multiplied by \( g \) to get

\[
A \leftarrow A + g(M - A)
\]

Think of \( A \) as a prediction of the next measurement. \( M - A \) is the error in that prediction and the expression above says we make a new prediction based on the old prediction plus some fraction of the prediction error. The prediction error is the sum of two components: (1) error due to ‘noise’ in the measurement (random, unpredictable effects like fluctuations in competing traffic) and (2) error due to a bad choice of \( A \). Calling the random error \( E_r \) and the estimation error \( E_e \),

\[
A \leftarrow A + gE_r + gE_e
\]

The \( gE_e \) term gives \( A \) a kick in the right direction while the \( gE_r \) term gives it a kick in a random direction. Over a number of samples, the random kicks cancel each other out so this algorithm tends to converge to the correct average. But \( g \) represents a compromise: We want a large \( g \) to get mileage out of \( E_e \) but a small \( g \) to minimize the damage from \( E_r \). Since the \( E_e \) terms move
The thin line shows the total bandwidth used by the four senders without congestion avoidance (fig. 8), averaged over 5 second intervals and normalized to the 25 KBps link bandwidth. Note that the senders send, on the average, 25% more than will fit in the wire.

The thick line is the same data for the senders with congestion avoidance (fig. 9). The first 5 second interval is low (because of the slow-start), then there is about 20 seconds of damped oscillation as the congestion control 'regulator' for each TCP finds the correct window size. The remaining time the senders run at the wire bandwidth. (The activity around 110 seconds is a bandwidth 're-negotiation' due to connection one shutting down. The activity around 80 seconds is a reflection of the 'flat spot' in fig. 9 where most of conversation two's bandwidth is suddenly shifted to conversations three and four — a colleague and I find this 'punctuated equilibrium' behavior fascinating and hope to investigate its dynamics in a future paper.)

A toward the real average no matter what value we use for \( g \), it's almost always better to use a gain that's too small rather than one that's too large. Typical gain choices are 0.1–0.2 (though it's a good idea to take long look at your raw data before picking a gain).

It's probably obvious that \( \bar{A} \) will oscillate randomly around the true average and the standard deviation of \( \bar{A} \) will be \( g \text{sdev}(M) \). Also that \( \bar{A} \) converges to the true average exponentially with time constant \( 1/g \). So a smaller \( g \) gives a stabler \( \bar{A} \) at the expense of taking a much longer time to get to the true average.

If we want some measure of the variation in \( M \), say to compute a good value for the TCP retransmit timer, there are several alternatives. Variance, \( \sigma^2 \), is the conventional choice because it has some nice mathematical properties. But computing variance requires squaring \( (M - \bar{A}) \) so an estimator for it will contain a multiply with a danger of integer overflow. Also, most applications will want variation in the same units as \( A \) and \( M \), so we'll be forced to take the square root of the variance to use it (i.e., at least a divide, multiply and two adds).

A variation measure that's easy to compute is the mean prediction error or mean deviation, the average of \( |M - \bar{A}| \). Also, since

\[
\text{mdev}^2 = \left( \sum |M - \bar{A}| \right)^2 \geq \sum |M - \bar{A}|^2 = \sigma^2
\]

mean deviation is a more conservative (i.e., larger) estimate of variation than standard deviation.\(^{15}\)

There's often a simple relation between mdev and sdev. E.g., if the prediction errors are normally distributed, \( \text{mdev} = \sqrt{\pi/2} \text{sdev} \). For most common distributions the factor to go from sdev to mdev is near

\(^{15}\)Mathematical purists may note that I elided a factor of \( n \), the number of samples, from the previous inequality. It makes no difference in the result.
Figure 10 showed the old TCPS were using 25% more than the bottleneck link bandwidth. Thus, once the bottleneck queue filled, 25% of the senders' packets were being discarded. If the discards, and only the discards, were retransmitted, the senders would have received the full 25 KBps link bandwidth (i.e., their behavior would have been anti-social but not self-destructive). But fig. 8 noted that around 25% of the link bandwidth was unaccounted for.

Here we average the total amount of data acked per five second interval. (This gives the effective or delivered bandwidth of the link.) The thin line is once again the old TCPS. Note that only 75% of the link bandwidth is being used for data (the remainder must have been used by retransmissions of packets that didn’t need to be retransmitted). The thick line shows delivered bandwidth for the new TCPS. There is the same slow-start and turn-on transient followed by a long period of operation right at the link bandwidth.

Figure 11: Effective bandwidth of old and new TCPS

Fast estimators for average $A$ and mean deviation $D$ given measurement $M$ follow directly from the above. Both estimators compute means so there are two instances of the RFC793 algorithm:

\[ E_{rr} = M - A \]
\[ A \leftarrow A + g E_{rr} \]
\[ D \leftarrow D + g(|E_{rr}| - D) \]

To compute quickly, the above should be done in integer arithmetic. But the expressions contain fractions ($g < 1$) so some scaling is needed to keep everything integer. A reciprocal power of 2 (i.e., $g = 1/2^n$ for some $n$) is a particularly good choice for $g$ since the scaling can be implemented with shifts. Multiplying through by $1/g$ gives

\[ 2^n A \leftarrow 2^n A + E_{rr} \]
\[ 2^n D \leftarrow 2^n D + (|E_{rr}| - D) \]

To minimize round-off error, the scaled versions of $A$ and $D$, $SA$ and $SD$, should be kept rather than the unscaled versions. Picking $g = .125 = 1/8$ (close to the .1 suggested in RFC793) and expressing the above in C:

\[
M := (SA >> 3); /* = E_{rr} */
SA += M;
if (M < 0)
   M = -M; /* = abs(E_{rr}) */
M := (SD >> 3);
SD += M;
\]

It’s not necessary to use the same gain for $A$ and $D$. To force the timer to go up quickly in response
Because of the five second averaging time (needed to smooth out the spikes in the old TCP data), the congestion avoidance window policy is difficult to make out in figures 10 and 11. Here we show effective throughput (dataacked) for TCP with congestion control, averaged over a three second interval.

When a packet is dropped, the sender sends until it fills the window, then stops until the retransmission timeout. Since the receiver cannot ack data beyond the dropped packet, on this plot we'd expect to see a negative-going spike whose amplitude equals the sender's window size (minus one packet). If the retransmit happens in the next interval (the intervals were chosen to match the retransmit timeout), we'd expect to see a positive-going spike of the same amplitude when receiver acks its cached data. Thus the height of these spikes is a direct measure of the sender's window size.

The data clearly shows three of these events (at 15, 33 and 57 seconds) and the window size appears to be decreasing exponentially. The dotted line is a least squares fit to the six window size measurements obtained from these events. The fit time constant was 28 seconds. (The long time constant is due to lack of a congestion avoidance algorithm in the gateway. With a ‘drop’ algorithm running in the gateway, the time constant should be around 4 seconds)

Figure 12: Window adjustment detail

To changes in the RTT, it's a good idea to give \( D \) a larger gain. In particular, because of window-delay mismatch, there are often RTT artifacts at integer multiples of the window size.\(^\text{16}\) To filter these, one would like \( 1/g \) in the \( A \) estimator to be at least as large as the window size (in packets) and \( 1/g \) in the \( D \) estimator to be less than the window size.\(^\text{17}\)

\(^\text{16}\)E.g., see packets 10–50 of figure 5. Note that these window effects are due to characteristics of the Arpa/Milnet subnet. In general, window effects on the timer are at most a second-order consideration and depend a great deal on the underlying network. E.g., if one were using the Wideband with a 256 packet window, \( 1/256 \) would not be a good gain for \( A \) (\( 1/16 \) might be).

\(^\text{17}\)Although it may not be obvious, the absolute value in the calculation of \( D \) introduces an asymmetry in the timer: Because \( D \) has

Using a gain of .25 on the deviation and computing the retransmit timer, \( rto \), as \( A+2D \), the final timer code looks like:

\[
\begin{align*}
M &\leftarrow (SA >> 3); \\
SA &\leftarrow M; \\
& \text{if } (M < 0) \\
& \quad M = -M; \\
M &\leftarrow (SD >> 2); \\
SD &\leftarrow M; \\
rto &\leftarrow ((SA >> 2) + SD) >> 1;
\end{align*}
\]

the same sign as an increase and the opposite sign of a decrease, more gain in \( D \) makes the timer go up quickly and come down slowly, 'automatically' giving the behavior suggested in [Mii83]. E.g., see the region between packets 50 and 80 in figure 6.
Note that $SA$ and $SD$ are added before the final shift. In general, this will correctly round $rto$: Because of the $SA$ truncation when computing $M - A$, $SA$ will converge to the true mean rounded up to the next tick. Likewise with $SD$. Thus, on the average, there is half a tick of bias in each. The $rto$ computation should be rounded by half a tick and one tick needs to be added to account for sends being phased randomly with respect to the clock. So, the 1.5 tick bias contribution from $A + 2D$ equals the desired half tick rounding plus one tick phase correction.

**B The combined slow-start with congestion avoidance algorithm**

The sender keeps two state variables for congestion control: a congestion window, $cwnd$, and a threshold size, $ssthresh$, to switch between the two algorithms. The sender's output routine always sends the minimum of $cwnd$ and the window advertised by the receiver. On a timeout, half the current window size is recorded in $ssthresh$ (this is the multiplicative decrease part of the congestion avoidance algorithm), then $cwnd$ is set to 1 packet (this initiates slow-start). When new data is acked, the sender does

```c
if (cwnd < ssthresh) /* if we're still doing slow-start */
  cwnd += 1
else /* otherwise do Congestion */
  cwnd += 1/cwnd
```

Thus slow-start opens the window quickly to what congestion avoidance thinks is a safe operating point (half the window that got us into trouble), then congestion avoidance takes over and slowly increases the window size to probe for more bandwidth becoming available on the path.

**C Window Adjustment Policy**

A reason for using $\frac{1}{2}$ as a the decrease term, as opposed to the $\frac{7}{6}$ in [JRC87], was the following handwav- ing: When a packet is dropped, you’re either starting (or restarting after a drop) or steady-state sending. If you’re starting, you know that half the current window size ‘worked’, i.e., that a window’s worth of packets were exchanged with no drops (slow-start guarantees this). Thus on congestion you set the window to the largest size that you know works then slowly increase the size. If the connection is steady-state running and a packet is dropped, it’s probably because a new connection started up and took some of your bandwidth. We usually run our nets with $\rho \leq 0.5$ so it's probable that there are now exactly two conversations sharing the bandwidth. I.e., you should reduce your window by half because the bandwidth available to you has been reduced by half. And, if there are more than two conversations sharing the bandwidth, halving your window is conservative — and being conservative at high traffic intensities is probably wise.

Although a factor of two change in window size seems a large performance penalty, in system terms the cost is negligible: Currently, packets are dropped only when a large queue has formed. Even with an [ISO86] ‘congestion experienced’ bit to force senders to reduce their windows, we’re stuck with the queue because the bottleneck is running at 100% utilization with no excess bandwidth available to dissipate the queue. If a packet is tossed, some sender shuts up for two RTT, exactly the time needed to empty the queue. If that sender restarts with the correct window size, the queue won’t reform. Thus the delay has been reduced to minimum without the system losing any bottleneck bandwidth.

The 1-packet increase has less justification than the 0.5 decrease. In fact, it’s almost certainly too large. If the algorithm converges to a window size of $w$, there are $O(w^2)$ packets between drops with an additive increase policy. We were shooting for an average drop rate of $<1\%$ and found that on the Arpanet (the worst case of the four networks we tested), windows converged to 8–12 packets. This yields 1 packet increments for a 1% average drop rate.

But, since we've done nothing in the gateways, the window we converge to is the maximum the gateway can accept without dropping packets. I.e., in the terms of [JRC87], we are just to the left of the cliff rather than just to the right of the knee. If the gateways are fixed so they start dropping packets when the queue gets pushed past the knee, our increment will be much too aggressive and should be dropped by about a factor of four (since our measurements on an unloaded Arpanet place its ‘pipe size’ at 4–5 packets). It appears trivial to implement a second order control loop to adaptively determine the appropriate increment to use for a path. But second order problems are on hold until we've spent some time on the first order part of the algorithm for the gateways.
References


