CS480/CS680 Linear Algebra Self-Assessment

Due: September 8 at 11:00

For related review, see: Appendix A1-A5 in Hearn & Baker textbook

1. (a) Given points \( p_1 = (1, 6, 5) \) and \( p_2 = (-2, 2, 5) \), solve for \( v_1 \) the vector from \( p_1 \) to \( p_2 \).
   (b) Given a third point \( p_3 = (0, 6, 5) \), solve for \( v_2 \) the vector from \( p_1 \) to \( p_3 \).

2. (a) Find the value for the magnitude of \( v_1 \).
    (b) Find the value for the magnitude of \( v_2 \).

3. (a) Solve for the unit vector in the direction of \( v_1 \).
    (b) Solve for the unit vector in the direction of \( v_2 \).

4. (a) Solve for the vector (cross) product \( v_1 \times v_2 \).
    (b) Solve for \( v_2 \times v_1 \).

5. Solve for the scalar (dot) product \( v_2 \cdot v_1 \).

6. If two vectors \( u, v \in \mathbb{R}^n \) are orthogonal, what is the value of their scalar (dot) product?

7. Which of the following are unit vectors?
   \[
   \left( \frac{1}{2}, -\frac{1}{2}, 0 \right), \quad (0, -1, 0), \quad \frac{1}{25}(-3, 0, 4)
   \]

8. We are given two non-zero vectors \( u, v \in \mathbb{R}^3 \). Assume the angle between \( u \) and \( v \) satisfies \( 0 < \theta < \frac{\pi}{2} \). Use dot products and/or cross products of \( u \) and \( v \) to give expressions for:
   (a) \( \cos \theta \)
   (b) \( \sin \theta \)
   (c) A vector perpendicular to both \( u \) and \( v \)

9. Given three square matrices \( Q, R, S \in \mathbb{R}^{n \times n} \), which statements are true in general?
   (a) \((QRS)^{-1} = S^{-1}R^{-1}Q^{-1}\)
   (b) \(QR = RQ\)
   (c) \((QRS)^T = Q^TR^TS^T\)
   (d) \(Q(R + S) = QS + QR\)

10. Given a square matrix \( A \in \mathbb{R}^{n \times n} \) whose columns form an orthonormal basis
    (a) What is the dot product of any pair of columns in \( A \)?
    (b) What is the inverse of \( A \)?