Robust Identification of Shared Losses Using End-to-End Unicast Probes*

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Errata

We present corrections to Fact 3 and (as a consequence) to Lemma 1 of BUCS Technical Report BUCS-TR-2000-013 (also published in IEEE ICNP’2000) [1]. These corrections result in slight changes to the formulae used for the identifications of shared losses, which we quantify.

Correction to Fact 3

Let $n_A$ denote the fraction of the 2-packet probes in which the first packet sent in the pair is destined to client $A$ and let $n_B$ denote the fraction of the 2-packet probes in which the first packet sent in the pair is destined to client $B$ ($n_A + n_B = 1$). The corrected statement of Fact 3 in [1] is the following:

**Fact 3** The quantity $b_{A,B}$ is an unbiased estimator for

$$
\left(1 - \prod_{i \in L_S} p_i^{1+}\right) + \left(\prod_{i \in L_S} p_i^{1+} - \prod_{i \in L_S} p_i^{2+}\right) (n_A q_A + n_B q_B) + \prod_{i \in L_S} p_i^{2+} q_A q_B.
$$

Correction to Lemma 1

The above correction of Fact 3 implies that the quantity $g_A + g_B + b_{A,B} - g_{A,B} - 1$ is no longer an unbiased estimate of $X$ as stated in Lemma 1 of [1]. Let $X^*$ be that quantity:

$$
X^* = g_A + g_B + b_{A,B} - g_{A,B} - 1
$$

The corrected unbiased estimate for $X$ is given in the corrected Lemma 1 (below) in terms of the quantity $X^*$.

**Lemma 1** For $n_A = n_B = \frac{1}{2}$ the quantity $\frac{2X^* g_A g_B (g_A + g_B - 2X^*)}{(g_A + g_B)^2 g_{A,B}}$ is an unbiased estimator of $X$.

**Proof:** Let $q_S = (1 - \prod_{i \in L_S} p_i^{1+})$. Also, note that $g_A$, $g_B$, $b_{A,B}$ and $g_{A,B}$ can all be written in terms of $q_A$, $q_B$, $q_S$ and $X$ as follows:

$$
g_A = (1 - q_S)(1 - q_A)
$$
$$
g_B = (1 - q_S)(1 - q_B)
$$
$$
g_{A,B} = (1 - q_S - X)(1 - q_A)(1 - q_B)
$$
$$
b_{A,B} = q_S + (1 - q_S - X)q_A q_B + X (n_A q_A + n_B q_B)
$$

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Substituting $g_A, g_B, g_{A,B}$ and $b_{A,B}$ in equation 1 we get
\[ X^* = X \left( n_B(1 - q_A) + n_A(1 - q_B) \right) \] (2)
Substituting $n_A = n_B = \frac{1}{2}$ in equation 2, we get
\[ X^* = \frac{1}{2} X \left( (1 - q_A) + (1 - q_B) \right) \]
Substituting $(1 - q_A)$ with $\frac{g_A}{(1 - q_s)}$ and $(1 - q_B)$ with $\frac{g_B}{(1 - q_s)}$ from the $g_A$ and $g_B$ equations we get the following:
\[ X^* = \frac{1}{2} X \left( \frac{g_A + g_B}{1 - q_s} \right) \]
\[ X = \left( \frac{2X^*}{g_A + g_B} \right) \left( 1 - q_s \right) \] (3)
This equation is in terms of $X$ and $q_s$. We can also get another equation in terms of $X$ and $q_s$ by substituting $(1 - q_A)$ with $\frac{g_A}{(1 - q_s)}$ and $(1 - q_B)$ with $\frac{g_B}{(1 - q_s)}$ in the $g_{A,B}$ equation.
\[ g_{A,B} = (1 - q_s - X) \frac{g_A g_B}{(1 - q_s)^2} \] (4)
Solving equations 3 and 4 we can get $X$. By substituting $X$ from 3 in 4 we get
\[ g_{A,B} = \left( 1 - q_s - \left( \frac{2X^*}{g_A + g_B} \right) \left( 1 - q_s \right) \right) \frac{g_A g_B}{(1 - q_s)^2} \]
\[ 1 - q_s = \frac{g_A g_B \left( g_A + g_B - 2X^* \right)}{(g_A + g_B) g_{A,B}} \]
Finally, substituting $1 - q_s$ in equation 3, we get what we wanted to prove:
\[ X = \frac{2X^* g_A g_B (g_A + g_B - 2X^*)}{(g_A + g_B)^2 g_{A,B}} \]

\[ \blacksquare \]

**Impact on Results**

In the above derivation, equation 2 relates the quantities $X$ and $X^*$. Recall that $X^*$ is precisely the (erroneous) quantity used in [1] as an unbiased estimator for $X$. Thus, equation 2 allows us to quantify the "error" introduced through the use of $X^*$ rather than $X$. Let $E = (X^* - X)/X$ denote this error (relative to $X$).

\[ E = 1 - \left( n_B(1 - q_A) + n_A(1 - q_B) \right) \]
Since $(n_B(1 - q_A) + n_A(1 - q_B))$ is at most 1 ( = 1 in case $q_A = 1$ and $q_B = 1$), then we are underestimating the value of $X$ and the error increases with increasing values of $q_A$ and $q_B$. We are currently evaluating the effect of this error on our simulation results. Initial evidence suggests that the corrected results (shown in Figures 4 through 8) of [1] are almost identical to (visually indistinguishable from) those reported in [1].

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**References**