TCP-friendly SIMD Congestion Control and Its Convergence Behavior

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Abstract

The increased diversity of Internet application requirements has spurred recent interests in flexible congestion control mechanisms. Window-based congestion control schemes use increase rules to probe available bandwidth, and decrease rules to back off when congestion is detected. The parameterization of these control rules is done so as to ensure that the resulting protocol is TCP-friendly in terms of the relationship between throughput and packet loss rate. In this paper, we propose a novel window-based congestion control algorithm called SIMD (Square-Increase/Multiplicative-Decrease). Contrary to previous memory-less controls, SIMD utilizes history information in its control rules. It uses multiplicative decrease but the increase in window size is in proportion to the *square* of the time elapsed since the detection of the last loss event. Thus, SIMD can efficiently probe available bandwidth. Nevertheless, SIMD is TCP-friendly as well as TCP-compatible under RED, and it has much better convergence behavior than TCP-friendly AIMD and binomial algorithms proposed recently.

Keywords: Congestion Control, TCP-friendly, Fairness, Convergence.

1 Introduction

In a shared network, end-hosts must react to network conditions and adapt their transmission rates to avoid severe congestion [1] while still maintaining high bandwidth utilization. The success of the Internet is due in part to the congestion control mechanisms [15] implemented in the dominant transport layer protocol TCP. A TCP connection uses additive-increase/multiplicative-decrease (AIMD), i.e., it probes available bandwidth by increasing its congestion window size linearly, and responds to increased congestion (indicated by packet losses) by decreasing the window size multiplicatively.

Recently proposed congestion control mechanisms include generalizations of TCP-like window-based schemes [3, 11, 26, 30] and equation-based schemes [12, 23, 29]. One common objective of these new schemes is to reduce variations in transmission rate. Such high variations may limit network utilization. In addition, they are not desirable for emerging applications such as real-time streaming applications on the Internet.

It is required that new protocols implement congestion control mechanisms that interact well with TCP [10]. That is, they should maintain *TCP-compatibility*, or fairness across connections using different protocols. To provide such fairness, *TCP-friendliness* is necessary, which means the (λ, p) relationship $\lambda = \sqrt{3/2}/(R\sqrt{p})$ should hold, where λ is the throughput of a flow, p is its packet loss rate, and R is the round-trip time.

In addition, there are other requirements for congestion control algorithms:

- *Smoothness* measures the transmission rate variation of a connection using the protocol. Smoothness is important in steady state. High smoothness is desirable for some applications, e.g., Internet real-time applications.

- *Aggressiveness* means how fast the connection probes extra bandwidth. In particular, when there is a sudden increase in available bandwidth, it is desirable that the connection acquires it quickly.

- *Responsiveness* means how fast the connection reacts to increased congestion and decreases its window size. It is desirable that the connection reduces its transmission rate to its fair share promptly. Both aggressiveness and responsiveness are measures of the transient behavior of congestion control protocols [30].

- *Convergence* means whether and how fast competing connections converge to their fair share of bandwidth. Certainly, convergence speed is related to the aggressiveness and responsiveness indices. More aggressive and responsive protocols usually converge faster.

Several recently proposed TCP-friendly congestion control schemes, including general AIMD [11, 30], binomial algorithms [3], TFRC [12], and TEAR [26], can provide smoother transmission rate than TCP. One problem is, these algorithms may lack the aggressiveness and responsiveness of TCP. Hence, when network conditions change drastically, these protocols can not react to the change promptly. Recent studies [11, 30] have compared TCP AIMD,[1] general AIMD, TFRC, and TEAR, and shown that typically higher smoothness means lower aggressiveness. Therefore, a question is, *can TCP-friendly congestion control algorithms maintain high smoothness in steady state and still have high aggressiveness when there are drastic changes in network conditions.*

Meanwhile, it is necessary to consider the convergence of congestion control schemes. Chiu and Jain [4] showed that AIMD control converges to fairness and efficiency. Recently, it was shown that additive increase of TCP and general AIMD control is inferior [14]. Binomial algorithms [3] with non-linear control (using non-additive increase) also possess the convergence property. Binomial algorithms are similar to AIMD in that they all use memory-less control. That is, their control rules use only the current window size. Therefore, a question is, *can we improve the convergence behavior by using history in window-based congestion control algorithms?*

This paper provides an answer to these two questions. We study TCP-like window-based congestion control algorithms. Contrary to the memory-less AIMD and binomial algorithms [3], we consider the case where connections utilize history information, in addition to the current window size. The only history we use is the window size at the time of detecting the last loss. To this end, we propose a novel algorithm called SIMD (Square-Increase/Multiplicative-Decrease). SIMD decreases the window size multiplicatively but increases it in proportion to the *square* of the time elapsed since the detection of the last loss event. SIMD can have high smoothness in steady state, and if network conditions change drastically, SIMD can grow aggressive. On the contrary, other control schemes increase the window size linearly or sub-linearly. We show that SIMD is TCP-friendly: connections using SIMD have approximately the same throughput as TCP connections, given the same packet loss rate and round-trip time. Furthermore, SIMD has better convergence behavior than that of memory-less AIMD and binomial algorithms. We use a synchronized feedback model [4] to illustrate the convergence behavior of SIMD. In addition, using the *ns* simulator [7], we show that SIMD can fully capitalize on the random loss property of RED [13] to improve convergence speed.

Our SIMD algorithm is the first step to explore a new space between memory-less window-based congestion control schemes and equation-based schemes which use more history information. Compared to memory-less window-based schemes, SIMD improves the transient behaviors by using history. Compared

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[1] We use $AIMD(\alpha, \beta)$ to refer to the general AIMD with additive constant α and multiplicative decrease parameter β. The term *TCP AIMD* refers to $AIMD(1, 0.5)$ or standard TCP. For simplicity, we also use *AIMD* for the general case.

to equation-based schemes, SIMD has several unique properties: the self-clocking nature of window-based schemes, and simple modifications to TCP's implementation. The remainder of this paper is organized as follows. We propose our algorithm in Section 2, and show its TCP-friendliness in Section 3. We analyze its convergence behavior in Section 4. Our simulation results are described in Section 5. We revisit related work in Section 6 and finally conclude the paper.

2 SIMD Congestion Control

A TCP-like window-based congestion control scheme increases the congestion window as a result of the successful transmission of a window of packets, and decreases the congestion window upon the detection of packet losses. We call such a sequence of window increments followed by one window decrement a *congestion epoch*. The congestion control scheme defines one control rule for window increase, and another rule for window decrease. For example, AIMD uses the following linear control rules:

$$Increase: \quad w_{t+R} \leftarrow w_t + \alpha, \quad \alpha > 0,$$
$$Decrease: \quad w_{t+\delta} \leftarrow w_t - \beta w_t, \quad 0 < \beta < 1.$$

where w_t is the window size at time t, R is the round-trip time, and δ is the time to detect packet loss since the last window update. That is, for AIMD, the window size is increased by a constant when a window of packets are transmitted successfully, and it is decreased by a constant factor once a packet loss event is detected. Binomial algorithms [3] generalize AIMD with non-linear controls. They use the following control rules:

$$Increase: \quad w_{t+R} \leftarrow w_t + \alpha/w_t^k, \quad \alpha > 0,$$
$$Decrease: \quad w_{t+\delta} \leftarrow w_t - \beta w_t^l, \quad 0 < \beta < 1.$$

That is, binomial algorithms generalize additive-increase by increasing inversely proportional to a power k of the current window (for TCP, $k = 0$), and generalize multiplicative-decrease by decreasing proportional to a power l of the current window (for TCP, $l = 1$).

We say that AIMD and binomial algorithms are memory-less since the increase and decrease rules use only the current window size w_t and constants (α, β, k, and l). Neither of them utilizes history information. On the contrary, we find the window size at the end of the last congestion epoch (before the decrease) handy and useful. Our scheme maintains such a state variable w_{max}, which is updated at the end of *each* congestion epoch. In addition, let w_0 denote the window size after the decrease. Given a decrease rule, w_0 can be obtained from w_{max}, and vice versa. For example, for TCP, $w_0 = (1 - \beta)w_{max}$. Henceforth, for clarity, we use both w_{max} and w_0. [2]

We define the control rules of SIMD as:

$$Increase: \quad w_{t+R} \leftarrow w_t + \alpha\sqrt{w_t - w_0}, \quad \alpha > 0,$$
$$Decrease: \quad w_{t+\delta} \leftarrow w_t - \beta w_t, \quad 0 < \beta < 1. \tag{1}$$

Like AIMD, SIMD uses multiplicative decrease. However, SIMD uses an increase rule very different from those used by AIMD and binomial algorithms. First, SIMD uses the history information of a connection since w_0 is the window size after the last decrease. (Later, we will also show that α itself depends on w_{max}, and changes from one congestion epoch to another.) Second, the increase pattern of the window size is super-linear. To elaborate on this point, next we show that SIMD's increase rule results in a quadratic function of time t since the detection of the last loss event.

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[2]When TCP slow start ends and congestion avoidance phase starts, we have the first value of w_0, i.e., the current window size. Then the first value of w_{max} is computed.
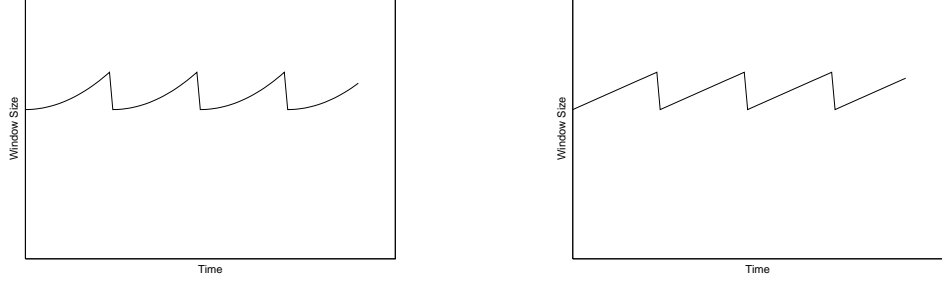
(a) Window size increases super-linearly (b) Window size increases linearly

Figure 1: Different increase patterns of congestion window

Let $w(t)$ be the continuous approximation of the window size at time t (in RTT's) elapsed since the window started to increase. By definition, $w_0 = w(0)$. Using linear interpolation and continuous approximation, from the increase rule in (1), we have

$$\frac{dw(t)}{dt} = \alpha\sqrt{w(t) - w_0}.$$

This gives us

$$\frac{1}{\sqrt{w(t) - w_0}}dw(t) = \alpha dt,$$

and then by integrating both sides, we have

$$2\sqrt{w(t) - w_0} = \alpha t + C,$$

Notice that the constant $C = 0$ since when $t = 0$, $w(t) = w_0$. We then rewrite it as

$$w(t) = w_0 + \frac{\alpha^2}{4}t^2. \tag{2}$$

Therefore, SIMD can grow aggressive with time, as shown in Figure 1(a). This property is important since it allows SIMD to efficiently probe extra bandwidth when it is available. For SIMD, it is possible to have high smoothness (low variation of window size) in steady state by using a small β, and high aggressiveness when there are drastic changes in network conditions. On the contrary, if $w(t)$ is a linear or sub-linear function of t, as shown in Figure 1(b), then the connections are unable to acquire bandwidth quickly. For example, TCP-friendly AIMD algorithm needs to parameterize its control rules by defining α as a function of β [11, 31]. In particular, without considering the effect of TCP's timeout mechanisms, $\alpha = 3\beta/(2-\beta)$. Although smoothness is possible by using moderate decrease, AIMD becomes insensitive to sudden increases in available bandwidth.

So for SIMD, a remaining question is, how can we define α in the increase rule (1) such that SIMD is TCP-friendly, given β and the state variable w_{max} (or w_0)? We assume the multiplicative decrease factor β is a constant. We define α as follows:

$$\alpha = \frac{3\sqrt{\beta}}{(1 - 2\beta/3)\sqrt{2w_{max}}}. \tag{3}$$

Thus, during a congestion epoch, α is inversely proportional to $\sqrt{w_{max}}$. This choice is justified in Section 3. From this, Equation (2) becomes:

$$w(t) = w_0 + \frac{9\beta}{8(1 - 2\beta/3)^2 w_{max}}t^2. \tag{4}$$

We can observe the following:

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- The increase term of the increase rule in (1) is proportional to $\sqrt{(w_t - w_0)/w_{max}}$. Since w_t, w_0, and w_{max} are dependent on the window size, the increase term is time-varying. Therefore, SIMD can be viewed as a special AIMD whose increase parameter α (in the control rules of AIMD) is always varying. The elegance of SIMD is, by doing this, it can provide high smoothness (using small β) in steady state, and still have high aggressiveness when there is a sudden increase of available bandwidth.

- The rate at which $w(t)$ increases is inversely proportional to w_{max}, as shown in Equation (4). Therefore, if there are two SIMD flows competing, then the flow with smaller window size is more aggressive. This property can result in better convergence behavior. AIMD does not have such property.

3 TCP-Friendliness

In this section, we explain why defining α by Equation (3) makes our SIMD algorithm TCP-friendly. The notion of TCP-friendliness refers to the throughput and packet loss rate relationship. We consider a random loss model, where the losses are Bernoulli trails, i.e., packets are dropped uniformly with a fixed probability p. In addition, we do not consider the effect of TCP's timeout mechanisms.
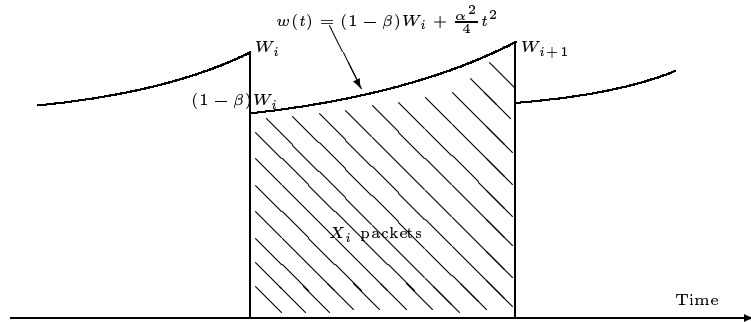

Figure 2: Window increases with time, and decreases when packet losses occur.

Consider many congestion epochs where the window increases and decreases alternately in steady state, as shown in Figure 2. Let W_i be the window size in the beginning of the ith epoch. In this epoch, the window size is decreased to $(1 - \beta)W_i$, then increased by, say I_i packets, to W_{i+1} before the first packet loss happens. Assume X_i packets are sent successfully in this epoch. Before we consider random losses, it will be helpful to consider the simpler case of periodic losses first.

Periodic Losses

Under a periodic loss model, the window size increase and decrease are deterministic. Both W_i and X_i are constants, denoted by W and X, respectively. I_i is a constant equal to βW. Assume in order for SIMD to be TCP-friendly under the periodic loss model, we need define α as α_P.

Given the window size increase function (2), we can compute the duration (in RTTs) of each congestion epoch:

$$T = \frac{2\sqrt{\beta W}}{\alpha_P},$$

and the number of packets in each epoch is given by:

$$
\begin{aligned}
X &= \int_0^T ((1 - \beta)W + \frac{\alpha_P^2}{4}t^2)dt \\
&= (1 - 2\beta/3)WT.
\end{aligned}
$$

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For the congestion control to be TCP-friendly, the throughput and loss rate relationship must hold. Without considering the effect of TCP's timeout mechanisms, the relationship is $\lambda = \sqrt{3/2}/(R\sqrt{p})$, where λ is the average throughput and R is the round-trip time. We have $\lambda = \frac{X}{TR}$, i.e., average throughput is the number of packets between two consecutive losses divided by the time (in seconds) between the two losses. We also have $p = \frac{1}{X}$. Plug them into the (λ, p) relationship, we get

$$\alpha_P = \frac{3\sqrt{\beta}}{(1 - 2\beta/3)\sqrt{w_{max}}} \tag{5}$$

Noticing, here w_{max} is equal to W, by definition. Therefore, under the periodic loss model, α_P provides TCP-friendliness.

Random Losses

Now we consider a random loss model where the losses are Bernoulli trails; packets are dropped uniformly with a fixed probability p. Assume in order for SIMD to be TCP-friendly under the random loss model, we need to define α as α_R.

Consider the random variable X_i, the number of packets sent in the ith epoch up to but not including the first packet lost. Given the random loss model, the probability that j packets are acknowledged successfully before the first loss is given by:

$$\begin{aligned} P[X_i = j] &= (1 - p)^j p, \quad j = 0, 1, 2, \dots \\ &\approx p e^{-pj}, \quad \text{for } p \ll 1 \end{aligned}$$

Let T_i denote the number of rounds (RTT's) between two consecutive loss events. T_i can be computed by X_i divided by the average window size in the ith epoch $\overline{w_i}$, i.e., $T_i = X_i/\overline{w_i}$. Using (2), this results in a window increase of size

$$I_i = \frac{\alpha_R^2}{4}\left(\frac{X_i}{\overline{w_i}}\right)^2.$$

Computing $E[I_i]$ is difficult since X_i and $\overline{w_i}$ are correlated. However, when the window size variation is small enough, we ignore such correlation and use the time-average window size $\overline{w}$ to approximate $\overline{w_i}$. Therefore,

$$I_i \approx \frac{\alpha_R^2}{4}\left(\frac{X_i}{\overline{w}}\right)^2.$$

Then the expected window increase is:

$$\begin{aligned} E[I_i] &= \sum_{j=0}^{\infty} I_i P[X_i = j] \\ &\approx \sum_{j=0}^{\infty} \frac{\alpha_R^2}{4}(j/\overline{w})^2(1 - p)^j p \\ &\approx \int_0^{\infty} \frac{\alpha_R^2}{4}(x/\overline{w})^2 p e^{-px} dx \\ &= \frac{\alpha_R^2}{2(p\overline{w})^2}, \end{aligned} \tag{6}$$

Note that, under the periodic loss model, $X_i = 1/p$, and $T_i = X_i/\overline{w} = \frac{1}{p\overline{w}}$. Therefore,

$$E[I_i] = \frac{\alpha_P^2}{4(p\overline{w})^2}. \tag{7}$$

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To obtain α_R under the random loss model, we equalize the expected window increases $E[I_i]$ under both loss models. [3] Specifically, equating (6) and (7), we obtain $\alpha_R = \alpha_P/\sqrt{2}$. This results in Equation (3). As α_P satisfies TCP-friendliness under the periodic loss model, we expect that Equation (3) makes SIMD TCP-friendly under the random loss model.

Considering that the random loss model is obviously more realistic, we henceforth use the definition of α in Equation (3). In Section 5, we use simulations to validate the TCP-friendliness of SIMD for a wide range of loss rate.

4   Convergence to Fairness and Efficiency

In this section, we first show that SIMD converges to fairness and efficiency under a synchronized feedback assumption. Then we show that SIMD converges faster than memory-less AIMD and binomial controls.

4.1   Convergence of SIMD

We adopt the ideal synchronized feedback assumption [4]. To show that multiple users with synchronized feedbacks using our control scheme converge to fairness, we use the vector space used by Chiu and Jain [4] to view the system state transitions as a trajectory. For ease of presentation, we show a two-user case. It is straightforward to apply the same technique to the multiple-user case to reach the same conclusion.

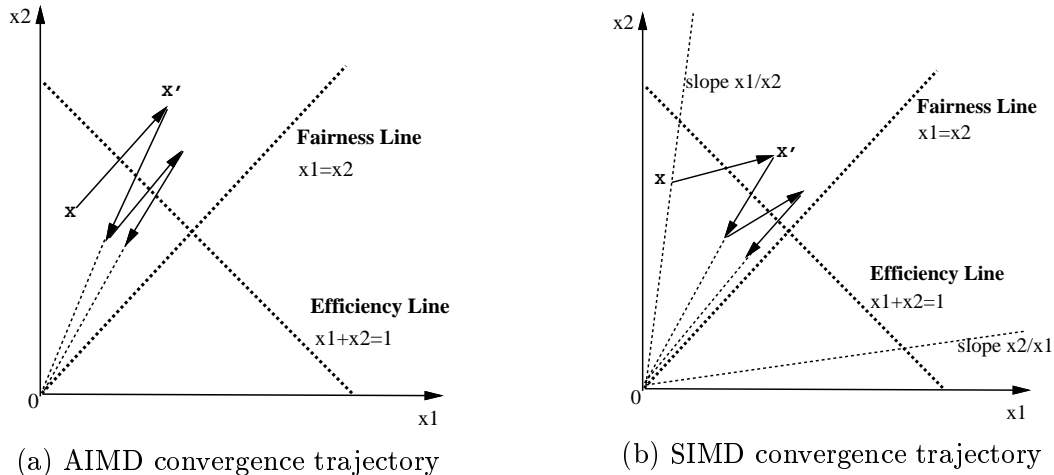(a) AIMD convergence trajectory   (b) SIMD convergence trajectory

Figure 3: Convergence of our congestion control schemes.

As shown in Figure 3(a), any two-user resource allocation can be represented by a point $X(x_1, x_2)$, where x_i is the resource allocation (normalized by total capacity) for the ith user, $i = 1, 2$. We define the fairness index as

$$\max(\frac{x_1}{x_2}, \frac{x_2}{x_1}).$$

If the fairness index is closer to unity, the resource allocation is more fair. The line $x_1 = x_2$ is the "fairness line". The line $x_1 + x_2 = 1$ is the "maximum utilization line" or "efficiency line". The goal of control schemes is to bring the system to the intersection of the fairness line and the efficiency line. When the system is under-utilized (assuming $x_1 \leq x_2$ without loss of generality), AIMD increases the

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[3]In steady state, the expected increase of the window size is equal to the expected decrease of the window size. Under both loss models, the expected decreases of the window size are roughly equal, given the same loss rate and roughly the same average window size. Therefore, we need only to equalize the expected increases under both loss models.

resource allocation of both users by a constant. Figure 3(a) shows the trajectory to X' parallel to the fairness line. This movement improves fairness (i.e., reduces the fairness index). Then both users use multiplicative decrease, which does not change fairness. Hence, as the system evolves, AIMD brings the resource allocation point towards the fairness line, finally oscillating around the efficiency line.

For SIMD control, we first observe Equation (4). We can see that the window size of a connection increases in proportion to $1/x_i, i = 1, 2$. Thus, as shown in Figure 3(b), the increase trajectory emanates from $X(x_1, x_2)$ with slope $\frac{x_1}{x_2}$. Indeed, at any point between the two lines emanating from the origin with slopes x_1/x_2 and x_2/x_1, the resource allocation X' is more fair than X as it reduces the value of the fairness index. Therefore, the increase phase of SIMD improves fairness. Since like AIMD, SIMD uses multiplicative decrease, the decrease phase of SIMD does not change fairness. Hence, SIMD converges to fairness and efficiency.

4.2 Convergence Speed

We first intuitively show that SIMD converges faster than AIMD. Then we analytically show the time for different control schemes to bring the difference between two user allocations within a certain small bound.

First, to intuitively show that SIMD converges faster than AIMD, we show that the increase trajectory of SIMD intersects the efficiency line at a point that is usually more fair than that of AIMD. Let $X(x_1, x_2)$ be the initial under-utilized allocation, $x_1 + x_2 < 1$ and assume $x_1 < x_2$. Using AIMD, the intersection of the trajectory and the efficiency line is $(\frac{1+x_1-x_2}{2}, \frac{1-x_1+x_2}{2})$. Using SIMD, the intersection is $(x_1 - x_2 + \frac{x_2}{x_1+x_2}, x_2 - x_1 + \frac{x_1}{x_1+x_2})$. [4] By comparing the fairness index of these two intersections, we found that our control scheme reaches a more fair intersection if $x_1 + x_2 > 1/3$. This condition is shown as area (1) in Figure 4(a). Since intuitively, the size of area (1) is much larger than area (2), we say SIMD usually converges faster than AIMD.

Then, we analytically compare the convergence time of SIMD, general AIMD [11, 31], and binomial control schemes [3]. Binomial algorithms are a family of algorithms generalizing AIMD. The control rules were shown in Section 2. We choose IIAD (Inverse-Increase/Additive-Decrease) as a representative. IIAD has an increase term inversely proportional to the current window size ($k = 1$) and a constant decrease term ($l = 0$). [5] We still assume synchronized feedback and use Figure 4(b) to illustrate the process of convergence to fairness. For ease of analysis, we choose the variables to be the actual window sizes (w_1, w_2). We also divide the convergence time into two parts: T_1, the time it takes the control mechanism to bring an arbitrary initial point (W_1, W_2), where $W_2 \geq W_1$ and $W_1 + W_2 < W$, close to the efficiency line $w_1 + w_2 = W$, and T_2, the time until the difference between the two user windows stays within a certain small bound, i.e., $|w_1 - w_2| < \epsilon$. T_1 and T_2 are measured in round-trip times. We also denote the difference between the two user windows after T_1 as Δ. The detailed analysis is given in Appendix A and we only present the main results here in Table 1.

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[4]We get these two intersections as follows. Let $\Delta x_i, i = 1, 2$ denote the increase of the ith user. For AIMD, to get Δx_1 and Δx_2, we solve the following:

$$(x_1 + \Delta x_1) + (x_2 + \Delta x_2) = 1,$$

$$\Delta x_1 = \Delta x_2.$$

For SIMD, since the increase Δx_i is inversely proportional to x_i, we solve the following:

$$(x_1 + \Delta x_1) + (x_2 + \Delta x_2) = 1,$$

$$\frac{x_1}{x_2} = \frac{\Delta x_2}{\Delta x_1}.$$

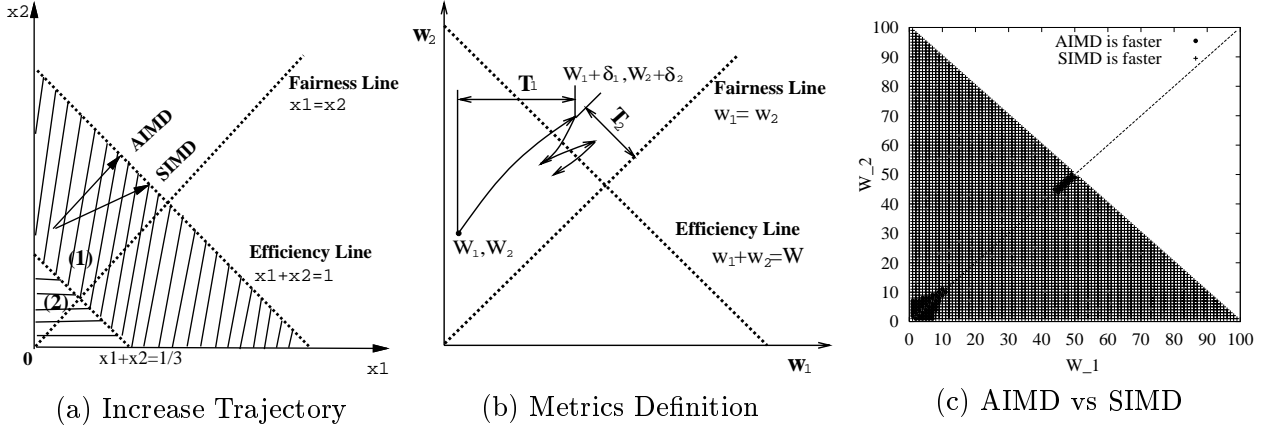[5]Another binomial algorithm SQRT with $k = l = 0.5$ lies between AIMD and IIAD. Thus we do not consider it here.

(a) Increase Trajectory	(b) Metrics Definition	(c) AIMD vs SIMD

Figure 4: Comparison of convergence speed

Algorithm	T_1 (RTT)	Δ	T_2 (RTT)
TCP	$\frac{W-W_1-W_2}{2}$	$W_2 - W_1$	$\frac{W}{4}\log_{1/2}\frac{\epsilon}{\Delta}$
AIMD	$\frac{(W-W_1-W_2)(2-\beta)}{6\beta}$	$W_2 - W_1$	$\frac{(2-\beta)W}{6}\log_{1-\beta}\frac{\epsilon}{\Delta}$
IIAD	$\frac{1}{12\beta}((\frac{(W_2^2-W_1^2)}{W})^2 - 2(W_1^2 + W_2^2) + W^2)$	$\frac{W_2^2-W_1^2}{W}$	$\frac{W}{3}\log_{1-2\beta/W}\frac{\epsilon}{\Delta}$
SIMD	$\frac{2}{3}(1-\frac{2\beta}{3})\sqrt{\frac{2}{\beta(1-\beta)}}\sqrt{\frac{W_1W_2(W-W_1-W_2)}{W_1+W_2}}$	$(2-\frac{W}{W_1+W_2})(W_2 - W_1)$	$\frac{\sqrt{2}W}{3}\log_{1-2\beta}\frac{\epsilon}{\Delta}$

Table 1: Performance measures on convergence to fairness and efficiency

We numerically solve the above equations for different initial points. Figure 4(c) shows the regions for which SIMD with $\beta = 1/16$ converges faster/slower (i.e., $T_1 + T_2$ is smaller/larger) than TCP-friendly AIMD with $\beta = 1/16$ for $\epsilon = 1$ and $W = 100$. In most cases SIMD converges faster than AIMD, which supports our intuitive claim (cf. Figure 4(a)). Numerical results also show that IIAD (with $\alpha = 1$ and $\beta = 2/3$ such that IIAD is TCP-friendly) is much slower than AIMD and SIMD in all cases.

5   Simulation Results

We use the *ns* simulator [7] to validate that with RED [13] queue management strategy (or randomized dropping), our proposed algorithm is TCP-friendly and TCP-compatible. We also investigate the way two homogeneous flows converge to their bandwidth fair share and found that our proposed algorithm outperforms other algorithms, including TCP [15], generalized AIMD [11, 31], and IIAD [3]. Details about the implementation of SIMD in the *ns* simulator are described in Appendix B.

Unless explicitly specified, in all of the experiments, RED was used as the queue management policy at the bottleneck link. The bottleneck queue configuration and other simulation parameters are listed in Table 2.

The bottleneck queue size and RED queue parameters are tuned as recommended in [5]. The "gentle_" option of RED queue is turned on as recommended in [9]. We choose $\beta = 1/16$ for SIMD and AIMD (and thus $\alpha \approx 1/10$ for AIMD to ensure TCP-friendliness). For IIAD, $\alpha = 1$ and $\beta = 2/3$. For ease of presentation, in the rest of this section, we will call these implementations by their family name, e.g., AIMD for AIMD(1/10,1/16) when there is no confusion. We use SACK [19] for congestion detection.

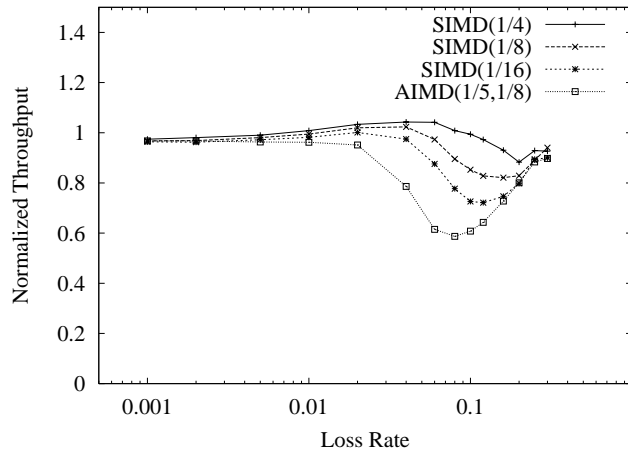| Description	Value
Packet size	1000 bytes
Maximum window	128 packets
TCP version	SACK
TCP timer granularity	0.1 seconds
RED queue limit Q	2.5 × B/W delay product
DropTail queue limit	1.5 × B/W delay product
RED parameters	min_{th}: 0.15Q, max_{th}: 0.5Q, w_q:0.002
	max_p:0.1, wait_ on, gentle_ on

Table 2: Network configuration

Figure 5: TCP Friendliness

We also obtained similar results for other mechanisms (e.g. Reno, newReno). We assume no delayed acknowledgments.

5.1 TCP-Friendliness

We conducted the following experiment to test the TCP-friendliness of our SIMD algorithm: A single flow under investigation is traveling through a single fat link (with infinite bandwidth and buffer size). However, the link drops an incoming packet uniformly with probability p. We varied the loss rate p and compared the normalized long-term throughput (with respect to standard TCP measured over 3000 RTT) of SIMD for different β values and plotted them in Figure 5. For comparison, we also plotted AIMD(1/5,1/8) throughput.

We notice that all the curves have a dip when the loss rate is moderate. A close look at the TCP-friendly equation [22] can reveal one possible explanation of this abnormality:

$$\lambda(p, \alpha, \beta) \approx \min(\frac{W_{max}}{R}, \frac{1}{R\sqrt{\frac{2\beta}{\alpha(2-\beta)}p} + T_0 \min(1, 3\sqrt{\frac{\beta(2-\beta)}{2\alpha}p})p(1 + 32p^2)}) \tag{8}$$

When loss rate is low, TCP mainly stays in the *congestion avoidance* stage, and the AIMD algorithm dominates Equation (8). When loss rate is very high, TCP spends most of its time retransmitting packets, and the *exponential back-off* algorithm dominates Equation (8). Since all TCP variants studied in this paper use the same timeout mechanism as standard TCP, and they carefully calibrate the values of parameters during congestion avoidance to match standard TCP, they can achieve comparable
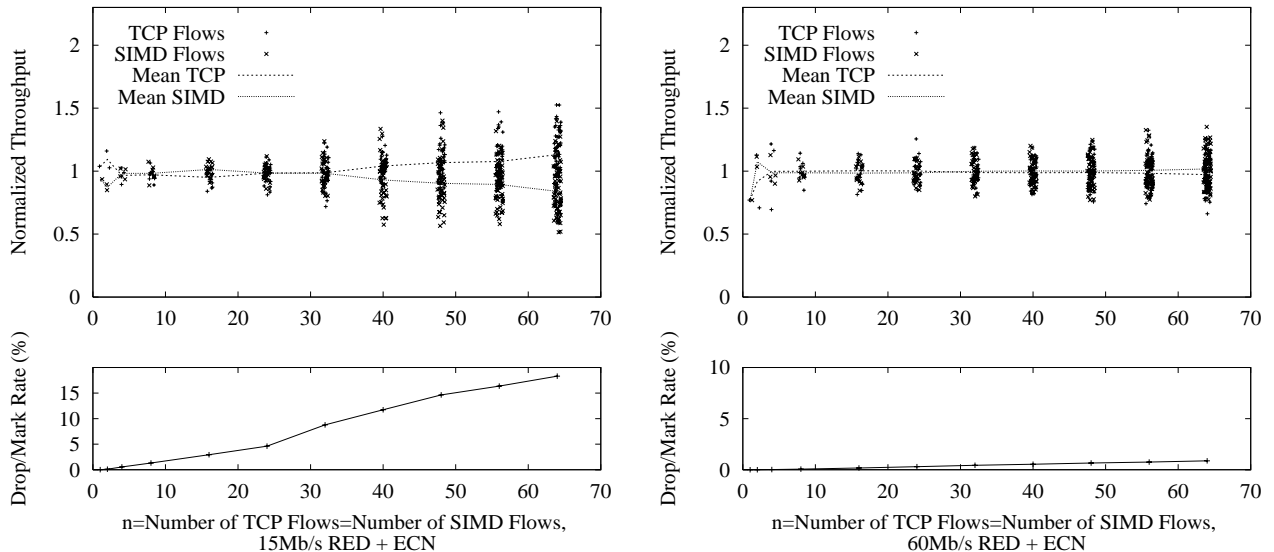
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Figure 6: TCP competing with SIMD(1/16), RED with ECN

throughput as standard TCP for very high and low loss rates. However, for the loss regime in between, it becomes hard, if not impossible, to obtain α and β values that would approximate well both the congestion avoidance and the exponential backoff components of the TCP-friendly equation [31].

Nevertheless, in the worst case (loss rate around 15%), SIMD(1/16), which is the worst among all the SIMD algorithms considered, can achieve at least 75% throughput as standard TCP, and performs much closer to standard TCP than AIMD(1/5,1/8)[6]. Given the fact that most parts of the Internet are experiencing less than 5% loss rate [6], our algorithm is TCP-friendly under these conditions.

5.2 TCP-Compatibility

We use the method described in [11] to test TCP-compatibility. n SIMD flows and n standard TCP SACK flows compete for bandwidth over a shared bottleneck link. There are also 4 background TCP flows transmitting packets in the opposite direction to introduce random ACK delays. We consider both RED and DropTail queues. Figure 6 and Figure 7 show the simulation results for RED queues, with and without ECN bit set, respectively. In each case, results are shown for a bottleneck link bandwidth of 15Mbps and 60Mbps. The measured average round-trip delay is around 0.1 second. Each point in the graph represents the throughput of an individual flow in the last 60 seconds, and the dashed lines represent the average throughput of SIMD and standard TCP flows. In the lower graphs, we also plot the packet loss rate for the RED without ECN case, and the rate of ECN early marking plus dropping due to queue overflow for the RED with ECN case.

As can be observed from the graphs, SIMD achieves a slightly lower average throughput than standard TCP when when the loss rate exceeds a certain level. This is partly due to the reason we illustrate in Figure 5. Another possible explanation is that when severe congestion happens, SIMD can not compete well against standard TCP since compared to TCP, SIMD opens its congestion window more conservatively at the beginning of each congestion epoch. Therefore, when the time between two consecutive

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[6]The weakness of AIMD(α, β) with small β under medium loss conditions is also reported in [11]. The authors try to compensate for the bandwidth loss by increasing the value of α. However, when loss rate is small (e.g. less than 3%), AIMD with large α could achieve significantly higher bandwidth than standard TCP and become less TCP-friendly. Therefore, we maintain the theoretical α values throughout our simulations.
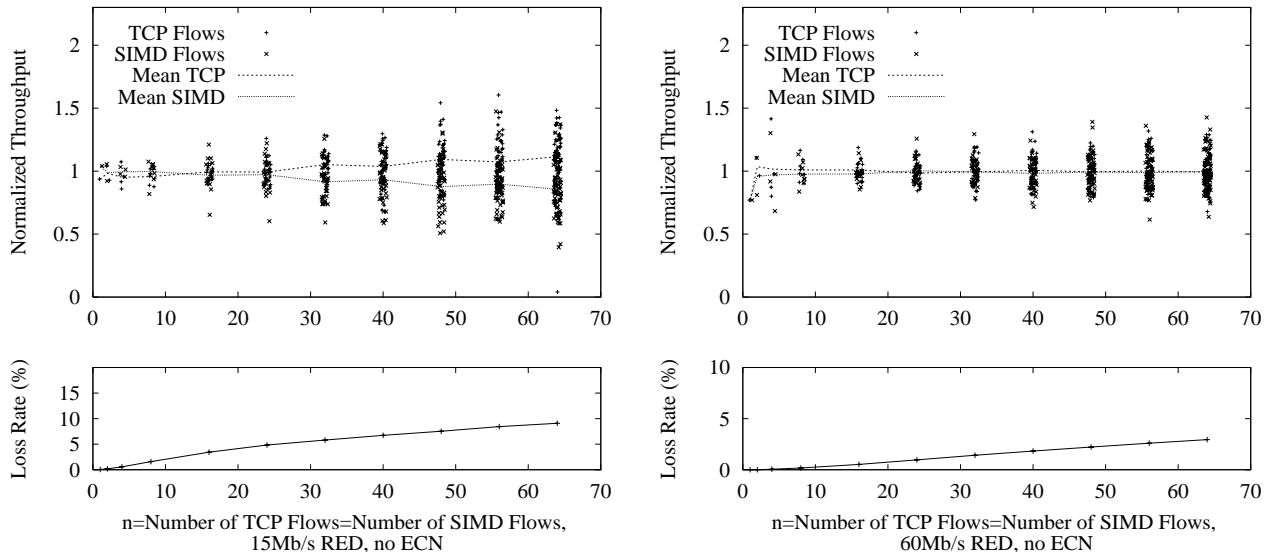
Figure 7: TCP competing with SIMD(1/16), RED without ECN

packet losses is short, the more aggressive TCP tends to gain more throughput. However, in a reasonable loss regime (loss rate below 10%), SIMD shows very impressive TCP-compatibility[7].

We also found that with DropTail queue management, as shown in Figure 8, SIMD can still be TCP-friendly and TCP-compatible. The difference, compared to the RED queue experiment, is that the variance becomes larger and SIMD now gets less bandwidth than standard TCP compared to the previous experiment. Note that the assumption of randomized packet losses made in our analysis does not apply to DropTail. Under DropTail, packet losses are more correlated (bursty drops). We conjecture that because the round-trip times of connections are randomized in the simulation, the chance of having synchronized packet arrivals is small, and the side effect of a DropTail queue (correlated drops for each flow) is thus not so significant.

5.3 Convergence to Fairness and Efficiency

In this section, we assume a homogeneous protocol environment, i.e., all flows use the same algorithm for congestion control. We then vary the network configuration to study the convergence time to efficiency and fairness of different algorithms.

We use the topology shown in Figure 9 to perform this experiment. In the beginning of the simulation, there are $c_1 + 1$ connections sharing link $(b1, b2)$, 2 connections sharing link $(b2, b3)$, $c_2 + 1$ connections between $b3$ and $b4$. Link bandwidths and delays are shown in the figure. At time 400, all background flows terminate and only two flows (s1-r1) and (s2-r2) stay to compete for the bottleneck link $(b2,b3)$.[8]

5.3.1 $W_1 < \frac{W}{2} < W_2$, $W_1 + W_2 = W$ (Convergence to Fairness)

We create this scenario to study the convergence time to fairness given that the initial point (W_1, W_2) is on the efficiency line $(w_1 + w_2 = W)$. To create this setup, we let $c_1 = 15$, $c_2 = 0$, $x = 6$Mbps, $y = 6$Mbps.

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[7]Note that in case of 60Mbps link and less than 4 flows, the length of the measurement period (60 seconds) is too short compared to the length of each congestion epoch (more than 40 seconds), thus the variance of the results appears to be large.

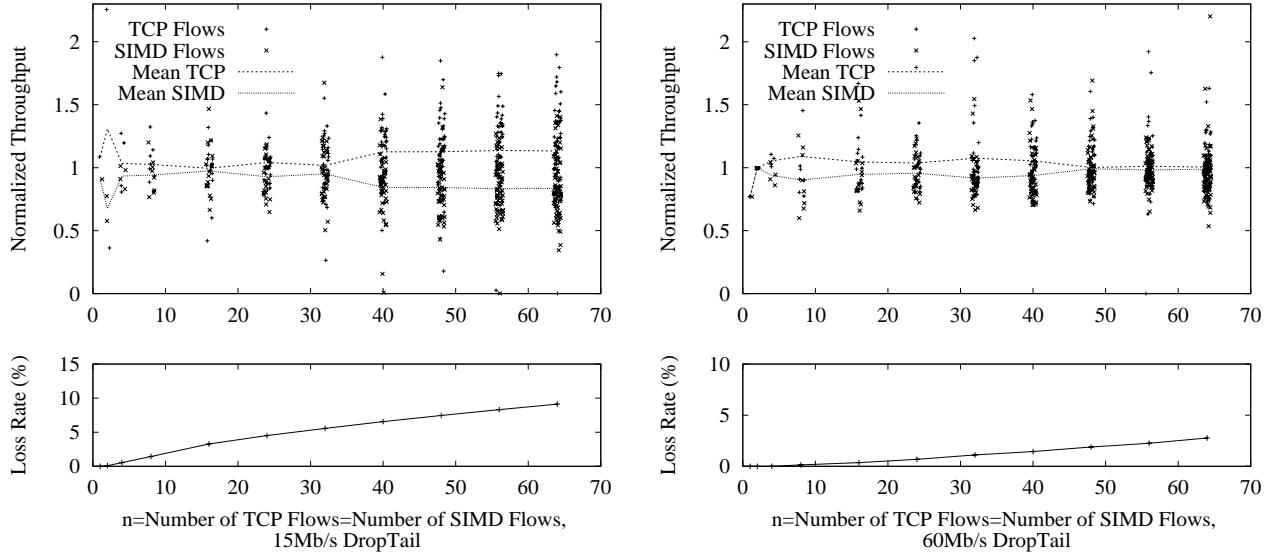[8]We use packet size of 500 bytes in these experiments.

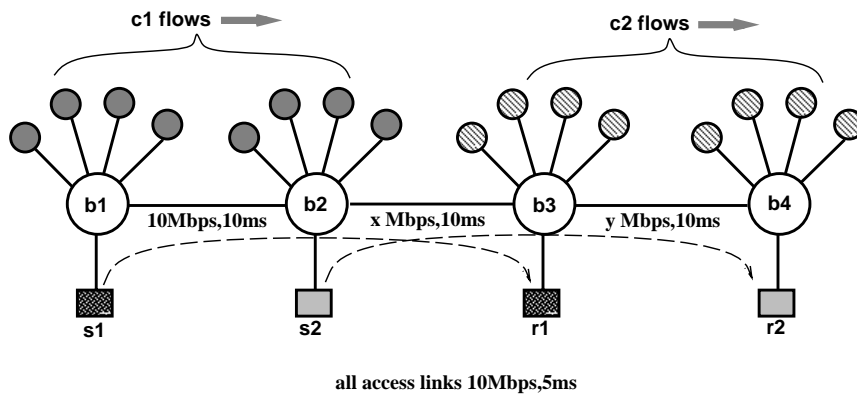Figure 8: TCP competing with SIMD(1/16), with DropTail

Figure 9: Simulation topology for convergence test

So the bottleneck link for flow (s2,r2) remains link ($b2$,$b3$), but for flow (s1,r1), the bottleneck changes from link ($b3$,$b4$) to ($b2$, $b3$) at time 400. We can also compute that: $W \approx 110$, $W_1 \approx 7$, and $W_2 \approx 100$. Figure 10 plots the transient behavior of the congestion window of different protocols.

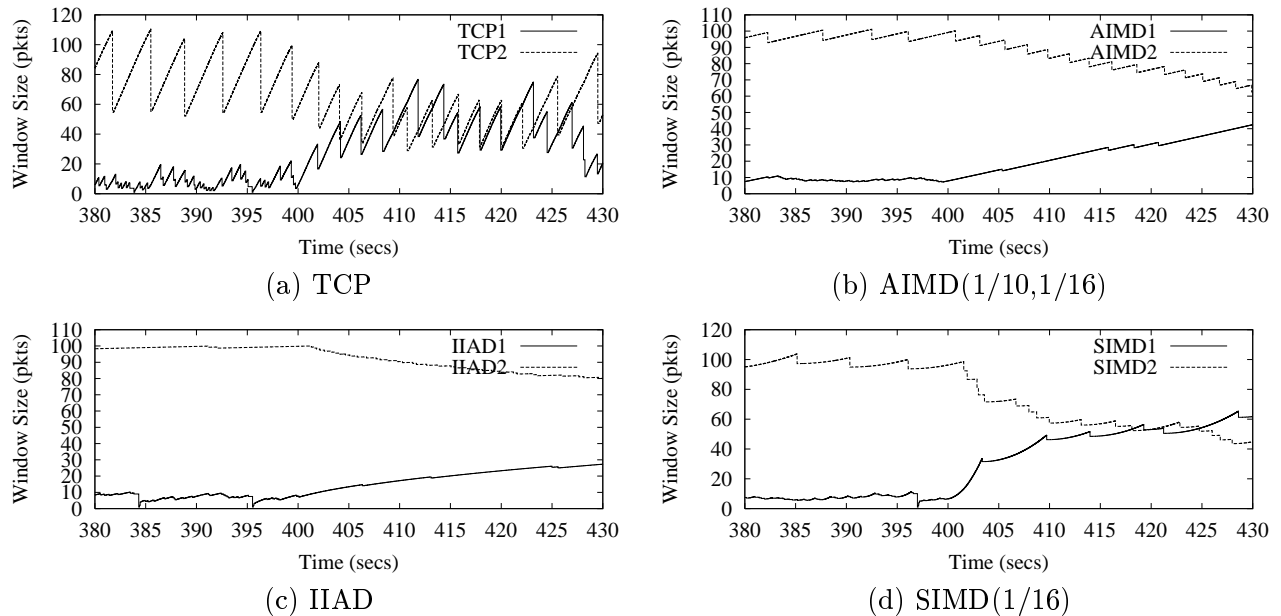(a) TCP (b) AIMD(1/10,1/16) (c) IIAD (d) SIMD(1/16)

Figure 10: Two flows converge to fair share of bandwidth

It can be observed from the graph that standard TCP has the highest convergence speed, and IIAD generates the smoothest but least responsive traffic. It is worth noticing that in this scenario, where significant bandwidth change happens, our proposed algorithm converges much faster than AIMD to the fair share of the bandwidth.

Algorithm	Experiment 1				Experiment 2					
	W_1	W_2	T_2 (RTT)		W_1	W_2	T_1 (RTT)		Δ (pkts)	
			simu	anal			simu	anal	simu	anal
TCP	6.1	99.6	68.0	88.7	8.8	13.8	55	43.7	5.8	6.0
AIMD	7.9	99.2	776	1217	12.7	31.0	349	342	18.6	18.3
IIAD	7.7	99.8	4232	6684	11.8	31.2	1284	1242	8.1	7.6
SIMD	6.6	96.3	**218**	**852**	10.2	33.2	90	85.1	13.6	12.3

Table 3: Quantitative measures on convergence time

Table 3 gives the convergence time to fairness (T_2). Here we use $\epsilon = 10$ packets (cf. Section 4.2). The theoretical value is also given in the table for comparison. The following observations can be made from the table:

- The simulation results agree with the theoretical analysis in the ranking of various protocols except that all measured convergence times are smaller than the corresponding theoretical values. This is expected since our analysis is based on synchronized feedback assumption, and routers that do not differentiate among flows when dropping packets. In contrast, in the simulation, we use RED, so

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flows with larger window sizes would see more packet drops. In other words, RED helps to enhance the convergence rate to fairness.

- SIMD benefits from RED much more than other schemes. The T_2 value from simulations is much smaller than the value obtained from analysis (shown in boldface). This is because RED allows SIMD flows with smaller windows to experience less packet losses, which gives them a better chance to become more aggressive. On the contrary, AIMD does not fully capitalize on the random loss property of RED since its aggressiveness does not change. As a result, SIMD converges to fairness much faster.

5.3.2 $W_1 < W_2 < \frac{W}{2}$ (Convergence to Efficiency)

To create such scenario, we let $c_1 = 11$, $c_2 = 3$, $x = 6$Mbps, $y = 10$Mbps. So initially the bottleneck link for flow (s1,r1) is ($b1,b2$), and for flow (s2,r2) the bottleneck is ($b3,b4$). But at time 400, both of them switch to link ($b2, b3$). Roughly, we have $W \approx 110$, $W_1 \approx 10$, and $W_2 \approx 30$. We can then study T_1, the convergence time to efficiency of different control schemes. Figure 11 plots the transient behavior of the congestion window of different protocols.

(a) TCP

(b) AIMD(1/10,1/16)
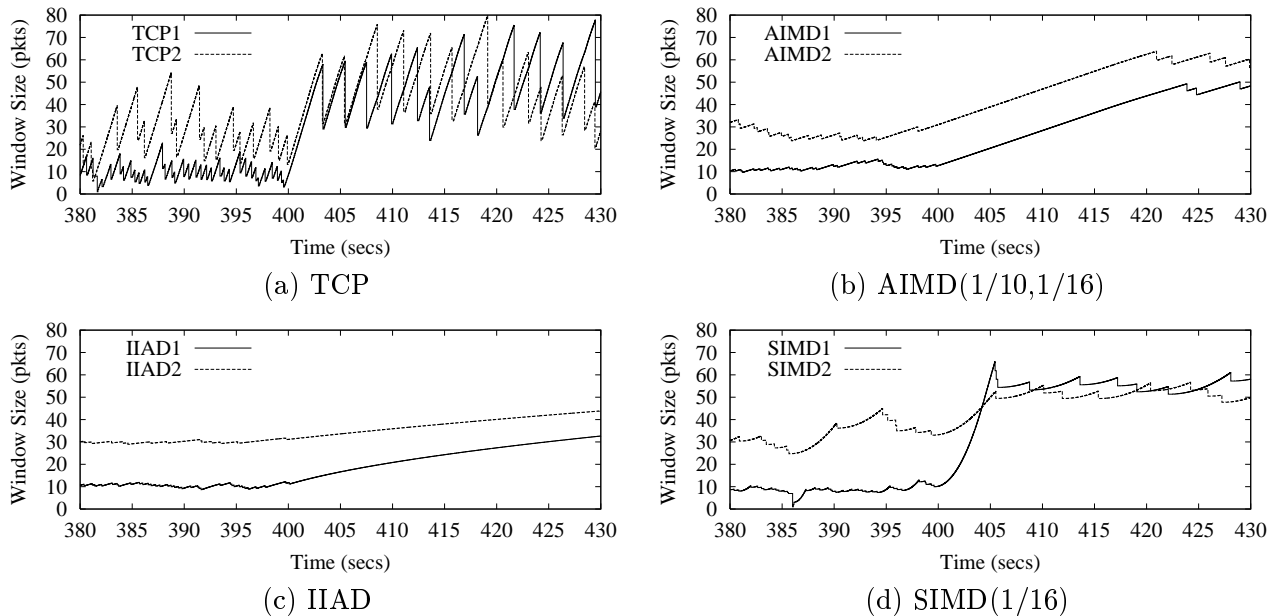
(c) IIAD

(d) SIMD(1/16)

Figure 11: Two flows converge to fair share of bandwidth

The advantage of our SIMD algorithm is more pronounced in this scenario. TCP is still the fastest responding protocol, but still at the expense of high variability. In addition, general AIMD suffers from the problem of convergence efficiency, i.e, all flows have the same window increments, so before packet loss happens, they increase their congestion window at the same rate and thus do not efficiently converge to the fair share. On the contrary, our SIMD algorithm allows the two competing flows to smoothly and quickly transit to the fair steady state, since the flow with smaller window grows more aggressive than the one with larger window. IIAD takes a much longer time to converge due to its inherent weak aggressiveness (sub-linear increase).

We also give convergence time to efficiency (T_1) in Table 3. Analytical results closely match the simulation results.

6   Related Work

Under a synchronized feedback assumption, Chiu and Jain [4] analyze AIMD control, thus provide a sound basis for Jacobson's TCP algorithm [15] and Ramakrishnan and Jain's DECbit scheme [24]. To provide smoother transmission rate than that given by TCP, several TCP-like window-based congestion control mechanisms have been proposed, including the general AIMD [11, 31] and TEAR [26]. These mechanisms use a moderate window decrease parameter to reduce rate variability, meanwhile use a matching window increase parameter to satisfy TCP-friendliness. There are tradeoffs between smoothness and reaction to changes in network conditions [11, 30].

Chiu and Jain also mentioned non-linear controls in [4]. They argued that non-linear controls reduce robustness and are not suitable for practical purposes. On the contrary, Bansal and Balakrishnan [3] proposed binomial algorithms that interact well with TCP AIMD. Binomial algorithms generalize additive-increase by increasing inversely proportional to a power k of the current window, and generalize multiplicative-decrease by decreasing proportional to a power l of the current window. Binomial algorithms can be TCP-friendly if and only if $k + l = 1$. Binomial controls are memory-less in that they use only the current window size in their control rules. SIMD is radically different from memory-less binomial algorithms. To our knowledge, SIMD is the first window-based TCP-friendly congestion control algorithm using history information in its control rules. By doing so, SIMD improves its transient behavior and convergence speed without sacrificing smoothness in steady state.

Another approach to provide smoother transmission rate is equation-based congestion controls [12, 23, 29], first proposed in [18]. In these schemes, the end-systems measure the packet loss rate and round-trip time, and use the TCP-friendly equation [22] to compute the transmission rate. Two comparisons [11, 30] of equation-based and window-based congestion controls have shown that equation-based schemes and window-based AIMD share similar transient behaviors but equation-based schemes provide higher smoothness. However, the aggressiveness of equation-based schemes is limited by the nature of rate-based control, which lacks a self-clocked mechanism for overload protection as in window-based control. Notably, equation-based schemes use more history information (up to 8 congestion epochs [12]). Therefore, SIMD is a step toward exploring the space between window-based memory-less control schemes and equation-based schemes that make use of longer history.

Applications can be adaptive to the congestion level of the network in a TCP-friendly way. Examples include RAP [25] and LDA [28]. Applications using RAP can adapt the quality of transmitted streams based on the estimated rate. RAP employs an AIMD algorithm, similar to TCP. LDA relies on RTP [27] for feedback information about packet losses and round-trip time. The additive increase rate is estimated using reported loss, delay, and bottleneck bandwidth values.

Much of the literature has focused on the modeling of TCP congestion control [2, 8, 17, 20, 21, 22]. Ott *et al.* showed that if packet losses are independent with small probability p, the average window size and long-term throughput are of the order of $1/\sqrt{p}$. A heuristic analysis in [8] shows that the throughput of a connection is inversely proportional to its round-trip time. Lakshman *et al.* [17] studied the properties of TCP in a regime where the bandwidth-delay product is high and losses are random. In [20], Mathis *et al.* studied the relationship between TCP throughput and packet loss rate when TCP is in congestion avoidance mode and came up with the well-known TCP-friendly equation. Padhye *et al.* [22] extend this method and use a stochastic model that also captures the effect of TCP's timeout mechanism on throughput, thus provide a more accurate prediction of TCP throughput when random loss probability is moderate. Altman *et al.* [2] analyze TCP throughput under a more general loss process which is assumed to be only stationary. The model thus can account for any correlation and inter-loss time distributions. They also show that the throughput is inversely proportional to round-trip time and the square root of packet loss probability.

7 Conclusion

We proposed a novel window-based congestion control algorithm called SIMD (Square-Increase/Multiplicative-Decrease). Contrary to previous memory-less controls, SIMD utilizes history information in its control rules. It uses multiplicative decrease but the window size increases in proportion to the square of the time elapsed since the detection of the last loss event. Thus, SIMD can maintain smoothness in steady state, while efficiently probing available bandwidth when there are drastic changes in network conditions. We have shown that SIMD is TCP-friendly as well as TCP-compatible under RED. We have also shown that SIMD has faster convergence than TCP-friendly memory-less AIMD and binomial algorithms. Our simulations using the *ns* simulator have demonstrated the superiority of SIMD.

To summarize, SIMD is the first example of window-based congestion control algorithms that uses history information in its control rules. It explores a new space between memory-less window-based congestion control schemes and equation-based schemes that use history spanning many congestion epochs. Indeed, this new space defines a new class of TCP-friendly window-based congestion control algorithms, of which SIMD is an instance [16]. Future work includes comparisons between equation-based schemes and SIMD under different conditions.

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A   Convergence Time

We use Figure 4(b) to illustrate how we compute the convergence time. We use the phase plot as in the convergence analysis. Assuming we start from an arbitrary point (W_1, W_2) in the graph, also assume this point is below the efficiency line $w_1 + w_2 = W$, where W is the bottleneck resource (measured in terms of packets). Without loss of generality, we assume $W_2 \geq W_1$. We assume synchronized feedback. We can then analyze the time it takes a control mechanism to bring this starting point close to the fairness line, or specifically, $|w_2 - w_1| < \epsilon$, and then oscillate around the efficiency line. For ease of analysis, we divide this convergence time into two parts: T_1, the time it takes to converge to the efficiency line, i.e., the time from the initial point to the first time loss is detected (window is then decreased), measured in number of round trip times; and T_2, the time needed to converge to the fairness line, measured in number of congestion epochs. We also derive T_2 in terms of number of RTT's at the end of this section.

A.1   Convergence Time to Efficiency

At time T_1, the trajectory crosses the efficiency line. We can thus compute T_1 as follows.

For AIMD, we have that the window increments of w_1 and w_2 are the same, i.e., $\delta_1 = \delta_2$. Since

$$w_1 + w_2 = W_1 + \delta_1 + W_2 + \delta_2 = W$$

we now have:

$$\delta_1 = \delta_2 = (W - W_1 - W_2)/2$$

Since in each RTT, windows are increased by 1 in TCP and by $\alpha = 3\beta/(2 - \beta)$ in general AIMD [11], we now have:

$$
\begin{aligned}
T_1^{TCP} &= \frac{W - W_1 - W_2}{2} \\
T_1^{AIMD} &= \frac{W - W_1 - W_2}{2\alpha} = \frac{(W - W_1 - W_2)(2 - \beta)}{6\beta}
\end{aligned}
$$

Note that at this moment, the difference between the two window values remains the same, i.e.,

$$\Delta^{TCP} = \Delta^{AIMD} = w_2 - w_1 = W_2 - W_1$$

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For SIMD, we have that the window increment is inversely proportional to W_i, so $\delta_1/\delta_2 = W_2/W_1$, therefore, we have:

$$\delta_1 = \frac{W_2}{W_1 + W_2}(W - W_1 - W_2)$$

$$\delta_2 = \frac{W_1}{W_1 + W_2}(W - W_1 - W_2)$$

Since the window increase function is $w(t) = w_0 + \frac{\alpha^2}{4}t^2$, where α is defined in Equation (3), the time it takes each user to come to this point is:

$$T_1^{SIMD} = \gamma\sqrt{\frac{W_1 W_2 (W - W_1 - W_2)}{W_1 + W_2}}$$

where $\gamma = 2(1 - \frac{2\beta}{3})\sqrt{\frac{2}{9\beta(1-\beta)}}$. And the difference between the two new values becomes:

$$\Delta^{SIMD} = |2(W_2 - W_1) - \frac{W}{W_1 + W_2}(W_2 - W_1)|$$

Given IIAD rules, since the trajectory is inversely proportional to the current window size, the window growth follows the function $w(t) = \sqrt{2\alpha t} = \sqrt{3\beta t}$.[9] By solving the equation $w_1 + w_2 = \sqrt{3\beta T_1 + W_1^2} + \sqrt{3\beta T_1 + W_2^2} = W$, we get:

$$T_1^{IIAD} = \frac{1}{12\beta W^2}(W_1^4 + W_2^4 + W^4 - 2W_1^2 W_2^2 - 2W^2 W_2^2 - 2W_1^2 W^2)$$

And the difference becomes:

$$\Delta^{IIAD} = w_2 - w_1 = \frac{|W_2^2 - W_1^2|}{W}$$

A.2 Convergence Time to Fairness

After time T_1, the trajectory will oscillate around the efficiency line. We now redefine the initial point (W_1, W_2) to be the starting point of each congestion epoch. We can then derive the change in Δ after each congestion epoch.

For TCP/AIMD, at the end of each congestion epoch, the window values evolve to $(W_1 + \delta_1, W_2 + \delta_2)$. Thus the starting point for the next congestion epoch will be: $((1-\beta)(W_1 + \delta_1), (1-\beta)(W_2 + \delta_2))$. Since the point is oscillating around the efficiency line, the sum of the two window values at the beginning of each congestion epoch should be the same. We therefore have:

$$(1 - \beta)(W_1 + \delta_1 + W_2 + \delta_2) = W_1 + W_2$$

$$\delta_1 = \delta_2$$

Therefore, $\delta_1 = \delta_2 = \frac{\beta}{1-\beta}\frac{W_1+W_2}{2}$. After each congestion epoch, the difference between the two window values becomes:

$$\Delta' = (1 - \beta)|(W_2 + \delta_2) - (W_1 + \delta_1)| = (1 - \beta)\Delta$$

For SIMD, since the window increments are inversely proportional to W_i's, we can then have the following relationships:

$$(1 - \beta)(W_1 + \delta_1 + W_2 + \delta_2) = W_1 + W_2$$

$$\delta_1/W_2 = \delta_2/W_1$$

---

[9]Since $\frac{dw}{dt} = \alpha/w$, and $w(0) = 0$, we get $w(t) = \sqrt{2\alpha t}$. Also, $\alpha = \frac{3}{2}\beta$ for TCP-friendliness [3].

We can then get:
$$\Delta' = (1 - 2\beta)|W_2 - W_1| = (1 - 2\beta)\Delta$$

For IIAD, it is hard to derive the relationship in steady state, but when W_1 and W_2 are large, we can use the window values at the initial point to approximate the current window sizes, and thus make the window increments again inversely proportional to W_i's. Note that this approximation is actually an upper bound on convergence rate, which implies IIAD will converge slower than this rate. Thus:

$$W_1 + \delta_1 - \beta + W_2 + \delta_2 - \beta = W_1 + W_2$$
$$\delta_1/W_2 = \delta_2/W_1$$

We can then get:
$$\Delta' = (1 - \frac{2\beta}{W_1 + W_2})\Delta \approx (1 - \frac{2\beta}{W})\Delta$$

The last approximation is valid when $W_1 + W_2 \approx W$, or $W \gg \beta$.

Assume we need to have some bounds on the difference between the two windows, i.e., $|w_1 - w_2| < \epsilon$, and also assume the initial window difference is Δ, we then have:

$$T_2^{TCP} = \log_{1/2} \frac{\epsilon}{\Delta}$$
$$T_2^{AIMD} = \log_{1-\beta} \frac{\epsilon}{\Delta}$$
$$T_2^{SIMD} = \log_{1-2\beta} \frac{\epsilon}{\Delta}$$
$$T_2^{IIAD} \geq \log_{1-2\beta/W} \frac{\epsilon}{\Delta}$$

Note that T_2's are measured in number of congestion epochs. To convert it to RTT's, we need to compute the length of congestion epoch L (in RTT's) for each mechanism given the initial window values of each congestion epoch. Given the window increase and decrease rules, it is straightforward to obtain the following results[10]:

$$L^{TCP} = W/4$$
$$L^{AIMD} = \frac{\beta W}{2\alpha} = \frac{(2-\beta)W}{6}$$
$$L^{SIMD} = \frac{2}{3}\frac{1 - \frac{2\beta}{3}}{1 - \beta}\sqrt{2W_1W_2}$$
$$L^{IIAD} = \frac{4}{3}\frac{W_1W_2}{(W - 2\beta)} \approx \frac{4}{3}\frac{W_1W_2}{W}$$

We can then iteratively compute the value of T_2 in terms of RTT's by summing up the length of each congestion epoch. However, note that in steady state, i.e., $W_1 \approx W_2 \approx \frac{W}{2}$. Assuming $\beta^{SIMD} \ll 1$, we have $L^{SIMD} \approx \sqrt{2}W/3$. Assuming $\beta^{IIAD} \ll W$, we have $L^{IIAD} \approx W/3$. If we assume that the length of the transient congestion epoch values are close to the steady state values, we can then approximate the convergence time T_2 (in RTT) to fairness as follows:

$$T_2^{TCP} = \frac{W}{4}\log_{1/2}\frac{\epsilon}{\Delta}$$
$$T_2^{AIMD} = \frac{(2-\beta)W}{6}\log_{1-\beta}\frac{\epsilon}{\Delta}$$

---

[10] For example, for TCP, since the total window size increases by $\frac{W}{2}$ in one congestion epoch at 2-packet increments (one per user), we get $L = \frac{W}{4}$ RTT's.

$$T_2^{SIMD} \approx \frac{\sqrt{2}W}{3}\log_{1-2\beta}\frac{\epsilon}{\Delta}$$

$$T_2^{IIAD} \geq \frac{W}{3}\log_{1-2\beta/W}\frac{\epsilon}{\Delta}$$

B   Implementation

To implement SIMD algorithm[11], we need only to change the way the congestion window is updated in standard TCP according to control rules (1). However, since now we need to know the value of the congestion window after the last packet loss, we have to add a special variable w_0 to record this value. We then divide the increment in each RTT by the current window size to approximate the window increment rule upon each acknowledgment packet. For SIMD(β), we have the following increase rule:

$$w_{new} = w_{old} + \alpha\frac{\sqrt{w_{old} - w_0}}{w_{old}}, \tag{9}$$

where α is given in Equation (3). Note that $w_0 = w_{max}(1 - \beta)$, where w_{max} is the window size right before the loss is detected.

There's one problem with this approximation rule: for the first acknowledgment, we have to use some other equation since the current window size $w_t = w_0$ and that will make the increment to be zero. We solved this problem by noticing that since $w(t) = w_0 + \frac{\alpha^2}{4}t^2$, we have $w(1) - w_0 = \frac{\alpha^2}{4}$. Thus, upon receiving the first ACK packet, we increment the window as:

$$w_{new} = w_0 + \frac{\alpha^2}{4w_0}$$

The value of w_0 is reset to the current congestion window size whenever the congestion window is decreased. And the decrement rule is as follows:

$$w_{new} = w_{old} - \beta w_{old}$$

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[11]A release of the code for *ns* SIMD implementation and the simulation scripts used for this paper will be available soon.