Adaptive Routing of QoS-constrained Media Streams over Scalable Overlay Topologies

Gerald Fry and Richard West

Computer Science Department
Boston University
Boston, MA 02215
{gfry,richwest}@cs.bu.edu

Abstract

Current research on Internet-based distributed systems emphasizes the scalability of overlay topologies for efficient search and retrieval of data items, as well as routing amongst peers. However, most existing approaches fail to address the transport of data across these logical networks in accordance with quality of service (QoS) constraints. Consequently, this paper investigates the use of scalable overlay topologies for routing real-time media streams between publishers and potentially many thousands of subscribers. Specifically, we analyze the costs of using k-ary n-cubes for QoS-constrained routing. *Given a number of nodes in a distributed system, we calculate the optimal k-ary n-cube structure for minimizing the average distance between any pair of nodes. Using this structure, we describe a greedy algorithm that selects paths between nodes in accordance with the real-time delays along physical links. We show this method improves the routing latencies by as much as* 40%, *compared to approaches that do not consider physical link costs.*

We present a method for adaptive node placement in the overlay topology based upon the locations of publishers, subscribers, physical link costs and per-subscriber QoS constraints. One such method for repositioning nodes in logical space is discussed, to improve the likelihood of meeting service requirements on data routed between publishers and subscribers. Experimental analysis shows that a reduction in average lateness with respect to per-subscriber deadlines is achievable for subscriber groups of varying sizes without a significant increase in physical link stress.

1. Introduction

Recent work in the area of Internet-scale distributed systems suggests that a carefully constructed overlay topology is beneficial for routing application-specific data. The NARADA protocol, for instance, provides strong evidence that implementing multicast functionality at the end-host level results in advantages that outweigh the delay penalties incurred over implementation in the network core [8]. Such advantages include: (1) the ability to scale to larger topologies without requiring that group state be kept at core network routers, (2) flexibility to adapt routing behavior to application-specific events, and (3) reliance only on unicast functionality implemented at the network layer, permitting the use of COTS-based systems on existing IP networks.

Although NARADA gives a convincing argument for the usefulness of end-system multicast routing, the protocol itself fails to scale as group sizes increase beyond a few hundred hosts, partly due to communication overheads introduced by random probe messages. In contrast, there have been efforts to generate more scalable overlays for storage and retrieval as well as routing of data items among peers using consistent hashing techniques. Such work includes Pastry [17], Scribe [4], CHORD [20], CAN [16] and Tapestry [22]. There has also been work in the domain of distributed computing involving *k-ary n-cube* structures for communication in parallel processing architectures [9, 6, 7]. However, unlike NARADA, these systems make no explicit attempt to route data in accordance with latency and bandwidth requirements.

For real-time routing, it is not enough to use scalable overlays such as those described above. In applications where streams of multimedia data must be transmitted to a large set of subscribers with real-time constraints, it is imperative that information about the underlying physical network be leveraged, in order to efficiently route the data over the logical topology. For example, consider a nationwide digital broadcast system (on the scale of Shoutcast [19]), in which hundreds of thousands of subscriber hosts receive live video feeds from one or more publishers. Such a system may require data to be delivered to each subscriber with its own unique QoS constraints. In the absence of information about physical "proximities" between nodes, data could be routed over links that have large latencies or low bandwidths.

Contributions: This work focuses on the scalable delivery of real-time media streams. We present an analysis of *k-ary n-cube* graphs as structures for overlay topologies [13]. In particular, we develop a method for determining the optimal values of k and n, to represent a logical topology supporting m physical hosts. We describe a greedy algorithm for routing over the overlay structure while taking physical network proximity measures into account. Additionally, we investigate methods for dynamic subscriber relocation in logical space based on network proximity and per-subscriber latency constraints. Simulation results show a significant reduction in delay penalties relative to unicast delays when using the greedy routing algorithm as opposed to random and ordered dimensional routing.

Sections 2 and 3 present an analysis of adaptive *k-ary n-cube* overlay topologies for use in QoS-aware P2P systems. In Section 4, we investigate several algorithms for routing data over a *k-ary n-cube* topology, including a comparison of simulation results. This is followed by Section 5 that discusses adaptive algorithms for re-assignment of hosts in logical space, in order to increase the probability of satisfying real-time latency constraints. Section 6 investigates the relationship between link stress and average lateness for routed messages. Implementation issues are then discussed in Section 7. Finally, related work is described in Section 8, followed by conclusions and future work in Section 9.

2. Analysis of *k-ary n-cube* Topologies

Scalable peer-to-peer (P2P) systems such as CHORD, CAN and Pastry use distributed hashing techniques for locating objects (and nodes) in logical space. These systems route in as little as $O(\lg M)$ hops along the overlay topology, where M denotes the number of logical hosts communicating in the system [4, 20, 16]. Furthermore, the lookup services associated with these systems require that hosts maintain up to $O(\lg M)$ sized routing tables.

We use undirected *k-ary n-cube* graphs to model logical overlays in a similar manner to the P2P systems described above. These graphs are specified using n as the *dimensionality* parameter and k as the *radix* (or *base*) in each dimension. Figure 1 shows an example of an overlay network structured as a *2-ary 3-cube* graph and a corresponding underlying physical network. A cost is associated with each edge in the physical network, and each edge in the logical overlay maps to the shortest path between the respective end-point nodes in the physical topology. The costs associated with logical edges are derived as the sum of the costs along the corresponding path taken in the physical network.

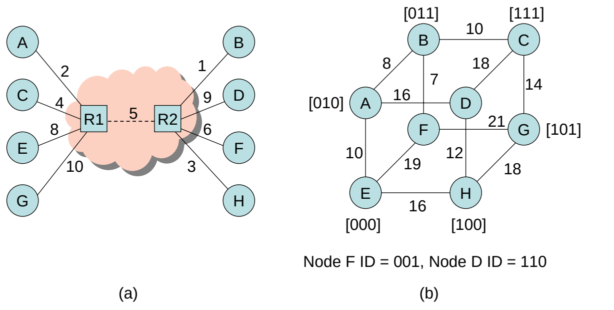


Figure 1. A sample overlay network

Note that the physical topology may contain router nodes that do not participate explicitly within the context of the overlay network (i.e., R1 and R2). The following properties of *k-ary n-cube* graphs are relevant to this work:

- $M = k^n$, where M is the number of nodes in the graph. Therefore, $n = \lg_k M$.
- Each node is of the same degree, with n neighbors if $k = 2$ or $2n$ neighbors if $k > 2$.
- The minimum distance between any pair of nodes in the graph is no more than $n\lfloor \frac{k}{2} \rfloor$ hops.
- The average routing path length between nodes in the graph is $A(k, n) = n\lfloor \frac{k^2}{4} \rfloor \frac{1}{k}$ hops.
- The optimal dimensionality of the graph is $n = \ln m$.
- Each node in the graph can be associated with a logical identifier consisting of n digits, where the ith digit (given $1 \leq i \leq n$) is a base-k integer representing the offset in dimension i.
- Two nodes are connected by an edge *iff* their identifiers have $n - 1$ identical digits, except for the ith digit in both identifiers, which differ by exactly 1 modulo k.

The regularity of *k-ary n-cube* graphs provides for a logical topology that is scalable in the sense that routing complexity increases less than linearly with the number of logical nodes in the system. Intuitively, the structure is regular and compact, with different values of k and n resulting in differing topology sizes and corresponding values of $A(k, n)$.

Worst Case Hop Count: Consider a *k-ary n-cube* graph having each node represented as a string of n coordinates, with each coordinate as a value in base k. We refer to the string of coordinates associated with a node as its *node identifier*. Given two nodes and their respective node identifiers, $S = (s_1, s_2, \ldots, s_n)$ and $D = (d_1, d_2, \ldots, d_n)$, the shortest path distance between them, $\delta(S, D)$, over the *k-ary n-cube* topology is calculated as:

$$\delta(S, D) = \sum_{i=1}^{n} \min\{(s_i - d_i) \bmod k, (d_i - s_i) \bmod k\}.$$

For all values of i, we have $0 \leq s_i, d_i < k$. Thus, the quantity, min$\{(s_i - d_i) \bmod k, (d_i - s_i) \bmod k\}$, can be no greater than $\lfloor \frac{k}{2} \rfloor$, and the worst case hop count between nodes is $\delta_{\max}(S, D) = n \cdot \lfloor \frac{k}{2} \rfloor$.

Average Hop Count: The remainder of this section derives an explicit formula for the average hop count between nodes in a *k-ary n-cube* graph, denoted $A(k, n)$. We assume $A(k, n)$ is calculated by considering the most direct path between a pair of nodes. The proof proceeds by induction on the number of dimensions, n, using the result from Lemma 1 as the base step. Lemma 2 provides a recursive formula used in completing the inductive step for the proof of Theorem 1. The formula $A(k, n)$ then follows directly as Corollary 1. Finally, the real-valued function, $A_{ext}(k, n)$, is minimized to find the ideal value of n with respect to average hop count between *k-ary n-cube* nodes.

Lemma 1. *For an undirected k-ary 1-cube graph, where k denotes the number of nodes in the first dimension, the sum of the distances, $H(k, 1)$, from any one node to every other node in the graph is given by:*

$$H(k, 1) = \lfloor \frac{k^2}{4} \rfloor \tag{1}$$

Proof. A k-ary 1-cube can be represented as an undirected cycle of k nodes. There are two cases to consider:

(i) If k is even, $H(k, 1)$ is given by:

$$H(k, 1) = \sum_{i=1}^{k/2} i + \sum_{i=1}^{k/2-1} i = \frac{k^2}{4} \tag{2}$$

(ii) If k is odd, $H(k, 1)$ is given by:

$$H(k, 1) = \sum_{i=1}^{(k-1)/2} 2i = \frac{k^2 - 1}{4} \tag{3}$$

Thus, by equations (2) and (3), for all $k \geq 2$, $H(k, 1) = \lfloor \frac{k^2}{4} \rfloor$. □

Lemma 2. *For an undirected k-ary n-cube graph, the following recursive identity holds:*

$$H(k, n) = H(k, n - 1) \cdot k + k^{n-1} \lfloor \frac{k^2}{4} \rfloor \tag{4}$$

Where $H(k, n)$ denotes the sum of the distances from any one node to every other node in a k-ary n-cube graph, k denotes the radix in each dimension, and n the number of dimensions.

Proof. A k-ary n-cube can be divided into k k-ary (n-1)-cubes. Each subcube of $n - 1$ dimensions is connected to one or two neighboring subcubes by k^{n-1} edges. Consider a reference node in a subcube. The sum of the distances between the reference node and every other node in the same subcube is $H(k, n - 1)$. The sum of distances between the reference node and each node in an adjacent subcube is $H(k, n - 1) + k^{n-1}$. Similarly, the sum of the distances between the reference node and each node in a subcube that is i cubes away from the subcube containing the reference node is given by $H(k, n-1) + i \cdot k^{n-1}$. There are two cases to consider:

(i) if k is odd, $H(k, n)$ is given by:

$$\begin{aligned} H(k, n) &= H(k, n - 1) + \\ &+ \sum_{i=1}^{\frac{k-1}{2}} 2(H(k, n - 1) + ik^{n-1}) \\ &= H(k, n - 1) \cdot k + k^{n-1} \frac{k^2 - 1}{4} \end{aligned} \tag{5}$$

(ii) if k is even, $H(k, n)$ is given by:

$$\begin{aligned} H(k, n) &= H(k, n - 1) + \\ &+ \sum_{i=1}^{\frac{k-2}{2}} 2(H(k, n - 1) + ik^{n-1}) + \\ &+ H(k, n - 1) + k^{n-1} \frac{k}{2} \\ &= H(k, n - 1) \cdot k + k^{n-1} \frac{k^2}{4} \end{aligned} \tag{6}$$

By equations (5) and (6),
$H(k, n) = H(k, n - 1) \cdot k + k^{n-1} \lfloor \frac{k^2}{4} \rfloor$. □

Theorem 1. *For an undirected k-ary n-cube, where $k \geq 2$ denotes the radix of each dimension, the sum of the distances, $H(k, n)$, from any one node to every other node in the graph is given by:*

$$H(k, n) = k^n \cdot n \lfloor \frac{k^2}{4} \rfloor \frac{1}{k} \tag{7}$$

Proof. The proof proceeds by induction on the number of dimensions, n. By Lemma 1,

$$H(k, 1) = \lfloor \frac{k^2}{4} \rfloor = k^1 \cdot 1 \lfloor \frac{k^2}{4} \rfloor \frac{1}{k} \tag{8}$$

Therefore, the formula holds for $n = 1$ and establishes a basis for the inductive argument. Next, suppose that the result holds for $n - 1$ dimensions:

$$H(k, n - 1) = k^{n-1} \cdot (n - 1) \lfloor \frac{k^2}{4} \rfloor \frac{1}{k} \tag{9}$$

Substituting the right side of the above equation for $H(k, n-1)$ in Equation (4) yields:

$$\begin{aligned} H(k,n) &= (k^{n-1} \cdot (n-1)\lfloor \frac{k^2}{4} \rfloor \frac{1}{k})k + k^{n-1}\lfloor \frac{k^2}{4} \rfloor \\ &= k^n \cdot n \lfloor \frac{k^2}{4} \rfloor \frac{1}{k} \end{aligned} \tag{10}$$

By induction, the result holds for all $n > 0$. □

Corollary 1. *In a k-ary n-cube graph, where n denotes the number of dimensions and k the radix in each dimension, the average hop count along a path between two nodes, $A(k, n)$, is given by the following:*

$$A(k,n) = n \lfloor \frac{k^2}{4} \rfloor \frac{1}{k} \tag{11}$$

Proof. The result is obtained by dividing the right side of Equation (7) by the number of nodes, k^n, in the graph and thus follows directly from Theorem 1 and the symmetry of the k-ary n-cube structure. □

The function $A(k, n)$ is defined over the domain $\{(k, n)|k, n \in Z \wedge k \geq 2 \wedge n \geq 1\}$, where Z denotes the set of integers. Consider an extension of this function, $A_{ext}(k, n)$, with domain $\{(k, n)|k, n \in R \wedge k \geq 2 \wedge n \geq 1\}$, where R denotes the set of *real* numbers. Assuming non-integer values are possible for parameters k and n, the following optimization problem can be solved to find the optimal dimensionality, n_{opt}, with respect to the cost function $A_{ext}(k, n)$:

Problem 1. *Find $n = n_{opt}$ which minimizes $A_{ext}(k, n) = n\frac{k}{4}$, given constraints $k \geq 2$, $n \geq 1$, and $M = k^n$, where M is constant.*

By the constraint $M = k^n$, it follows that $k = M^{\frac{1}{n}}$. Substituting for k, the cost function becomes:

$$A_{ext}(n) = n\frac{M^{\frac{1}{n}}}{4} \tag{12}$$

Since M is held constant, taking the derivative of $A_{ext}(n)$ with respect to the single variable n yields:

$$A_{ext}'(n) = \frac{M^{\frac{1}{n}} \cdot (n - \ln M)}{4n} \tag{13}$$

As a result of the constraint $n \geq 1$, the only relevant critical point occurs when $n = \ln M$. To see that this point is indeed a minimum, the second derivative with respect to n is examined:

$$A_{ext}''(n) = \frac{M^{\frac{1}{n}}(\ln M)^2}{4n^3} \tag{14}$$

For all $n \geq 1$, $A_{ext}''(n) > 0$. In particular, $A_{ext}''(n) > 0$ for $n = \ln M$. Therefore, the function $A_{ext}(k, n)$ is minimized with respect to the constraints exactly when $n = n_{opt} = \ln M$, giving $A_{ext}(k, n) = \frac{1}{4}M^{\frac{1}{\ln M}} \ln M$. The above analysis suggests that the number of dimensions, n, of a *k-ary n-cube* graph with M nodes should be chosen as close as possible to $n_{opt} = \ln M$, in order to minimize the average path length between pairs of nodes.

3. *M-region* Analysis

In practice, using $n = \ln M$ as the number of dimensions in a *k-ary n-cube* graph is not feasible, since n must be a positive integer. This section introduces an integer-based analysis of the choices for parameters k and n to construct a *k-ary n-cube* system that is optimal with respect to average path length.

Given a range of values for the number of *physical* hosts, m, in the system, a corresponding pair of values (k, n) is determined for defining an overlay network with $M = k^n$ *logical* hosts. We refer to this range of sizes for the *physical* network, $[m_l, m_u]$, whose members correspond to the same chosen values of k and n, as an *M-region*.

In this paper, the following assumptions are maintained for the purpose of *M-region* analysis:

- There is not necessarily a one-to-one mapping between *physical* hosts and nodes in the *k-ary n-cube* graph representing the overlay network. However, for such a logical structure to be useful for routing, we require that the number of *physical* hosts, m, be less than or equal to the number of *k-ary n-cube* nodes, M, representing the *logical* hosts. The case in which $m < M$ requires that some *physical* hosts be responsible for performing the routing functions of multiple *logical* nodes, including maintenance of the corresponding routing tables and proximities of immediate neighbors in the overlay topology.
- We optimize the structure of the overlay with respect to:
    - the *approximate* behavior of the function $n \cdot k \simeq A(k, n)$, and
    - the *actual* value of the average path length, $A(k, n)$, calculated to floating point precision.

  Results for each case confirm that fewer *M-region* calculations are necessary when the floating point value of the function $A(k, n)$ is minimized.
- Consider two *k-ary n-cube* graphs corresponding to parameters (k_1, n_1) and (k_2, n_2), such that $k_1 \cdot n_1 = k_2 \cdot n_2$ and ${k_1}^{n_1} > {k_2}^{n_2}$. Given two such choices for the structure of the overlay, the graph corresponding to parameters (k_1, n_1) is desirable since it can support a larger number of *physical* hosts without increasing the

```
Calculate_M-Region(int m) {
  i = 1
  k = j = 2;
  while(M[i,j] < m)
    i++;
  n = i;

  max_M = M[i,j];
  min_A = A[i,j];

  inc_j = 1;
  while(i > 0) {
    j += inc_j;
    i--;
    if((A[i,j] <= min_A) && (M[i,j] > max_M)) {
      inc_j = 1;
      max_M = M[i,j];
      min_A = A[i,j];
      n = i;
      k = j; }
    else inc_j = 0; }
  return k, n; }
```

Figure 2. Algorithm for calculating optimal M-regions

n \ k	2	3	4	…
1	2	3	4	…
2	4	6	8	…
3	6	9	12	…
4	8	12	16	…
5	10	15	20	…
6	12	18	24	…
7	14	21	28	…
8	16	24	32	…
9	18	27	36	…
10	20	30	40	…
⋮	⋮	⋮	⋮	⋱

Figure 3. Matrix A

n \ k	2	3	4	…
1	2	3	4	…
2	4	9	16	…
3	8	27	64	…
4	16	81	256	…
5	32	243	1024	…
6	64	729	4096	…
7	128	2187	16384	…
8	256	6561	65536	…
9	512	19683	262144	…
10	1024	59049	1048576	…
⋮	⋮	⋮	⋮	⋱

Figure 4. Matrix M

average path length.

- In terms of this analysis, there exist some *k-ary n-cube* graphs that are inherently suboptimal *for all values of* m. For example, $k = 2$ and $n = 11$ results in a graph with $2^{11} = 2048$ nodes and $A(2, 11) \simeq 2 \cdot 11 = 22$. This graph is considered suboptimal since a *3-ary 7-cube* results in more *logical* nodes ($M = 2187$) and thus can support more *physical* hosts than in the former configuration, and at the same time the latter results in a lower value for average path hop count between *logical* hosts ($A(3, 7) \simeq 21$).

Problem 2. *Given a value for the number of physical hosts in the system, m, find $k \in Z$ and $n \in Z$ that minimize the function $A(k, n) = O(k \cdot n)$ while simultaneously maximizing the number of nodes, $M = k^n$, comprising the corresponding k-ary n-cube graph.*

An iterative algorithm is presented in Figure 2 for solving the above problem. The pseudocode involves two matrices A and M, each indexed by $k \geq 2$ and $n \geq 1$. The element at the nth row and the kth (beginning with $k = 2$) column of matrix M is denoted $M[n, k] = k^n$. Similarly, $A[n, k] = n \cdot k \simeq A(k, n)$ is the value of the element at the nth row and kth (beginning with $k = 2$) column of matrix A.

Figures 3 and 4 show part of the matrices A and M, respectively, in tabular form. As an example, suppose the algorithm in Figure 2 is called with the argument $m = 100000$. At the end of the first *while* loop, $j = 2$ and $i = 17$. Intuitively, variables i and j store the indices of the elements currently being examined in matrices A and M. Upon reaching the *if* statement in the first iteration of the second *while* loop, we have $i = 16$ and $j = 3$. In subsequent iterations of the loop, the third columns of A and M are examined. The *if* condition fails upon each iteration until i has been decremented to 11. At this point the values $i = 11$ and $j = 3$ are saved in variables k and n. The loop then continues to examine all entries in the fourth columns of A and M, but the *if* condition is never again satisfied. The loop terminates when i has been decremented to 0 and the previously stored values of k and n are returned. The optimal *k-ary n-cube* structure for an overlay with 100000 *physical* hosts is the graph corresponding to $k = 3$ and $n = 11$. We can also deduce from this result that the structure will remain optimal until the number of *physical* hosts increases beyond the upper bound of the *M-region*, $m_u = 3^{11} = 177147$ *logical* hosts.

The discussion of *M-regions* in this paper corresponds roughly to *realities* in CAN. However, the algorithm in Fig-

$[m_l, m_u]$	k	n	$M = k^n$
$[2, 2]$	2	1	2
$[3, 4]$	2	2	4
$[5, 9]$	3	2	9
$[10, 27]$	3	3	27
$[28, 32]$	2	5	32
$[33, 81]$	3	4	81
$[82, 243]$	3	5	243
$[244, 729]$	3	6	729
$[730, 2187]$	3	7	2187
$[2188, 6561]$	3	8	6561
$[6562, 19683]$	3	9	19683
$[19684, 59049]$	3	10	59049
$[59050, 177147]$	3	11	177147
$[177148, 531441]$	3	12	531441
$[531442, 1594323]$	3	13	1594323
$[1594324, 4782969]$	3	14	4782969

Figure 5. Table of M-regions

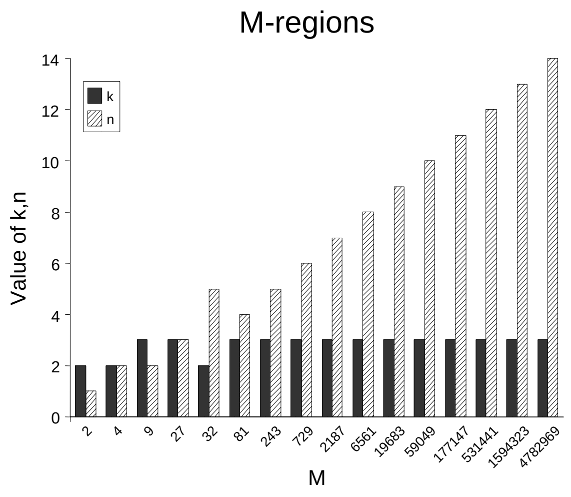


Figure 6. M-region Graph

ure 2 is explicit in determining overlay configurations that are optimal with respect to network size and average path between pairs of *logical* hosts. The optimal *k-ary n-cube* graph can be computed each time the size of the *physical* network changes, or a number of *M-regions* can be pre-calculated and stored in memory, along with the state information to maintain the structures of several alternative routing topologies.

Figure 5 lists the first sixteen *M-regions* as a table with four columns, using the floating point value for average hop count, $A[n, k] = n\lfloor\frac{k^2}{4}\rfloor\frac{1}{k}$. The first column specifies the range of *physical* network sizes, $[m_l, m_u]$, for the corresponding optimal values of k, n, and M. This table is constructed by examining the output of the algorithm in Figure 2 for each $m = 2^i$, over the range $2 \leq m \leq 4782969$. Additionally, Figure 6 shows a bar chart of these *M-regions*. Columns for k and n are shown side-by-side for each size of the logical network corresponding to the appropriate *M-region*. This is defined as the range of integer values, $[m_l, m_u]$, corresponding to the *physical* network sizes for which the *k-ary n-cube* overlay having M nodes is optimal. Given m *physical* hosts, the *M-region* can be found on the bar chart in Figure 6 by reading the values of k and n for the largest value of M on the horizontal axis such that $m \leq M$.

4. Proximity-based Greedy Routing

For the purposes of QoS-constrained routing, this work investigates the performance of three algorithms that leverage *k-ary n-cube* logical topologies, built on top of a physical network:

- *Ordered Dimensional Routing*: For a destination identifier, $d_1d_2\cdots d_n$, a message is initially routed to a node that matches d_1 in the first digit of its logical node ID. For each dimension $i \mid 1 \leq i \leq n$, the message passes to a node whose ith digit of its ID matches d_i. This is the method for routing used by systems based on Pastry, such as Scribe and PeerCQ [4, 11].
- *Random Ordering of Dimensions*: This is similar to *ordered dimensional routing* except messages are forwarded along randomly selected dimensions towards the destination. We make sure that messages are always routed closer to the destination at each hop.
- *Greedy Routing*: As a main contribution of this work, greedy routing is performed using some measure of physical proximity. It is assumed that each host maintains a measured cost (i.e., latency) to each of its direct neighbors in the *k-ary n-cube*. A message is forwarded to the neighbor along the logical edge which results in the lowest cost among all other neighbors for which forwarding reduces the distance to the destination node. Since there are $n\lfloor\frac{k^2}{4}\rfloor\frac{1}{k}$ hops on average along the overlay network between two hosts, and finding the next hop requires searching $O(n)$ neighbors, the resulting complexity of the greedy algorithm is $O(n^2k)$.

Figure 7 illustrates a case in which the greedy routing algorithm results in a path with lower cost compared with ODR. The figure shows two paths through a *3-ary 2-cube* graph from source node S to destination node D. The path taken by the greedy algorithm results in an end-to-end cost of 19, whereas the path corresponding to ODR has a total cost of 21. Note that the edges labeled 8 and 5 are never used as next hop edges since traversing them would not bring a message logically closer to its destination.

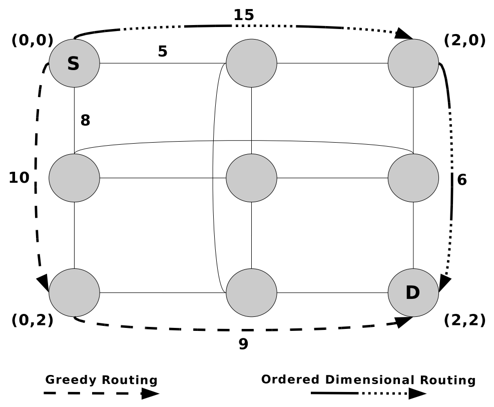


Figure 7. Example of Greedy Routing and ODR

Experimental Analysis: Experimental analysis was done via a simulation written in C, while leveraging gt-itm for generating random transit-stub physical topologies [21]. The physical topology contains 5,050 routers, and the system is comprised of 65,536 hosts each randomly assigned to a router. The experiment proceeds by choosing one host at random to be a publisher, and all other hosts are assumed to be subscribers. A message is then routed from the publisher host to each subscriber host and end-to-end latencies are recorded, as well as the unicast latency of a message routed directly between the publisher and each subscriber (as if the hosts are logically directly connected). The delay penalty of routing over the overlay relative to the unicast (IP layer) delay is calculated as the logical end-to-end latency divided by the unicast delay for each subscriber host.

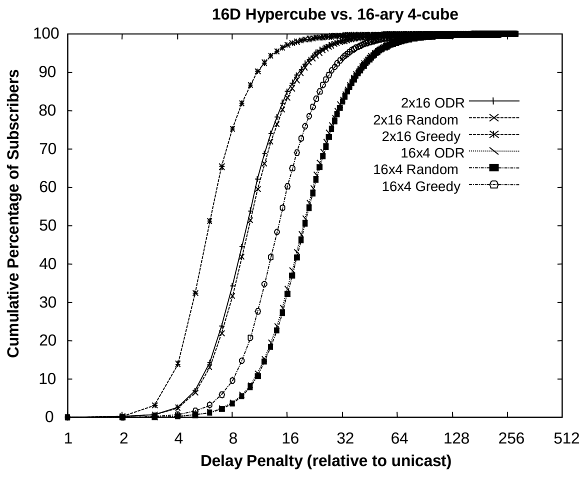


Figure 8. Comparison of routing algorithms

Figure 8 shows the cumulative distribution of delay penalties for the three algorithms using two different configurations of k and n. The values on the y-axis represent the percentage of subscribers which incur a delay penalty no more than the corresponding value on the x-axis. Simulation results indicate a significant improvement in delay penalty for greedy routing compared with random and ordered dimensional routing for both structures, whereas ordered dimensional routing performs no better than in the random case. We also see that the greedy algorithm performs better relative to the other routing methods when the node degree is greater, since this gives a higher probability of finding next hops with closer proximity in the underlying physical network. Additionally, the results show that the topology in which $k = 2$ performs better than in the case where $k = 16$, which is consistent with the analysis of *M-regions* in the previous section. As can be seen from Figure 8, there is as much as a 40% reduction in the relative delay penalty when using the greedy algorithm compared to the ODR or random approaches. This difference in performance is most noticeable when $k = 2$ and $n = 16$ for the 80th cumulative percentile delay penalty value.

5. Adaptive Node ID Assignment

During system initialization, it is assumed that the *k-ary n-cube* overlay is constructed by assigning logical node identifiers, chosen uniformly at random, to all participating physical hosts. Initially, all hosts function equally as routing agents forwarding messages across the logical topology. Once a host has received a node identifier corresponding to a position in the logical network, it can request to become a *publisher* of a new data stream or a *subscriber* to an already existing data stream. Such requests may take the form of messages routed over the optimal *k-ary n-cube* structure using the greedy algorithm described in the previous section.

Since there may be more nodes in the optimal *k-ary n-cube* overlay than physical hosts in the system, it is important that hosts are initially assigned to positions in the logical network according to a uniform random distribution. This method of mapping physical hosts to logical positions in the overlay reduces the chances of organizing hosts into a linear logical topology (ie, a partial *k-ary n-cube* network making use of only one of the available dimensions), which should be avoided to maintain an optimal average hop count between logical nodes.

In the absence of information about how *publisher* and *subscriber* hosts are associated with QoS-constrained data streams, random placement of physical hosts in the logical network is appropriate. However, as sets of hosts begin to specify interest in receiving particular data streams with corresponding service constraints, it becomes possible to reassign such *subscriber* hosts to more appropriate locations in logical space. Re-assignment of a host to a new location in the *k-ary n-cube* overlay based on the requested QoS constraints is accomplished by *swapping* the logical node

```
Subscribe(Subscriber S, Publisher P, Depth d)

 If d = D return

 Find a neighbor i of P such that
 i.cost(P) is maximal for all neighbors

 If S.cost(P) < i.cost(P)
   then swap logical positions of i and S
 else Subscribe(S, i, d+1)
```

Figure 9. Adaptive node re-assignment algorithm

identifier, as well as routing table information, with some other host in the system.

We investigate an algorithm that swaps the positions of joining subscribers with other hosts in order to increase the likelihood of satisfying QoS constraints as well as to decrease the delay penalties relative to unicast. One such algorithm works as shown in Figure 9. This algorithm takes three arguments. S represents the new subscriber which is assumed to advertise its interest in receiving a data stream from the publisher host P. The notation $i.cost(P)$ denotes the total end-to-end cost of routing a message between host P along the *physical* topology to host i.

The algorithm checks for positions appropriate for reassignment of subscribers in the overlay starting from the publisher node. Each subscriber host is swapped into a position d *logical hops* away from the publisher host if it achieves a low enough *physical* delay to the source of the published data. Intuitively, the algorithm minimizes the maximal delay along the set of direct logical links to the publisher node by considering each subscriber host in turn. For some constant depth, D, the algorithm recursively checks for appropriate logical positions with increasing hop counts from the original publisher for relocation of the subscriber host. The algorithm is thus a branch-and-bound approach, and seeks to minimize a linear cost along a particular path in the search space. Since $O(n)$ neighbors must be examined for each time the function is called, the resulting complexity of the adaptive algorithm is $O(s \cdot n \cdot D)$, where s denotes the number of subscribers in the group, and D is the maximal depth from the original publisher host.

A simulation of this algorithm was run using a randomly generated transit stub topology consisting of 5050 routers. Hosts are initially randomly assigned to routers and are organized into a 16-dimensional hypercube as the overlay topology. One publisher host is randomly chosen, and all other hosts are initially considered as neither publishers nor subscribers. As the experiment proceeds, each host is processed as a new subscriber in random order, with uniformly randomly generated latency constraints in the range bounded below by the minimal physical link latency and bounded above by the mean physical latency multiplied by the worst case logical hop count ($n \cdot \lfloor \frac{k}{2} \rfloor$). Each new subscriber host in turn is considered for re-assignment to a new location in the logical overlay using the algorithm stated in Figure 9 with $D = 1$ and d initialized to 0. Before and after adaptation, the total cost of greedy routing over the logical topology to each subscriber host is recorded, and this cost is compared with the corresponding host's latency constraint. A *success* is recorded if the achieved cost does not surpass the constraint, for each subscriber, and the resulting count of such *successes* is divided by the number of subscribers comprising the group to obtain a *success* ratio.

Figure 10(a) plots the success ratios for subscriber group sizes of 512, 1024, 2048, 4096, 8192, 16384, and 65535 hosts. Results are given for groups of subscribers that are dynamically re-assigned in the overlay topology as well as for the case in which no adaptation is performed. Success ratios are consistently greater when the adaptive algorithm is used, and it is apparent that QoS constrained data streams can be more successfully delivered to subscribers when adaptive node ID assignment is leveraged.

For a given subscriber host, S, and its corresponding latency constraint, c, a *normalized lateness* value, $L(S, c)$, is calculated using the following formula:

$$L(S,c) = \begin{cases} 0 & \text{if } S.cost(P) \leq c \\ \frac{S.cost(P)-c}{c} & \text{if } S.cost(P) > c \end{cases},$$

where $S.cost(P)$ denotes the total cost of routing a message along the logical network from publisher host P to subscriber S. The lateness values are normalized in order to eliminate bias towards subscribers with large latency constraints, relative to other subscriber hosts in the group, and all subscriber hosts with satisfied constraints are assigned a *normalized lateness* of zero. Figure 10(b) compares the *average normalized lateness* values obtained for varying group sizes before and after relocation of hosts in logical space.

Figure 11 shows a plot of the cumulative distribution of delay penalties for subscriber hosts in a group of 65535 nodes arranged in a *16-dimensional hypercube* overlay for both the adaptive and non-adaptive cases. The $90th$ percentile delay penalty in the adaptive case in this experiment is approximately 10, whereas over 90% of subscribers suffer a delay penalty of 12 (relative to unicast) in the non-adaptive case.

Note that, the algorithm in Figure 9 may be improved by allowing subscriber hosts to be swapped into new positions in the *k-ary n-cube* structure that are more than one logical hop away from the publisher host. Simulation experiments are currently under development to determine the effects of this extension to the adaptive algorithm on relative delay penalty and success ratio metrics.

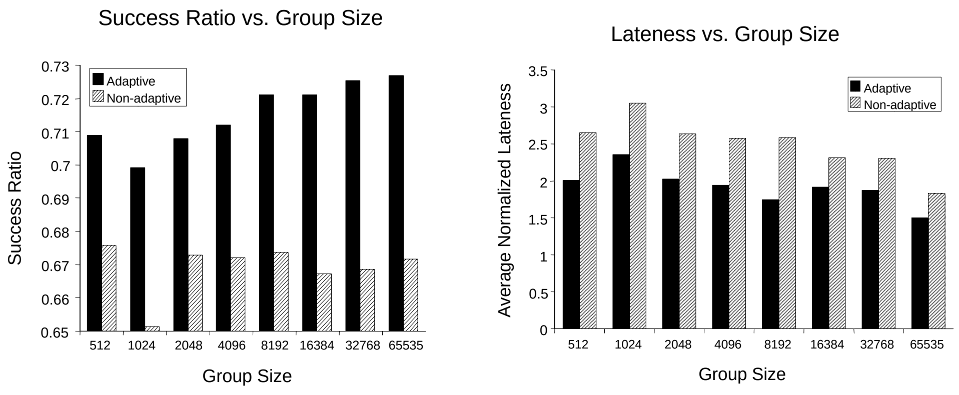


Figure 10. Effect of adaptive node relocation on (a) success ratio, and (b) normalized lateness

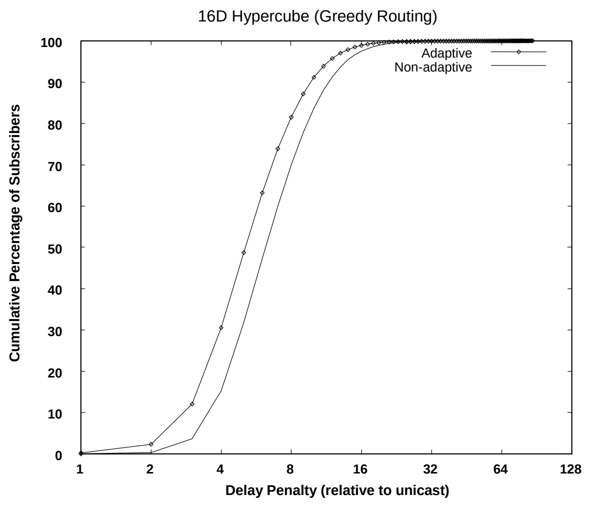


Figure 11. Adaptive vs. Non-adaptive Delays

6. Link Stress

The approaches considered in the preceding sections focus mainly on reducing the average total delay of routing messages between publisher and subscriber hosts. However, it is also useful to consider the effects of such algorithms on *average physical link stress*, which we define as the average number of times a message must be forwarded over each physical link, in order to multicast a distinct message from a publisher host to each of its subscribers.

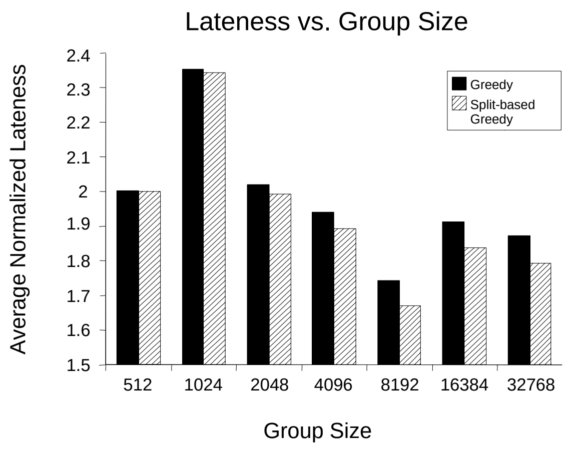
An alternative algorithm, that we call "split-based greedy routing", is used to further reduce average normalized lateness values, without unduly increasing the *physical link stress*. The algorithm extends the greedy routing approach developed in Section 4. At each hop along the path taken by the greedy algorithm, each neighbor is checked to see if it is already a subscriber. The path of a message through the *logical overlay* is redirected via an existing subscriber, if such a host exists that decreases the total end-to-end delay, compared with simply routing via the greedy algorithm.

A simulation was run, to investigate the effects of the split-based greedy algorithm. A group of subscribers is formed by selecting hosts at random positions in the logical topology. For varying group sizes in a 16D hypercube topology, a single message is multicast from a randomly selected publisher to each subscriber host. Each group comprises a set of subscribers that are randomly assigned to nodes in the overlay. For both of the greedy and split-based routing algorithms, a multicast tree is constructed using the *union* of paths generated by the respective routing approaches. Average normalized lateness values are calculated using the formula in Section 5, and *average physical link stress* values are obtained for each group size by counting the number of times a a message is forwarded over a physical link and dividing the result by the number of unique physical links *involved* in multicasting the message to all subscribers.

Figure 12(a) includes a chart comparing average normalized lateness values in each routing algorithm, and suggests that the split-based greedy approach reduces lateness values. Figure 12(b) shows a comparison of link stress, where the split-based approach is *usually* slightly lower in the *average physical link stress* involved in multicasting a single message. In some cases, however, it is possible for link stress to be increased, if the mapping between *physical paths* and *logical links* is such that there is a larger intersection of *physical links*. This is more likely to occur in larger group sizes since more *k-ary n-cube* edges must be traversed in order to multicast the message to all subscribers. Result from these experiments imply that average lateness can be reduced without significant change to the physical link stress.

7. Implementation Issues

The implementation of the overlay system features a set of "supernodes", that are responsible for maintaining



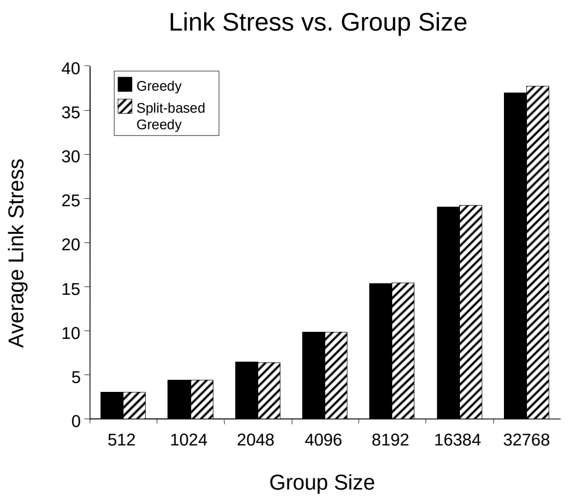


Figure 12. (a) Lateness and (b) link stress versus group size

a database of logical node IDs assigned to corresponding end-host IP addresses. That is, each host joining the system contacts one of a number of known supernodes that assigns a new logical ID. The supernode responsible for a new host could be a default and globally-known machine, or it could be one of a set of known machines selectable by a joining host. In either case, supernodes can communicate with one another to redistribute the set of (client [1]) hosts they manage. It is desirable to have each supernode manage approximately the same number of clients, since each supernode needs to communicate with clients to reconfigure routing tables due to newly-joining and/or departing hosts.

Initially, each node ID is randomly and uniformly selected such that a host has a one-to-one mapping of its IP address to a unique logical identifier. Supernodes exchange control messages amongst one another to maintain a consistent view of all hosts in the system. When a host wishes to leave the system it contacts its target supernode (or the default machine), which communicates with other supernodes to maintain a consistent view of system membership.

Supernodes collectively maintain information about the active k-ary n-cube topology based on the number of hosts, m, in the system. Depending on the current value of m, a joining or leaving host may trigger a reconfiguration of the overlay topology. That is, if the change in the value of m leads to a switch of the best-case *M-region*, then all supernodes dispatch control messages to existing clients to reorganize the logical layout of the system for routing purposes. These control messages include tables of mappings of IP addresses to node IDs for neighbors of each client in the new overlay topology. For a k-ary n-cube with $k > 2$, a control message includes remappings for up to $2n$ neighbors. However, over time it may be possible for clients to cache a number of routing tables for different topologies, only requiring supernodes to inform them of which one is active and any changes to individual entries. Since Figure 6 indicates relatively few distinct topologies for up to several million hosts, a client should not have to cache too much information. In fact, for a constant number of *M-regions*, a client still only stores $O(\log M)$ routing table entries. It is also worth noting that, to avoid oscillating between two *M-regions*, each supernode maintains a timer that tracks the reconfiguration rate. Only if it is absolutely necessary to move to a new topology, because m is greater than the current value of M, do we allow a reconfiguration before a minimum interval of time has expired. This avoids reconfiguring a system too rapidly, because the costs of doing so could outweigh the benefits. Overall, we envision the reconfiguration of a topology to be a fairly course-grained event, especially as the number of hosts in the system increases.

Once a client has registered with the system, it may be used as an intermediate host for routing and data processing purposes. An any point in time, it may contact a supernode to declare a new data channel on which it wishes to publish information. Alternatively, a client may contact a supernode to obtain a list of currently available channels, to which it may subscribe. If a client wishes to subscribe to a channel, it sends a subscription request to the channel publisher. The overlay topology can serve as a multipurpose communication substrate, as in the combined Scribe/Pasty system [4, 17], by supporting the transportation of *control messages* as well as data streams. As a result, a subscribing client does not need to contact a supernode to register with a data channel. Instead, it routes a request to the publisher, specifying its QoS requirements on the corresponding stream.

[1]Here, a client refers to any end-host in the system, and may even include a supernode if so desired.

As with Scribe, a publishing node need not be the actual source of information. This would be the case if several *real producers* of data all generated information for the same channel. In such a case, a publisher could be a *rendezvous* point for multiple data streams that need to be delivered across a single channel to subscribing nodes. Here, each producer might contact a supernode to declare interest in the same channel. Based on the physical proximities of each producer, a rendezvous point can be established at some node in the active overlay topology using a triangulation technique that minimizes the cost between each producer and the rendezvous node. This is certainly an area for further study and we intend to evaluate various approaches that are as decentralized as possible, while minimizing the amount of control state needed to be exchanged.

Additional control messages exchanged between logically neighboring clients includes physical proximity information. That is, for a given overlay topology, each client exchanges with its neighbors information about the communication costs between the corresponding hosts. This information includes values such as end-to-end latency, as well as available link bandwidth. Monitoring agents running on each host sample latency and bandwidth information at some predetermined (possibly adaptive) rate. Care must be taken to ensure the system monitors link costs fast enough to capture significant changes (e.g., when cross traffic over the Internet suddenly consumes a large amount of bandwidth and/or affects latency). Similarly, care must be taken not to sample link measurements too frequently, thereby leading to numerous redundant control messages being exchanged between hosts.

8. Related Work

A number of systems have been developed in the recent years that focus on methods for distributing data among hosts participating in an overlay network. The taxonomy of these systems lies largely along the extremities of two dimensions: scalability and QoS awareness.

For example, systems such as Pastry/Scribe, Chord, CAN, and Tapestry provide a lookup service that can scale to thousands of peers. However, these works are different in the formulation of their overlay topologies as well as their application. CAN identifies logical host positions with coordinates in a Cartesian space, whereas Chord arranges hosts in a logical ring. To further illustrate the difference in application, consider Pastry, which provides the functionality for routing arbitrary messages between hosts in the overlay. In contrast, CAN and Chord are systems designed for distributed storage and retrieval of data in large P2P systems.

Although it is desirable for a distributed system to scale to support a large number of hosts, it is essential to systems such as NARADA that physical proximities are taken into account in order to provide QoS constrained service to subscribers. For the NARADA protocol, this involves constructing a *mesh* using information collected from random probe messages. In this case, scalability is sacrificed to obtain lower delay penalties and a higher rate of meeting service constraints.

SkipNet is a recent work which builds scalable overlay topologies and includes methods for organizing data by string names so that routing locality can be guaranteed with respect to administrative domains [15]. Data can either be located at a particular node or distributed uniformly over a tree of names. Although SkipNet is similar to the work presented in this report in that scalable overlay topologies are used for construction of the logical network, our work focuses more on optimization of a regular graph structure with respect to average hop count and adaptive re-assignment of positions in the logical overlay.

While a number of other projects [1, 10, 5, 12, 2, 18, 3, 14] have implemented scalable multicast solutions at the application-level, most have either not addressed per-subscriber QoS requirements, or take a very different approach to ours. For example, OMNI [2] implements an overlay multicast network infrastructure that attempts to minimize the latency of real-time data using a two-tier approach. While facing similar objectives to our system, OMNI differs in that it divides end-hosts into two classes: (1) special Multicast Service Nodes (or MSNs) and, (2) subscribing clients. MSNs form an overlay backbone and each client subscribes with a single MSN. Routing trees connecting MSNs are continuously adapted, based on network conditions and the distribution of clients. In our approach, we treat all end-hosts as equivalent peers, forming a unifying k-ary n-cube overlay for the purposes of data delivery. We select paths through this overlay based on network latencies, bandwidth availability and per-subscriber service requirements and adapt the overlay topology based on the number of end-hosts present in the system.

Our *k-ary n-cube* topology seeks to relinquish the tradeoff between scalability and QoS awareness. In fact, the *k-ary n-cube* system generalizes the method of consistent hashing in a graph-theoretic perspective, which is crucial for analysis of average hop count between pairs of nodes in the overlay as derived in this work. By using the *k-ary n-cube* as a model for the overlay network and combining this structure with policies that take *physical* proximity into account (greedy routing, adaptive node re-assignment, etc.), a system can be developed that is QoS aware and scalable to the degree of lookup services that employ consistent hashing.

9. Conclusions and Future Work

This work analyzes the use of k-ary n-cubes for routing real-time media streams between publishers and potentially hundreds of thousands of subscribers, in keeping with per-subscriber service constraints. We analyze the minimal average hop-count between any pair of nodes in a k-ary n-cube and use this as the basis for constructing an overlay topology for real-time transport of data. This work extends the concept of *realities*, first described in the context of CAN [16], to determine M-regions. These are regions describing, for a given number of physical hosts in a system (m) the optimal values for k and n in the corresponding overlay structure. Using our greedy algorithm, which leverages physical proximity information, we are able to route over such topologies with significantly lower delay penalties than existing approaches based on peer-to-peer routing.

Future work includes further analysis and simulation of extensions to the algorithm outlined for adaptive reassignment of subscriber nodes in logical space. We plan to investigate how changing the overlay structure affects per-subscriber QoS constraints for real-time media streams. Specifically, the algorithm presented in Figure 9 will be augmented to allow a subscriber host to be displaced from its position in the overlay a constant number of times, and experimental analysis will be conducted via simulation.

References

[1] S. Banerjee, B. Bhattacharjee, and C. Kommareddy. Scalable application layer multicast. In *Proceedings of ACM SIGCOMM*, August 2002.

[2] S. Banerjee, C. Kommareddy, K. Kar, B. Bhattacharjee, and S. Khuller. Construction of an efficient overlay multicast infrastructure for real-time applications. In *IEEE INFOCOM*, San Francisco, CA, April 2003.

[3] M. Castro, P. Druschel, A. Kermarrec, A. Nandi, A. Rowstron, and A. Singh. Splitstream: High-bandwidth multicast in cooperative environments. In *19th ACM Symposium on Operating Systems Principles*, 2003.

[4] M. Castro, P. Druschel, A. Kermarrec, and A. Rowstron. SCRIBE: A large-scale and decentralized application-level multicast infrastructure. *IEEE Journal on Selected Areas in communications (JSAC)*, 2002.

[5] Y. Chawathe. *Scattercast: An Architecture for Internet Broadcast Distribution as an Infrastructure Service*. PhD thesis, University of California, Berkeley, December 2000.

[6] A. A. Chien. A cost and speed model for k-ary n-cube wormhole routers. *IEEE Transactions on Parallel and Distributed Systems*, 9(2):150–162, 1998.

[7] A. A. Chien and J. H. Kim. Planar-adaptive routing: Low-Cost adaptive networks for multiprocessors. *Journal of the ACM*, 42(1):91–123, 1995.

[8] Y.-H. Chu, S. G. Rao, and H. Zhang. A case for end system multicast. In *ACM SIGMETRICS 2000*, pages 1–12, Santa Clara, CA, June 2000. ACM.

[9] W. J. Dally. Performance analysis of k-ary n-cube interconnection networks. *IEEE Transactions on Computers*, 39(6):775–785, 1990.

[10] P. Francis. Yoid: Extending the multicast Internet architecture, 1999. On-line documentation: http://www.aciri.org/yoid/.

[11] B. Gedik and L. Liu. PeerCQ: A decentralized and self-configuring peer-to-peer information monitoring system. In *Int. Conf. on Dist. Comp. Systems (ICDCS)*, 2003.

[12] J. Jannotti, D. Gifford, K. Johnson, M. Kaashoek, and J. O'Toole. Overcast: Reliable multicasting with an overlay network. In *Proceedings of the 4th Symposium on Operating Systems Design and Implementation*, October 2000.

[13] M. Kang, C. Yu, H. Y. Youn, B. Lee, and M. Kim. Isomorphic strategy for processor allocation in k-ary n-cube systems. *IEEE Transactions on Computers, Vol. 52, No. 5*, pages 645–657, May 2003.

[14] D. Kostic, A. Rodriguez, J. Albrecht, and A. Vahdat. Bullet: High bandwidth data dissemination using an overlay mesh. In *19th ACM Symposium on Operating Systems Principles*, October 2003.

[15] N.J.A.Harvey, M. Jones, S. Saroiu, M. Theimer, and A. Wolman. Skipnet: A scalable overlay network with practical locality properties. In *Proceedings of the 4th USENIX Symposium on Internet Technologies and Systems*, March 2003.

[16] S. Ratnasamy, P. Francis, M. Handley, R. Karp, and S. Schenker. A scalable content-addressable network. In *Proceedings of the 2001 conference on applications, technologies, architectures, and protocols for computer communications*. ACM Press, 2001.

[17] A. Rowstron and P. Druschel. "Pastry: Scalable, distributed object location and routing for large-scale peer-to-peer systems. In *IFIP/ACM International Conference on Distributed Systems Platforms*, November 2001.

[18] S. Shi and J. Turner. Routing in overlay multicast networks. In *IEEE INFOCOM*, June 2002.

[19] Shoutcast: http://www.shoutcast.com.

[20] I. Stoica, R. Morris, D. Karger, M. F. Kaashoek, and H. Balakrishnan. Chord: A scalable peer-to-peer lookup service for Internet applications. In *Proceedings of the 2001 conference on applications, technologies, architectures, and protocols for computer communications*. ACM Press, 2001.

[21] E. W. Zegura, K. L. Calvert, and S. Bhattacharjee. How to model an internetwork. In *IEEE INFOCOM*, volume 2, pages 594–602, San Francisco, CA, March 1996.

[22] B. Zhao, J. Kubiatowicz, and A. Joseph. Tapestry: An infrastructure for fault-tolerant wide-area location and routing. Technical Report UCB/CSD-01-1141, Computer Science Division, U. C. Berkeley, Apr. 2001.