

## Practice Problem Set 2: Integrity (MACs & Signatures)

March 19, 2017

### MAC security

The following is the security game for message authentication codes (MACs).

- The game master chooses a random  $k$  to the MAC.
- The adversary has access to a  $MAC_k()$  oracle, that computes MACs on messages of the adversary's choice.
- The adversary has access to a  $VER_k(,)$  oracle, that Verifies that a tag  $t$  is a valid MAC on a message  $m$ ; both  $m$  and  $t$  can be chosen by the adversary.
- The adversary wins if outputs  $m^*, t^*$  such that  $m^*$  has not been queried to the  $MAC_k()$  oracle and  $VER_k(m^*, t^*) = 1$ .

We say the MAC is secure if no (polynomial time) adversary can win this game with probability better than about  $\frac{1}{2^\ell}$ , where  $\ell$  is the length of the MAC tag.

### Signature security

The following is the security game for digital signatures.

- The game master chooses a random asymmetric key  $(PK, SK)$  for the signature and gives  $PK$  to the adversary.
- The adversary has access to a  $Sign_{SK}()$  oracle, that computes signatures on messages of the adversary's choice.
- The adversary wins if outputs  $m^*, \sigma^*$  such that  $m^*$  has not been queried to the  $Sign_{SK}()$  oracle and  $VER_{PK}(m^*, \sigma^*) = 1$ .

We say the digital signature is secure if no (polynomial time) adversary can win this game with non-negligible probability.

### Questions.

**Exercise 1.** Show that

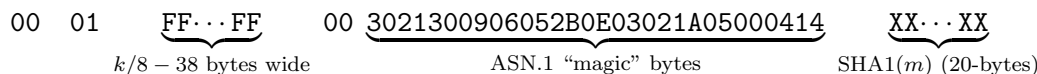
$$MD5(k||m)$$

is not a secure MAC. That is, present an attack that allows the adversary to win the MAC security game described above.

(Hint: Recall the length extension attack from Lab 1.)

**Exercise 2.** On February 23, 2017, researchers announced that they found a collision in SHA1. The collision was two files  $f_1$  and  $f_2$  such that  $SHA1(f_1) = SHA1(f_2)$ . See [shattered.io](http://shattered.io).

Consider PKCS #1 v1.5 RSA digital signatures. To sign a message  $m$ , the message is hashed and padded as shown below to obtain the padded value  $p(m)$ :



Then, the signature is

$$p(m)^d \pmod N$$

where  $N$  is the RSA modulus,  $d$  is the secret RSA decryption exponent, and  $e$  is the public encryption exponent. Thus, the public key is  $(e, N)$  and the secret key is  $(d, N)$ .

Present an attack that proves that PKCS #1 v1.5 RSA is not a secure digital signatures when SHA1 is used as the hash function. You must use the two files  $f_1$  and  $f_2$  in your attack.

**Exercise 3.** Dr Snakeoil markets a new product that he claims protects the integrity of messages.

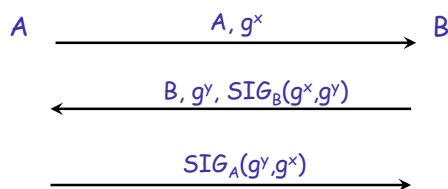
This product requires Alice and Bob to share a secret key 128-bit key  $k$  that they will use to authenticate every message they send.

Then, if Alice wants to send a message  $m$  to Bob, she breaks the message  $m$  up into blocks  $m_1, m_2, \dots, m_n$  and outputs the tag  $t_1, t_2, \dots, t_i, \dots, t_n$  where each  $t_i = HMAC_k(m_i)$ .

Alice then sends  $m_1, m_2, \dots, m_n, t_1, t_2, \dots, t_n$  to Bob.

1. Write down the verification algorithm for this scheme.
2. Prove that this scheme is not a secure MAC.

**Exercise 4.** (Key exchange). Consider the following diffie-helman key-exchange protocol. Recall that the shared key is  $k = g^{xy}$ , and that  $SIG_A(m)$  is the (public-key) digital signature on message  $m$  signed by the secret key of  $A$ . Suppose that  $A, B$  and  $E$  all know each other's correct public keys.



After this protocol runs, Alice and Bob send each other messages encrypted and authenticated under the key  $k$ .

Suppose there is a man-in-the-middle adversary  $E$  that can intercept, add, drop, and the modify the traffic that  $A$  sends to  $B$ .

1. Suppose that Alice and Bob are running software that has the following implementation flaw: it forgets to validate digital signatures and just accepts any messages it receives as valid.  
Show how Eve  $E$  can launch an man-in-the-middle attack, where she can read any of the encrypted and authenticated messages that Alice sends Bob.

2. Now suppose  $E$  can launch an “identity misbinding attack” where she convinces  $B$  that he shares the key  $k = g^{xy}$  with  $E$ , while convincing  $A$  that she shares  $k = g^{xy}$  with  $B$ . Explain exactly how  $E$  does this. (What messages does she send, and to who?) [Note, with this attack,  $E$  doesn't know  $k = g^{xy}$  but  $B$  considers anything sent by  $A$  as coming from  $E$ ]
3. Give an example of a scenario where your identity misbinding attack might create problems.