Problem Set #1 (Logic & Sets)
Due: Thursday, September 13 by 3:30 pm

To be completed individually. No late submission will be accepted

Reading: Chapter 1, particularly sections 1.3 and 1.4.
Chapter 2, and Chapter 3, section 3.1.

Problems:

1. Let \( A = \{ \text{all odd integers bigger than 11} \} \) and \( B = \{ 15, 25, 35 \} \)
   i. Is \( A \subseteq B \) ? Explain why or why not.
   ii. Is \( B \subseteq A \) ? Explain why or why not.

2. Give an example of sets \( A, B \) and \( C \) where,
   \( A \cap B = A \cap C \) but \( B \neq C \).

3. Give an example of sets \( A \) and \( B \) where,
   \( A \cap B \subseteq A-B \).

4. Prove that for any sets \( A \) and \( B \), \( A \cap B \subseteq A \cup B \).

5. Let \( A = \{ v, w,x,y,z, \} \), \( B = \{ v,w \} \), \( C = \{ v,w,x,y \} \), \( D = \{ y \} \), \( E = \) the empty set
   For each pair of sets above tell if one is a subset of the other.

6. Use the same sets \( A,B,C,D,E \) above and answer the following questions.
   i. Name 2 of the sets above which are disjoint.
   ii. Name 2 of the sets above which are not disjoint, but neither is a subset of the other.
   iii. Define 2 sets \( F \) and \( G \) which are subsets of \( C \) but not of \( B \).
   iv. Define a set \( H \) which is disjoint from \( C \) and a subset of \( A \).
   v. Define a set \( I \) whose intersection with \( A \) is equal to \( B \), but \( I \) and \( B \) are not equal.

7. Write the negation of these statements.
   Do NOT use the word “not” in your answer, as in “It is not the case that \( x+4 = 7 \).”
   (a) \( (n > 4) \land (7 = n + 2) \)
   (b) If \( a > b \) and \( b \geq c \), then \( a > c \) is an integer.
   (c) \( \exists a \forall b (a + b = b) \)
8. Prove that

\[ 2^n + 2^n = 2^{n+1} \]

for all \( n \geq 1 \).

Note: There are several ways to do this, one is a proof by induction, but any convincing proof method will do.