Problem Set #4
Due: Thursday, October 27

Reading: Chapters 6 and 7 of the on-line text, but only pages 79-91.
Chapter 6, section 1 of the regular text, pages 260-265. Do not read section 2, but you might want to read sections 3 and 4 eventually.

Problems:

1. Let $B_{m,n}$ be the complete bipartite graph with $m$ vertices on the left side and $n$ on the right. How many edges does $B_{m,n}$ have? For which $m,n$ does $B_{m,n}$ have an Euler cycle? Explain your reasoning.

2. Give an example of a graph which has one path which is both an Euler cycle and a Hamiltonian cycle.
   (A picture is fine here.)

3. Give an example of a graph which has an Euler cycle and but does not have a Hamiltonian cycle.
   (A picture is fine here.)

4. Consider the following directed graph. Note: Here consider an edge without an arrow on it as two edges, one in each direction.
   \[ a \rightarrow \rightarrow \rightarrow b \leftarrow \leftarrow \leftarrow c \]
   \[ \left| \quad \left| \quad \left| \right. \right. \]
   \[ d \leftarrow \leftarrow \leftarrow e \rightarrow \rightarrow \rightarrow f \]
   i. Write out this graph $G$ as $G = (V,E)$ where you need to specify exactly what the sets $V$ and $E$ are.
   ii. Does this graph contain a cycle? Why or why not?
   iii. Is this graph strongly connected? Strongly connected means that there is a path from every vertex to every other vertex?
   iv. Write the adjacency matrix for this graph.

5. True or false - explain your reasoning.
   For the graph in problem 2, $\forall a \in V \exists b \in V (\exists$ a path from $a$ to $b \lor \exists$ a path from $b$ to $a)$.

6. Page 265, problem 2
7. Prove by induction that there are infinitely many prime numbers. So formally you need to prove the “forall” statement, \((\forall a \in \mathbb{N})(\exists b \in \mathbb{N}) (b > a \land b \text{ is prime}).\)

8. Prove by induction that for any \(n \in \mathbb{N},\)

\[3^0 + 3^1 + 3^2 + ...3^n = (3^{n+1} - 1)/2\]

9. Page 266, problem 17(a)