

Boston University
CAS CS 330
Fall 2006

Lecture 1

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CS 330 course information

RTFM: Read the syllabus and live by it

RTFM stands for: _____



Learn to read the book: it's your friend

why are we here?

- Speed
- Elegance
- To avoid doing really stupid things out in real world

Q: What's really stupid?

A: Spending ~28,000,000 times longer than you have to to sort a database

E.g., database of 1,000,000,000 social security numbers:
radix sort ~18,000,000,000 ops
insertion sort ~500,000,000,000,000 ops

topic 1: multiplication

- How to compute $X \times Y$? Recall: grade school

$$\begin{array}{r} 7325 \\ \times 1956 \\ \hline 43950 \\ + 36625 \\ 65925 \\ 7325 \\ \hline 14327700 \end{array}$$

- n^2 digit multiplications (if X and Y are n digits each)
- n^2 (+ at most n) digit additions

does binary make it easier?

$$\begin{array}{r} 1011 \\ \times 1101 \\ \hline 1011 \\ + 0000 \\ 1011 \\ \hline 1011 \\ 10001111 \end{array}$$

- multiplications are trivial (no need to write out)
- still need to add all 1 digits ($\sim n^2$)

perhaps divide and conquer?

- $X = A 2^{n/2} + B$ $X = \underbrace{100011010111}_A \underbrace{}_B$
- $Y = C 2^{n/2} + D$
- $XY = AC 2^n + (AD + BC) 2^{n/2} + BD$
- Total work: 4 half-size multiplications (recursion!) + $3n$ for 3 additions
- How to find reasonable formula for total work?
- Denote work to multiply two n -bit integers by $T(n)$
- Then $T(n) \leq 4T(n/2) + 3n$

finding the running time

- Know: $T(n) \leq 4T(n/2) + 3n$
- Need to find $T(n)$
 - in particular, how does $T(n)$ compare with n^2 ?
- This problem is known as recurrence
- Postpone it for a bit

better? divide and conquer (Karatsuba 1962)

- $X = A 2^{n/2} + B$
- $Y = C 2^{n/2} + D$
- $XY = AC 2^n + (AD + BC) 2^{n/2} + BD$
- **Idea:** compute $AD + BC$ as $(A+B)(C+D) - AC - BD$
- Total work: 3 half-size multiplications
+ $6n$ for 6 additions/subtractions
- Then $T(n) \leq 3T(n/2) + 6n$

which one is better?

- Grade school: n^2
- Simple divide and conquer: $T(n) \leq 4T(n/2) + 3n$
- Karatsuba divide and conquer: $T(n) \leq 3T(n/2) + 6n$
- $T(n) \leq 4T(n/2) + 3n$ solves to: _____
- $T(n) \leq 3T(n/2) + 6n$ solves to: _____

asymptotic notation review

| $\lim_{n \rightarrow \infty} f(n) / g(n)$ | Notation | Informal Analogue |
|---|-------------------|-------------------|
| $= 0$ | $f \in o(g)$ | $f < g$ |
| ≥ 0 and $< \infty$ | $f \in O(g)$ | $f \leq g$ |
| > 0 and $< \infty$ | $f \in \Theta(g)$ | $f = g$ |
| > 0 and $\leq \infty$ | $f \in \Omega(g)$ | $f \geq g$ |
| $= \infty$ | $f \in \omega(g)$ | $f > g$ |