BU CAS CS 320 (Fall Semester, 2006)
Concepts of Programming Languages

Assignment 2

Out: Tueday, 19 September 2006
Due: Tueday, 26 September 2006

Total: 100 points

Exercise 1 *(50 points)*

1. *For each natural number n, let $vFib(n)$ be the vector $(Fib(n), Fib(n+1))^T$, that is, the transpose of the vector $(Fib(n), Fib(n+1))$. Clearly, we have the following equation for each natural number n.*

$$vFib(n+1) = A \cdot vFib(n), \text{ where } A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

Hence, we have the equation $vFib(n) = A^n \cdot vFib(0)$ for each natural number n. Please use this equation to implement a $\Theta(\log n)$-time and $\Theta(1)$-space procedure in SML that computes the Fibonacci numbers $Fib(n)$.

2. *Please find the last 8 digits of $Fib(buid)$, where buid is the 8-digit number in your BU id.*

Exercise 2 *(50 points) A positive natural number n is congruent if n is the area of a right triangle whose three sides are rational numbers. For instance, 5 is congruent since it is the area of the right triangle with three sides $3/2, 20/3$ and $41/6$; 6 is congruent since it is the area of the right triangle with three sides $3, 4$ and 5 (6 is also the area of the triangle with three sides $7/10, 120/7$ and $1201/70$). For your amusement, the right triangle with the following sides X, Y and Z, which is computed by D. Zagier, attests to 157 being congruent (You can readily verify that $X^2 + Y^2 = Z^2$ and $\frac{1}{2} X \cdot Y = 157$ in SML for the following X, Y and Z).*

$$\begin{aligned} X &= \frac{6803298487826435051217540}{411340519227716149383203} \\ Y &= \frac{411340519227716149383203}{21666555693714761309610} \\ Z &= \frac{224403517704336969924557513090674863160948472041}{8912332268928859588025535178967163570016480830} \end{aligned}$$

Please implement a program in SML to find 12 congruent numbers between 10 and 50. Note that you need to provide at least one right triangle for each congruent number such that the three sides of the right triangle are rationals and the area of the right triangle equals the congruent number.