

BU CAS CS 320 (SPRING SEMESTER, 2002)  
CONCEPTS OF PROGRAMMING LANGUAGES

## Assignment 2

Out: Monday, 28 January 2002

Due: Monday, 4 February 2002

Total: 120 points

**Exercise 1** (40 points) A Braun tree is a binary tree such that for each node the size of its left son equals the size of its right son or 1 plus the size of its right son. Note that the size of a binary tree is the number of branch nodes in the tree.

1. (20 points) Please use mathematical induction to prove that for each natural number  $n$ , there is exactly one Braun tree of size  $n$ .
2. (20 points) Please implement a program in Scheme that returns the height of the Braun tree of size  $n$  when  $n$  is given.

**Exercise 2** (20 points) The Fibonacci sequence is defined as follows.

$$Fib(n) = \begin{cases} 0 & \text{if } n = 0; \\ 1 & \text{if } n = 1; \\ Fib(n-1) + Fib(n-2) & \text{otherwise.} \end{cases}$$

Please show by induction on  $n$  that  $Fib(n) = (\phi^n - \psi^n)/\sqrt{5}$  for  $\phi = (1 + \sqrt{5})/2$  and  $\psi = (1 - \sqrt{5})/2$ .

**Exercise 3** (20 points) Each of the following two procedures defines a method for adding two nonnegative integers in terms of the procedure `inc`, which increases the value of its argument by 1, and `dec`, which decreases the value of its argument by 1.

```
(define (+1 a b)
  (if (zero? a) b (inc (+1 (dec a) b))))
```

```
(define (+2 a b)
  (if (zero? a) b (+2 (dec a) (inc b))))
```

Using the substitution model, please illustrate the process generated by each procedure in evaluating `(+1 4 5)` and `(+2 4 5)`. Are these processes tail-recursive, that is, iterative?

**Exercise 4** (40 points) Given a natural number  $n > 1$ ,  $n$  is square-free if there is no prime number  $p$  such that  $p^2$  divides  $n$ . For instance, 21 is square-free but 24 is not. Please implement a procedure to check if a given natural number is square-free.