BU CAS CS 320 (Spring Semester, 2002)

Concepts of Programming Languages

Assignment 3

Out: Monday, 4 Feburary 2002
Due: Monday, 11 Feburary 2002

Total: 160 points

Exercise 1 *(80 points)*

1. *(60 points) For each natural number n, let $vFib(n)$ be the vector $(Fib(n), Fib(n+1))^T$, that is, the transpose of the vector $(Fib(n), Fib(n+1))$. Clearly, we have the following equation for each natural number n.*

$$vFib(n+1) = A \cdot vFib(n), \text{ where } A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

   Hence, we have the equation $vFib(n) = A^n \cdot vFib(0)$ for each natural number n. Please use this equation to implement a $\Theta(\log n)$-time and $\Theta(1)$-space procedure in Scheme that computes the Fibonacci numbers $Fib(n)$.

2. *(20 points) Please find the last 8 digits of $Fib(buid)$, where buid is the 8-digit number in your BU id.*

Exercise 2 *(10 points) The idea of smoothing a function is an important concept in signal processing. If f is a function and dx is some small number, the the smoothed version of f is the function whose value at x is the average of $f(x - dx)$, $f(x)$ and $f(x + dx)$. Please write a procedure* `smooth` *that computes the smoothed f.*

Exercise 3 *(10 points) If f is a numerical function and n is a natural number, then we can form nth repeated application of f, which is defined to be the function whose value at x is $f(f(\cdots(f(x))\cdots))$, where there are n occurrences of f. Write a procedure that takes a function f and a naturnal number n, and returns a procedure that computes the nth repeated application of f.*

Exercise 4 *(60 points) A positive natural number n is congruent if n is the area of a right triangle whose three sides are rational numbers. For instance,* 5 *is congruent since it is the area of the right triangle with three sides* $3/2, 20/3$ *and* $41/6$*;* 6 *is congruent since it is the area of the right triangle with three sides* $3, 4$ *and* 5 *(*6 *is also the area of the triangle with three sides* $7/10, 120/7$ *and* $1201/70$*). For your amusement, the right triangle with the following sides X, Y and Z, which is computed by D. Zagier, attests to* 157 *being congruent (You can readily verify that $X^2 + Y^2 = Z^2$ and $\frac{1}{2}X \cdot Y = 157$ in Scheme for the following X, Y and Z).*

$$\begin{aligned} X &= \frac{6803298487826435051217540}{411340519227716149383203} \\ Y &= \frac{411340519227716149383203}{21666555693714761309610} \\ Z &= \frac{224403517704336969924557513090674863160948472041}{8912332268928859588025535178967163570016480830} \end{aligned}$$

Please implement a program in Scheme to find 12 *congruent numbers between* 10 *and* 50. *Note that you need to provide at least one right triangle for each congruent number such that the three sides of the right triangle are rationals and the area of the right triangle equals the congruent number.*