

BU CAS CS 320 (SPRING SEMESTER, 2002)  
CONCEPTS OF PROGRAMMING LANGUAGES

## Assignment 4

Out: Monday, 11 February 2002

Due: Tuesday, 19 February 2002

Total: 160 points

**Exercise 1** (20 points) Show that we can represent pairs of natural numbers using only natural numbers and arithmetic operations if we represent the pair  $a$  and  $b$  as the natural number  $2^a 3^b$ . Please give the corresponding definitions of the procedures `cons`, `car` and `cdr`.

**Exercise 2** (25 points) Let us represent a poker card as a pair `(cons s v)`, where  $1 \leq s \leq 4$  represents the suit of the card and  $1 \leq v \leq 13$  represents the value of the card. Please implement a procedure that returns a randomly shuffled deck of 52 distinct cards.

**Exercise 3** (15 points) Please define the following function in a continuation-passing style.

```
(define (map f xs)
  (if (null? xs) '() (cons (f (car x)) (map f (cdr xs)))))
```

**Exercise 4** (20 points) Let `deep-reverse` be the function that takes a given list as its argument and returns as a value the list with the elements reversed and each element deep-reversed if the element is also a list. For instance, reversing the list `((1 2) (3 4))` yields `((3 4) (1 2))`, while deep-reversing it gives `((4 3) (2 1))`.

**Exercise 5** (20 points) What would the interpreter print in response to evaluating each of the following expressions?

```
(list 'a 'b '(c d))
```

```
(list (list 'Abraham))
```

```
(list '(list Abraham))
```

```
(list '(list 'Abraham))
```

```
(cdr '((x1 y1) (x2 y2)))
```

```
(cadr '((x1 y1) (x2 y2)))
```

```
(pair? (caddr '(a beautiful rose)))
```

```
(memq 'red '(red shoes blue socks))
```

```
(memq 'red '((red shoes) (blue socks)))
```

```
(equal? (list 'a 'short 'list) '(a short list))
```

**Exercise 6** (60 points) Suppose that we represent a vector  $v = (v_i)$  as a sequence of numbers, and a matrix  $A = (a_{ij})$  as sequences of vectors (the rows of the matrix). For example, the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

is represented as the following sequence.

```
((1 2 3) (4 5 6) (7 8 9) (10 11 12))
```

For a matrix  $A$  of dimension  $m \times n$ , we use `(cons m (cons n A))` for its representation, where `m`, `n` and `A` are the representations of  $m$ ,  $n$  and  $A$ , respectively.

1. (10 points) Implement a procedure that checks whether its argument is a correct representation of a matrix with dimension information.
2. (20 points) Implement a procedure that transposes a matrix with dimension information. For instance, given the argument below,

```
(4 3 (1 2 3) (4 5 6) (7 8 9) (10 11 12))
```

the procedure should return the following.

```
(3 4 (1 4 7 10) (2 5 8 11) (3 6 9 12))
```

In particular, if you transpose a matrix twice, you obtain the original matrix.

3. (30 points) Implement a procedure that takes two matrices and returns their product. Note that two matrices can be multiplied only if they are of dimensions  $p \times q$  and  $q \times r$  for some natural numbers  $p, q, r$ .