

BU CAS CS 520 (FALL SEMESTER, 2005)
PRINCIPLES OF PROGRAMMING LANGUAGES

Assignment 1

Out: Wednesday, 09 Septmeber 2005

Due: Monday, 30 September 2005

Academic Integrity We pledge strict adherence to the university guidelines.

- All work you turn in must be *your own* unless specified otherwise.
- You are allowed to discuss problems with your classmates but you must write your own code and solutions.
- Please always remeber that every student deserves a chance to achieve a fair grade!

Total: 200 points

Exercise 1 (20 pts) Prove the following statement using mathematical induction.

$(2n + 1)^2 - 1$ is a multiple of 8 for every natural number n .

Exercise 2 (90 pts) Binary trees are defined as follows and we use $B(t)$ for the number of branch node B in t .

binary trees $t ::= E \mid B(t, t)$

Let us define function h on binary trees inductively.

$h(E) = 0$ $h(B(t_l, t_r)) = 1 + \max(h(t_l), h(t_r))$

Evidently, $h(t)$ computes the height of t . Braun trees are binary trees defined as follows.

- E is a Braun tree.
- $B(t_l, t_r)$ is a Braun tree if t_l and t_r are Braun trees and $B(t_r) \leq B(t_l) \leq B(t_r) + 1$.

Please do the following.

- (30 pts) Prove by structural induction on t that for every nonempty Braun tree t , $2^{h(t)-1} \leq B(t) < 2^{h(t)}$ holds.
- (30 pts) Implement in SML a procedure that lists all Braun trees of height n when given a natural number n .

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datatype BraunTree = E | B of BraunTree * BraunTree
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fun listBraunTrees (height: int): BraunTree list = (* your code is here *)
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- (30 pts) In general, what is the number of Braun trees of height n for a given natural number n ? Please justify your answer by mathematical induction.

Exercise 3 (40 points) MacCarthy's 91-function is defined as follows.

$$f_{91}(n) = \begin{cases} 91 & \text{if } n = 91; \\ f_{91}(f_{91}(n + 11)) & \text{if } 0 \leq n < 91 \text{ or } 91 < n \leq 100; \\ f_{91}(n - 10) & \text{if } n > 100; \end{cases}$$

Please use inductive reasoning to prove that $f_{91}(n) = 91$ for every natural number n .

Exercise 4 (50 points) Please implement in SML the Game-of-24 described as follows.

Given four natural numbers n_1, n_2, n_3 and n_4 , one chooses two of them and generates a rational number r_1 using either addition, subtraction, multiplication or division; one mixes r_1 with the remaining two numbers and chooses two of them to generate a rational number r_2 using either addition, subtraction, multiplication or division; one then takes r_2 and the last remaining number to get a rational number r_3 using addition, subtraction, multiplication, or division; if there is a way to make r_3 equal to 24, then we say that (n_1, n_2, n_3, n_4) is a good quad. For instance, $(10, 10, 4, 4)$ is a good quad since we have

$$(10 * 10 - 4) / 4 = 24$$

Similarly, $(5, 7, 7, 11)$ is a good quad since we have

$$(5 - 11/7) * 7 = 24$$

Game of 24 is a game that determines whether four given natural numbers are a good quad.

Please implement a program in SML that takes four given natural numbers and returns 1 or 0 according to whether the four natural numbers are a good quad; if they are a good quad, the program should also print out an arithmetic expression that attests to their being a good quad.