



BU CAS CS 520 (FALL SEMESTER, 2008)
PRINCIPLES OF PROGRAMMING LANGUAGES

Assignment 2

Out: Wednesday, 17 September 2008
Due: Tuesday, 30 September 2008

Total: 120 pts

Exercise 1 (5 pts) *Exercise 5.2.1 on page 59 in the textbook.*

Exercise 2 (5 pts) *Exercise 5.2.4 on page 61 in the textbook.*

Exercise 3 (5 pts) *Exercise 5.2.5 on page 62 in the textbook.*

Exercise 4 (5 pts) *Exercise 5.2.7 on page 63 in the textbook.*

Exercise 5 (30 pts) *Assume $t \equiv_{\alpha} t'$, that is, $nf_{\alpha}(t) \equiv nf_{\alpha}(t')$. Then we have the following.*

1. $FV(t) = FV(t')$.
2. If $y \notin \text{vars}(t) \cup \text{vars}(t')$, then $t[y/x] \equiv_{\alpha} t'[y/x]$ for any variable x .

Exercise 6 (30 pts) *Given a λ -abstraction $\lambda x.t$ and a variable $y \notin FV(t)$, please prove that $\lambda x.t$ and $\lambda y.t[x \mapsto y]$ are α -equivalent, that is, $nf_{\alpha}(\lambda x.t)$ and $nf_{\alpha}(\lambda y.t[x \mapsto y])$ are syntactically the same.*

Exercise 7 (40 pts) *Let us use the following datatypes `term` and `term1` for representing untyped (pure) λ -terms and their α -normal forms, respectively:*

`datatype term =`

`TmVar of string | TmLam of (string, term) | TmApp of (term, term)`

`datatype term1 =`

`TmVar1 of string | TmInd1 of int | TmLam1 of term1 | TmApp1 of (term1, term1)`

- (10 pts) *Please implement a function `nf_alpha` that translates a given λ -term into its alpha-normal form:*

```
extern fun nf_alpha (t: term): term1
```

- (30 pts) *Please implement a function `subst` of the following type:*

```
extern fun subst (t: term, x: string, s: term): term
```

Given a term t , a variable x and a term s , `subst(t, x, s)` should return $t[x \mapsto s]$.