Assignment 2

Out: Wednesday, 17 September 2008
Due: Tuesday, 30 September 2008

Total: 120 pts

Exercise 1 (5 pts) Exercise 5.2.1 on page 59 in the textbook.

Exercise 2 (5 pts) Exercise 5.2.4 on page 61 in the textbook.

Exercise 3 (5 pts) Exercise 5.2.5 on page 62 in the textbook.

Exercise 4 (5 pts) Exercise 5.2.7 on page 63 in the textbook.

Exercise 5 (30 pts) Assume $t \equiv \alpha t'$, that is, $nf_{\alpha}(t) \equiv nf_{\alpha}(t')$. Then we have the following.

1. $FV(t) = FV(t')$.

2. If $y \notin \text{vars}(t) \cup \text{vars}(t')$, then $t[y/x] \equiv_{\alpha} t'[y/x]$ for any variable $x$.

Exercise 6 (30 pts) Given a $\lambda$-abstraction $\lambda x.t$ and a variable $y \notin FV(t)$, please prove that $\lambda x.t$ and $\lambda y.t[x \mapsto y]$ are $\alpha$-equivalent, that is, $nf_{\alpha}(\lambda x.t)$ and $nf_{\alpha}(\lambda y.t[x \mapsto y])$ are syntactically the same.

Exercise 7 (40 pts) Let us use the following datatypes `term` and `term1` for representing untyped (pure) $\lambda$-terms and their $\alpha$-normal forms, respectively:

```plaintext
datatype term =
  TmVar of string | TmLam of (string, term) | TmApp of (term, term)

datatype term1 =
  TmVar1 of string | TmInd1 of int | TmLam1 of term1 | TmApp1 of (term1, term1)
```

- (10 pts) Please implement a function `nf_alpha` that translates a given $\lambda$-term into its alpha-normal form:

  ```plaintext
  extern fun nf_alpha (t: term): term1
  ```

- (30 pts) Please implement a function `subst` of the following type:

  ```plaintext
  extern fun subst (t: term, x: string, s: term): term
  ```

  Given a term $t$, a variable $x$ and a term $s$, $\text{subst}(t,x,s)$ should return $t[x \mapsto s]$.