BU CAS CS 520 (Fall Semester, 2009)
Principles of Programming Languages

Assignment 7

Out: Tuesday, 22 November 2009
Due: Thursday, 10 December 2009

Total: 60 points + 20 extra points

Exercise 1 *(20 points) The pure simply typed λ-calculus can also be formed as follows in the Curry style. The syntax is given below, where A ranges over base types:*

$$\begin{array}{llll} \textit{terms} & t & ::= & x \mid \lambda x.t \mid t_1(t_2) \\ \textit{types} & T & ::= & A \mid T_1 \rightarrow T_2 \\ \textit{contexts} & \Gamma & ::= & \emptyset \mid \Gamma, x : T \end{array}$$

The typing rules are given below:

$$\frac{\Gamma(x) = T}{\Gamma \vdash x : T} \ \textbf{(T-var)}$$

$$\frac{\Gamma, x : T_1 \vdash t : T_2}{\Gamma \vdash \lambda x.t : T_1 \rightarrow T_2} \ \textbf{(T-abs)}$$

$$\frac{\Gamma \vdash t_1 : T_1 \rightarrow T_2 \quad \Gamma \vdash t_2 : T_1}{\Gamma \vdash t_1(t_2) : T_2} \ \textbf{(T-app)}$$

We now extend the pure simply typed λ-calculus in the Curry style with the following language constructs:

$$\begin{array}{llll} \textit{types} & T & ::= & \ldots \mid \textit{cont}(T) \\ \textit{terms} & t & ::= & \ldots \mid \textbf{callcc}(t) \mid \textbf{throw}(t_1, t_2) \end{array}$$

and the following typing rules:

$$\frac{\Gamma \vdash t : \textit{cont}(T) \rightarrow T}{\Gamma \vdash \textbf{callcc}(t) : T} \ \textbf{(T-callcc)}$$

$$\frac{\Gamma \vdash t_1 : \textit{cont}(T_1) \quad \Gamma \vdash t_2 : T_1}{\Gamma \vdash \textbf{throw}(t_1, t_2) : T_2} \ \textbf{(T-throw)}$$

Let us define as follows two functions $|\cdot|$ and $\|\cdot\|$ from types to types, where ans is some fixed base type.

$$\begin{array}{rcl} \|A\| & = & A \\ \|T_1 \rightarrow T_2\| & = & \|T_1\| \rightarrow |T_2| \\ \|\textit{cont}(T)\| & = & \|T\| \rightarrow \textit{ans} \\ |T| & = & (\|T\| \rightarrow \textit{ans}) \rightarrow \textit{ans} \end{array}$$

In addition, let us define two functions $CPS(\cdot)$ and $CPS_0(\cdot,\cdot)$ mutually recursively as follows, where k ranges over λ-terms.

$$
\begin{array}{rcl}
CPS_0(k, x) & = & k(x) \\
CPS_0(k, \lambda x.t) & = & k(\lambda x.CPS(t)) \\
CPS_0(k, t_1(t_2)) & = & CPS_0(\lambda x_1.CPS_0(\lambda x_2.x_1(x_2)(k), t_2), t_1) \\
CPS_0(k, \mathbf{callcc}(t)) & = & CPS_0(\lambda x.x(k)(k), t) \\
CPS_0(k, \mathbf{throw}(t_1, t_2)) & = & CPS_0(\lambda x_1.CPS_0(x_1, t_2), t_1) \\
CPS(t) & = & \lambda k.CPS_0(k, t) \;\; (k \notin FV(t))
\end{array}
$$

Assume that $\Gamma \vdash t : T$ is derivable. Please show that $\|\Gamma\| \vdash CPS(t) : |T|$ is derivale, where $\|\Gamma\|$ is the context such that $\|\Gamma\|(x) = \|\Gamma(x)\|$ for each $x \in \mathbf{dom}(\|\Gamma\|) = \mathbf{dom}(\Gamma)$. Note that this question requires no knowledge of the dynamic semantics of **callcc** *and* **throw**.

Exercise 2 *(40 points + 20 extra points) See the posted file* `assignment7.dats` *for the rest of this assignment.*