

## Assignment 3

Out: Monday, 14 February 2005

Due: Tuesday, 22 February 2005

Total: 120 points

**Exercise 1** (30 points) Given a context-free grammar  $G = (V, \{a, b\}, R, S)$ , where  $V$  is

$$S \rightarrow aSb \mid SS \mid \epsilon$$

Please give a short and precise description of the language  $L(G)$  and then prove it by rigorous induction.

**Exercise 2** (30 points) Given a language  $L$ , let  $L_{half} = \{x \mid xy \in L \text{ for some } y \text{ satisfying } |x| = |y|\}$ . Prove that  $L_{half}$  is regular if  $L$  is.

**Exercise 3** (30 points) Let  $L = \{x\#y \mid x, y \in \{0, 1\}^* \text{ and } x \neq y\}$ . Prove that  $L$  is a context-free language.

**Exercise 4** (30 points) Let  $L = \{xy \mid x, y \in \{0, 1\}^* \text{ and } |x| = |y| \text{ and } x \neq y\}$ . Prove that  $L$  is a context-free language.