Problem 1. Sign up for the course mailing list, as described on the webpage.

Problem 2. Write down the truth tables for NOT ($\neg$), AND ($\land$), OR ($\lor$), XOR ($\oplus$), NAND, and IMPLIES ($\Rightarrow$) operators.

Problem 3. Write the following statements using the formal notation ($\exists, \forall, \neg, ...$):

(a) For every student in our class there is a seat

Extra credit: Every student in our class has a seat which no other students are seating in.

(b) Not every seat in our class has a student (write it in two ways: first, using the $\neg$ operator, and then without it)

(c) For every action $a$ there is an equal but opposite reaction $b$.

Problem 4. Translate into English: $\forall x \in \mathbb{N} \exists y, z \in \mathbb{R}. (y^2 = z^2 = x) \land (y \neq z)$. Is the statement true? Why or why not? (Here justification should be very brief.)

Problem 5. Express OR ($\lor$) operator in terms of NAND operators.

Problem 6. Express XOR ($\oplus$) operator in terms of NAND operators.

Problem 7. Express NOR (NOT OR: $\neg(a \lor b)$) operator in terms of NAND operators.

Below, assume that any Boolean function can be implemented as a Boolean circuit with $\neg, \lor, \land$ operators ($\forall f \exists C \forall x. C(x) = f(x)$, where $f$ is a Boolean function; $C$ is a Boolean circuit built out of $\neg, \lor, \land$ gates; and $x$ is an input).

Problem 8. Is NOR operator universal? (Can you express $\neg, \lor, \land$ operators in terms of NOR? If yes, show how; if not, which operator cannot be expressed in terms of NOR? Why? Is there a shorter way to do get to the answer than doing each of the three operators separately?)

Problem 9. Same as above, but now about XOR ($\oplus$): Is XOR operator universal? (Can you express $\neg, \lor, \land$ operators in terms of XOR? If yes, show how; if not, which operator cannot be expressed in terms of XOR? Why?)