There are 7 problems on the exam. The first and last are mandatory, and you may eliminate any one of problems 2 – 6 by drawing an X through it. Problem 1 is worth 10 points, and all other problems are worth 18. Please write in pen if possible. If you need more room, use the back of the sheet and tell me this on the front sheet of that problem.

**Problem 1 (10 pts -- Mandatory).** (True/False) Write True or False to the LEFT of the statement.

1. The assignment double x = (double) 4; is an example of a narrowing conversion. **False**
2. In the boolean expression ( A && B ), if the expression A evaluates to true, the second expression B will not be evaluated. **False**
3. The break statement quits the current iteration of a loop and starts the next one. **False**
4. Local variables in a class can never be defined as public. **True**
5. Generic classes in Java can only be instantiated by reference types. **True**
6. From the point of view of $\Theta(...)$ complexity, the worst-case complexity of inserting a new element into an array is the same whether it is ordered or unordered. **True**
7. If $f(n) = \sim g(n)$ then it must be the case that $f(n) = \Theta(g(n))$. **True**
8. The average case complexity of an algorithm can never be worse (i.e., take longer) than the worst-case complexity. **True**
9. The scope of a member of a class is the same whether it is declared static or not static. **True**
10. Strings in Java are immutable, that is, you can not change the characters in a String once you have created it. **True**
**Problem 4.** Suppose you have a stack for integers with the normal `push()` and `pop()` operations.

Now suppose you have a sequence of `push(...)` and `pop()` operations in the following form:

```java
// 0 or more statements of the form System.out.print( pop() + " ");
push( 0 );
// 0 or more statements of the form System.out.print( pop() + " ");
push( 1 );
// 0 or more statements of the form System.out.print( pop() + " ");
push( 2 );

etc.
// 0 or more statements of the form System.out.print( pop() + " ");
push( 9 );
// 0 or more statements of the form System.out.print( pop() + " ");
```

That is, you push the digits 0 ... 9 in order, and then before, in between, and after these pushes, you do some number of pops (possibly 0).

Which of the following sequences could possibly be printed out by such a sequence (there may be more than one)? Write “possible” or “not possible” next to each one.

(A) 4 6 8 7 5 3 2 9 0 1 **NOT Possible!**

(B) 8 7 6 5 4 3 2 1 0 9 **Possible**

(C) 4 3 2 1 0 5 6 7 8 9 **Possible**

(D) 9 8 7 6 5 4 3 2 1 0 **Possible**
Problem 5. For each function \( f \) from the following list of functions, determine the \textit{simplest} function \( g \) that makes the equality \( f(N) = \Theta(g(N)) \) true. Represent your answer as an equality (e.g., \( p(N) = \Theta(N^2) \)). The first one is done for you as a model for the rest of your answers.

\[
\begin{align*}
a(N) &= 2^N + N^2 - 5 & a(N) &= \Theta(2^N) \\
b(N) &= \log(N^3) + \log(2N) & b(N) &= \Theta(\log(N)) \\
c(N) &= 8N + 5\sqrt{N} & c(N) &= \Theta(N) \\
d(N) &= 12N + N\sqrt{N} + N\log(N) & d(N) &= \Theta(N^{1.5}) \\
e(N) &= \log^3(N) + \log(N) + 5 & e(N) &= \Theta(\log^3(N)) \\
f(N) &= 50 + \frac{1}{N} & f(N) &= \Theta(1)
\end{align*}
\]
Problem 6. This is a question about Java and has two parts.

(A) What is the output of the following lines of Java code?

```java
int a = 9;
double b = a / 2.0;
int c = a / 2;
double d = a / 2;
String e = c + "c";
a = a % 6;

System.out.println("a = " + a); // a = 3
System.out.println("b = " + b); // b = 4.5
System.out.println("c = " + c); // c = 4
System.out.println("d = " + d); // d = 4.0
System.out.println("e = " + e); // e = 4c
```

(B) Consider the program shown below.

```java
public class Problem3 {
    public static void main(String[] args) {
        int g = 1;
        if (g > 0) {
            int h = foo(g);
        }
        System.out.println(______); // blank for part a
    }

    public static int foo(int r) {
        int s;
        for (s = 0; s < 5; s++) {
            System.out.println("bar");
        }
        return ______; // blank for part b
    }
}
```

a. Which of the variables (g, h, r, s) could be used in the first blank?

   g

b. Which of the variables (g, h, r, s) could be used in the second blank?

   r, s
**Problem 7. (MANDATORY)** For the following algorithms, for the first set, state the worst-case and average-case time (in terms of $\Theta$) for performing the indicating operation. For the second set, simply give the $\Theta(...N...)\) estimate for the indicated situation. The number of items stored in each data structure is N. Assume for each row that you are using the algorithms we discussed in class.

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Worst Case $\Theta(......)$</th>
<th>Average Case $\Theta(......)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Find an element in an unordered array.</td>
<td>$\Theta( N )$</td>
<td>$\Theta( N )$</td>
</tr>
<tr>
<td>Delete an element from an ordered array.</td>
<td>$\Theta( N )$</td>
<td>$\Theta( N )$</td>
</tr>
<tr>
<td>Dequeue a key from a queue implemented as a ring buffer with resizing</td>
<td>$\Theta( 1 )$</td>
<td>$\Theta( 1 )$</td>
</tr>
<tr>
<td>Enqueue a key into a queue implemented as a ring buffer with resizing</td>
<td>$\Theta( N )$</td>
<td>$\Theta( N )$</td>
</tr>
<tr>
<td>Find an element in an ordered array.</td>
<td>$\Theta( \log(N) )$</td>
<td>$\Theta( \log(N) )$</td>
</tr>
<tr>
<td>Partition (as in Quicksort) a array of size N.</td>
<td>$\Theta( N )$</td>
<td>$\Theta( N )$</td>
</tr>
<tr>
<td>Merge two ordered lists, each of size N/2.</td>
<td>$\Theta( N )$</td>
<td>$\Theta( N )$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Algorithm and Data Input</th>
<th>$\Theta(......)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sort an already-sorted list using Selection Sort</td>
<td>$\Theta( N^2 )$</td>
</tr>
<tr>
<td>Sort an already-sorted list using Insertion Sort</td>
<td>$\Theta( N )$</td>
</tr>
<tr>
<td>Sort an already-sorted list using Mergesort</td>
<td>$\Theta( N \log(N) )$</td>
</tr>
<tr>
<td>Sort an already-sorted list using Quicksort</td>
<td>$\Theta( N^2 )$</td>
</tr>
</tbody>
</table>