Name:____________________

CS 132—First Midterm

You must do 3 of the 4 problems on this test. The exam will be graded out of 20 points: each problem is worth 6 points, and I will give you 2 points if you successfully write your name on the top of the exam. Please draw a large X through the problem you are NOT doing. You may do all the problems, but you MUST tell me which one I am not including in your score.

Problem One. (6 points) Given \( u = (1,2,0), \ v = (0, 1, 4), \) and \( w = (-3, 2, 7), \) give the following:

(a) \( 2u + 3v - w \)

(b) \( 3(-u + 2w - 3(u - v)) \)

\[
\begin{align*}
&(a) \quad 2(1,2,0) + 3(0,1,4) - (-3,2,7) \\
&= (2,4,0) + (0,3,12) - (-3,2,7) \\
&= (2,7,12) - (-3,2,7) \\
&= (5,5,5) \\
&(b) \quad 3\left( (1,0,2) + (0,2,8) - 3((1,2,0) - (0,1,4)) \right) \\
&= 3\left[ (-1,0,2) - 3(1,1,-4) \right] \\
&= 3(-4,-3,20) \\
&= \begin{pmatrix}
-30 \\
-30 \\
78
\end{pmatrix}
\]

\]
Problem Two. (6 points) Use Gauss-Jordan Elimination to solve the following system of equations; state whether there is a unique solution (and give it), no solution, or an infinite number of solutions (give the schema of these solutions).

\[ x_1 + 2x_2 + 3x_3 = -1 \]
\[ -x_1 - 2x_2 - 2x_3 = 2 \]
\[ 2x_1 + 4x_2 + 8x_3 = 4 \]
Problem Three. (6 points) Use Gaussian Elimination to solve the following system of equations; state whether there is a unique solution (and give it), no solution, or an infinite number of solutions (give the schema of these solutions).

\[ 2x_1 + x_2 + x_3 = -1 \]
\[ 3x_1 - x_2 + x_3 = 2 \]
\[ 2x_1 + 4x_2 + 8x_3 = 4 \]
**Problem Four.** (6 points) Describe with a diagram (in the case of 2 space) or best you can with words and diagrams (in the case of 3 space) the *solution* to each of the following sets of equations (each is separate):

(a) In 2 space:

\[ y = x - 5 \]

\[ x - y = 5 \]

(b) In 2 space:

\[ x + y = 5 \]

\[ 2x + 2y = 6 \]

(c) In 2 space:

\[ x + y = 5 \]

\[ 2x - y = 0 \]

(d) In 3 space:

\[ x - y = 0 \]

(e) In 3 space:

\[ x + y = 5 \]

\[ z = 1 \]