Problem One. Consider the following signal:

(a) Using the function \( \sin(\ldots) \) and parameters \( A, f, \) and \( \phi \) (for amplitude, frequency, and phase), give as many significantly different expressions for this signal as you can; by “significantly different” I mean using the concepts of negative and positive amplitude, negative and positive frequency, and various phases. Phases 3\( \pi/2 \) and -\( \pi/2 \) would be considered significantly different expressions, but for example 2\( \pi \), 4\( \pi \), 6\( \pi \), etc. would not be. Your goal is to show me you understand the various interesting ways of specifying this signal.

Solution: We can vary the frequency (positive and negative), phase (negative and positive), and the amplitude (negative and positive):

\[
\begin{align*}
\sin(2\pi*20*t + \pi/2) & \quad - \sin(2\pi*20*t - \pi/2), \\
\sin(2\pi*20*t - 3\pi/2) & \quad - \sin(2\pi*20*t + 3\pi/2) \\
\sin(2\pi* - 20*t + \pi/2) & \quad - \sin(2\pi* - 20*t - \pi/2) \\
\sin(2\pi* - 20*t - 3\pi/2) & \quad - \sin(2\pi* + 20*t + 3\pi/2) \\
\end{align*}
\]

[ Another possibility, which I did not ask for, is to use frequencies above the Nyquist Limit, instead of negative frequencies:

\[
\begin{align*}
\sin(2\pi* 44120*t + \pi/2) & \quad - \sin(2\pi* 44120*t - \pi/2) \\
\sin(2\pi* 44120*t - 3\pi/2) & \quad - \sin(2\pi* 44120*t + 3\pi/2) \\
\end{align*}
\]

(b) Do the same problem as (a) but use phasor notation instead of \( \sin(\ldots) \), and do not use negative amplitudes (i.e., use only positive and negative frequencies and various phases). Again, your answers should be “significantly different.”

Solution: Only the first column in (a) would be translated into exponential notation, so:

\[
\begin{align*}
e^{i*2\pi*20*t + \pi/2} & \quad e^{i*2\pi*20*t - 3\pi/2} \\
e^{i*2\pi* -20*t + \pi/2} & \quad e^{i*2\pi* -20*t - 3\pi/2} \\
\end{align*}
\]
Problem Two. Consider the following “attack-hold-decay” amplitude envelope, which is similar to the one you developed in homework two, except that there is an additional parameter which gives the amplitude of the hold portion of the envelope:

![Amplitude Envelope](image)

(a) Give a “piece-wise” mathematical definition of the function which modifies a signal according to this envelope.

\[ y = \begin{cases} 
A \times \frac{x}{B} & \text{for } x < B \\
A & \text{for } B \leq X \leq C \\
A \times 2^{-(x-C)/h} & \text{otherwise} 
\end{cases} \]

Essentially, you just take the exponentialAHD function from HW 02, and multiply all values by A.

(b) Give a Python implementation of this function, similar to what you did in homework two.

```python
def exponentialAHDMidterm(i, A, B, C, h):
    B = B*SR
    C = C*SR
    if(i < B):
        return A*i/B
    elif(i < C):
        return A
    else:
        return A*2**(-(i-C)/(SR*h))
```
Problem Three. In this problem we assume, as usual, a sample rate of 44100 and consider the relationship between window size and the frequencies of integral frequencies in the context of the Fourier Transform. Consider the following (integral) wave in a window of 2205 samples:

(a) Give the frequency of this wave as a “window frequency” $F_W$ and an “absolute frequency,” i.e., in Hz.

**Solution:** $F_W = 3$ $f_{abs} = 60$ Hz

(b) What is the smallest frequency detectable in this window? Give as a window frequency and an absolute frequency in Hz.

**Solution:** Smallest (i.e., least) frequency is $F_W = 1$ $f_{abs} = 20$ Hz

(c) The Nyquist Limit will put an upper bound on the frequencies that can be detected with this window of 2205 samples. What is the window frequency (e.g., $k*f$, where $f$ is the fundamental frequency of the window) of the highest positive frequency detectable? Give this frequency in relative terms (specifying $k$) and also in Hz.

**Solution:** In terms of window frequencies, the Nyquist Limit in a window of size $W$ is always $W/2$. So we have a limit of $2205/2 = 1102.5$, but since window frequencies must be integers, that means $k = F_W = 1102$. This corresponds to $f_{abs} = 1102*20 = 22040$ Hz.

(d) Generalizing the previous questions, if we have a window consisting of $W$ samples, what is the frequency in Hz of the fundamental frequency (give as a function of $W$)?

**Solution:** Fundamental frequency in window is $44100/W$ Hz.

(e) What is the highest frequency (less than the Nyquist Limit) detectable by a window of size $W$?

**Solution:** ceiling($W/2 - 1$)
Problem Four. Consider a window of length $W = 4410$ containing a signal created from 3 component sine waves as shown:

```
In [9]: displaySignal(makeSignal([[20,0.5,0],[40,0.3,3.1415],[60,0.2,0]], 4410,'Samples'))
```

(a) Draw the spectrum of this signal (i.e., the graph of frequency against amplitude).

If you were to plot absolute amplitude, you would have a scale of 0 - 32767 on the Y axis.
(b) Suppose you were to run the Discrete Sine Transform on this signal. What would be the output, assuming it outputs frequencies from 0 up to (but not including) the Nyquist Limit? (You may of course abbreviate the large number of frequency bins with zero amplitudes, but do show the largest and smallest frequencies output.)

Solution: (showing all the non-zero frequency bins):

<table>
<thead>
<tr>
<th>Freq</th>
<th>Amp</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.5</td>
</tr>
<tr>
<td>40</td>
<td>-0.3</td>
</tr>
<tr>
<td>60</td>
<td>0.2</td>
</tr>
</tbody>
</table>

// or could show 32767/2 as amplitude, etc.

// advancing phase by pi is same as negative amp!

(c) Repeat (b) but showing the output for the complex Fourier Transform, assuming it outputs all 4410 frequency bins; indicate the correspondence between frequencies over the Nyquist Limit and negative frequencies. (You will of course abbreviate the large number of frequency bins with zero amplitudes.)

<table>
<thead>
<tr>
<th>Freq</th>
<th>Amp</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.25</td>
</tr>
<tr>
<td>40</td>
<td>0.15</td>
</tr>
<tr>
<td>60</td>
<td>0.1</td>
</tr>
<tr>
<td>3990</td>
<td>0.1</td>
</tr>
<tr>
<td>3970</td>
<td>0.15</td>
</tr>
<tr>
<td>3950</td>
<td>0.25</td>
</tr>
</tbody>
</table>

// same as frequency -20

// same as frequency -40

// same as frequency -60
Problem Five (Mandatory Essay). Suppose we want to determine the unknown fundamental frequency of a signal X, presented in a window of length \( W = 4410 \) samples. To this point we could imagine three ways to do this:

(a) Use the Zero-Crossing Rate algorithm;
(b) Use the Auto-Correlation algorithm; or
(c) Apply the Fourier Transform and examine the spectrum that results.

Discuss the advantages and disadvantages of each of these approaches to the problem of pitch determination of such a signal.

Solution not provided…. talk to me about your answer if you wish.