CS480/CS680 Problem Set 1: 2D Transforms
Due: September 27 at beginning of lecture.

1. (20 points) Prove that the product of two successive reflections about either of the coordinate axes is equivalent to a single rotation about the coordinate origin. Be sure to handle all possible pairs of successive reflections.

2. (20 points) Prove that scaling and translation do not commute in general. We are given homogeneous transformation matrices for translation $T(t_x, t_y)$ and scaling $S(s_x, s_y)$. Under what special conditions will translation and scaling commute?

3. (20 points) Derive the homogeneous transform matrix for scaling along an arbitrary axis defined by a line. The line end points are $[x_1, y_1, 1]^T$ and $[x_2, y_2, 1]^T$.

4. (20 points) Prove that in general, the product of $N$ 2D rotations can be rewritten as a single rotation matrix; i.e.,

$$R(\theta_1)R(\theta_2)R(\theta_3)\ldots R(\theta_N) = R(\sum_{i=1}^{N} \theta_i).$$

5. (20 points) There are $M$ 2D objects. Each has its own Cartesian reference frame. The $i^{th}$ object’s reference frame is centered at a point $c_i$, and its frame’s $x$ and $y$ axes are defined by the orthonormal vectors $\mathbf{x}_i$ and $\mathbf{y}_i$ respectively. Give the series of homogeneous transforms needed to transform from the $i^{th}$ to the $j^{th}$ Cartesian reference frame. Write out the series of specific $3 \times 3$ homogeneous transform matrices in the order they are applied.

6. (CS680 required 20 points, CS480 extra credit 10 points) We can specify a 2D affine transformation by considering the location of a small number of points both before and after the points have been transformed. How many points are needed to specify the transform uniquely? Are there any special conditions that these points must satisfy? Outline an algorithm for estimating the affine transform given such a set of points.