

Recursive Backtracking II: Map Coloring

- Using just four colors (e.g., red, orange, green, and blue), we want color a map so that no two bordering states or countries have the same color.
- Sample map (numbers show alphabetical order in full list of state names):



- This is another example of a problem that can be solved using recursive backtracking.

Applying the Template to Map Coloring

```
boolean findSolution(n, perhaps other params) {  
    if (found a solution) {  
        this.displaySolution();  
        return true;  
    }  
    for (val = first to last) {  
        if (this.isValid(val, n)) {  
            this.applyValue(val, n);  
            if (this.findSolution(n + 1, other params)) {  
                return true;  
            }  
            this.removeValue(val, n);  
        }  
    }  
    return false;  
}
```

<i>template element</i>	<i>meaning in map coloring</i>
n	
found a solution	
val	
isValid(val, n)	
applyValue(val, n)	
removeValue(val, n)	

Applying the Template to Map Coloring

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            this.applyValue(val, n);  
            if (this.findSolution(n + 1, other params)) {  
                return true;  
            }  
            this.removeValue(val, n);  
        }  
    }  
    return false;  
}
```

<i>template element</i>	<i>meaning in map coloring</i>
n	state number
found a solution	
val	
isValid(val, n)	
applyValue(val, n)	
removeValue(val, n)	

Applying the Template to Map Coloring

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    if (found a solution) {  
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            this.applyValue(val, n);  
            if (this.findSolution(n + 1, other params)) {  
                return true;  
            }  
            this.removeValue(val, n);  
        }  
    }  
    return false;  
}
```

<i>template element</i>	<i>meaning in map coloring</i>
n	state number
found a solution	state number > num of last state
val	state
isValid(val, n)	
applyValue(val, n)	
removeValue(val, n)	

Applying the Template to Map Coloring

```
boolean findSolution(n, perhaps other params) {  
    if (found a solution) {  
        this.displaySolution();  
        return true;  
    }  
    for (val = first to last) {  
        if (this.isValid(val, n)) {  
            this.applyValue(val, n);  
            if (this.findSolution(n + 1, other params)) {  
                return true;  
            }  
            this.removeValue(val, n);  
        }  
    }  
    return false;  
}
```

<i>template element</i>	<i>meaning in map coloring</i>
n	state number
found a solution	state number > num of last state
val	color (iterates over the four colors)
isValid(val, n)	
applyValue(val, n)	
removeValue(val, n)	

Applying the Template to Map Coloring

```
boolean findSolution(n, perhaps other params) {  
    if (found a solution) {  
        this.displaySolution();  
        return true;  
    }  
    for (val = first to last) {  
        if (this.isValid(val, n)) {  
            this.applyValue(val, n);  
            if (this.findSolution(n + 1, other params)) {  
                return true;  
            }  
            this.removeValue(val, n);  
        }  
    }  
    return false;  
}
```

<i>template element</i>	<i>meaning in map coloring</i>
n	state number
found a solution	state number > num of last state
val	color (iterates over the four colors)
isValid(val, n)	no bordering states have the color
applyValue(val, n)	
removeValue(val, n)	

Applying the Template to Map Coloring

```
boolean findSolution(n, perhaps other params) {  
    if (found a solution) {  
        this.displaySolution();  
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            this.applyValue(val, n);  
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                return true;  
            }  
            this.removeValue(val, n);  
        }  
    }  
} return false;  
}
```

<i>template element</i>	<i>meaning in map coloring</i>
n	state number
found a solution	state number > num of last state
val	color (iterates over the four colors)
isValid(val, n)	no bordering states have the color
applyValue(val, n)	apply the color to the state
removeValue(val, n)	

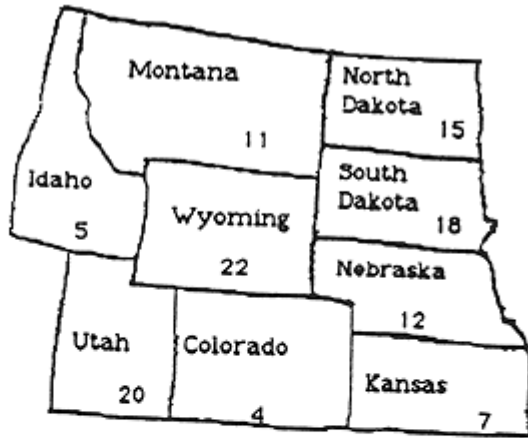
Applying the Template to Map Coloring

```
boolean findSolution(n, perhaps other params) {  
    if (found a solution) {  
        this.displaySolution();  
        return true;  
    }  
    for (val = first to last) {  
        if (this.isValid(val, n)) {  
            this.applyValue(val, n);  
            if (this.findSolution(n + 1, other params)) {  
                return true;  
            }  
            this.removeValue(val, n);  
        }  
    }  
}   
return false;  
}
```

<i>template element</i>	<i>meaning in map coloring</i>
n	state number
found a solution	state number > num of last state
val	color (iterates over the four colors)
isValid(val, n)	no bordering states have the color
applyValue(val, n)	apply the color to the state
removeValue(val, n)	remove the color from the state

Map Coloring Example

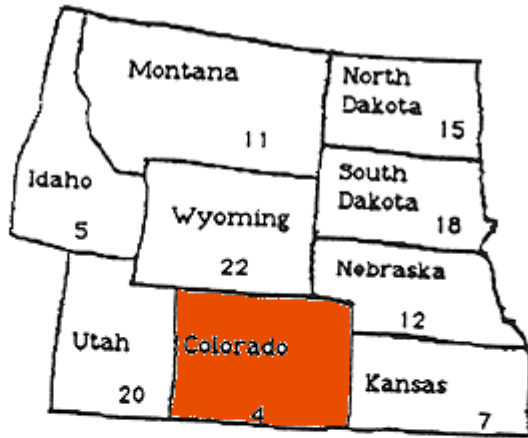
consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.

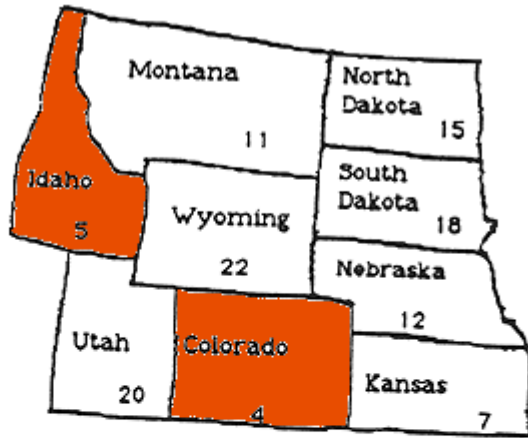


We color Colorado through
Utah without a problem.

Colorado: red

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



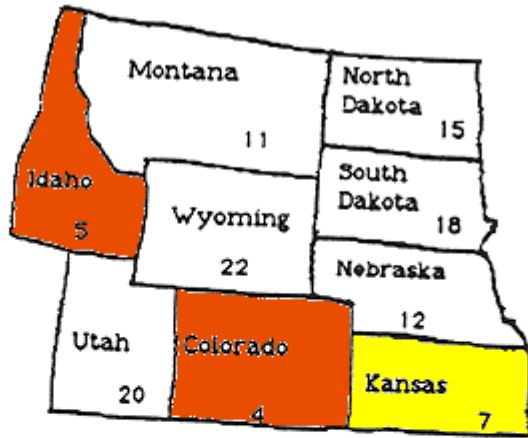
We color Colorado through Utah without a problem.

Colorado: red

Idaho: red

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

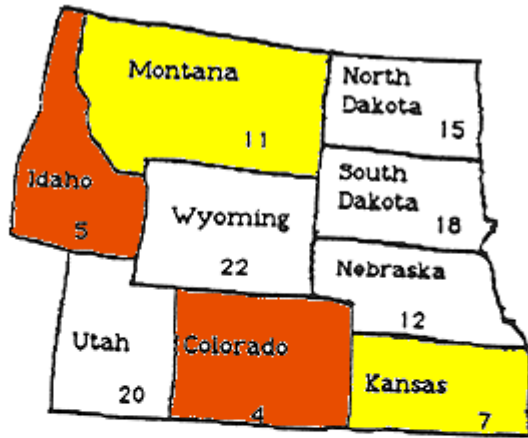
Colorado: red

Idaho: red

Kansas: yellow

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.

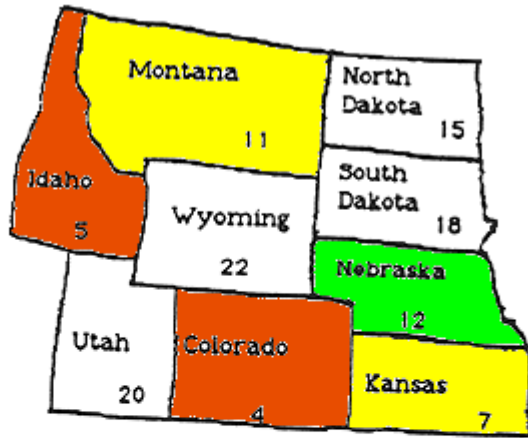


We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

Colorado: red

Idaho: red

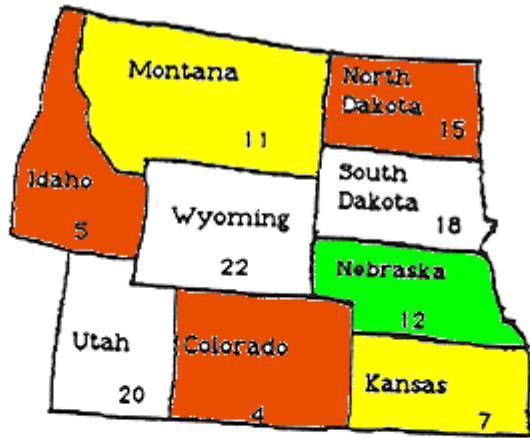
Kansas: yellow

Montana: yellow

Nebraska: green

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.

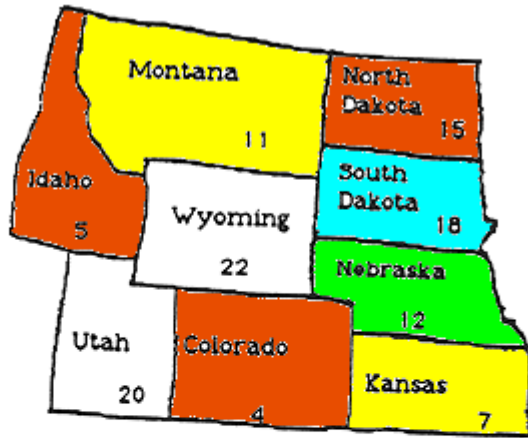


We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.

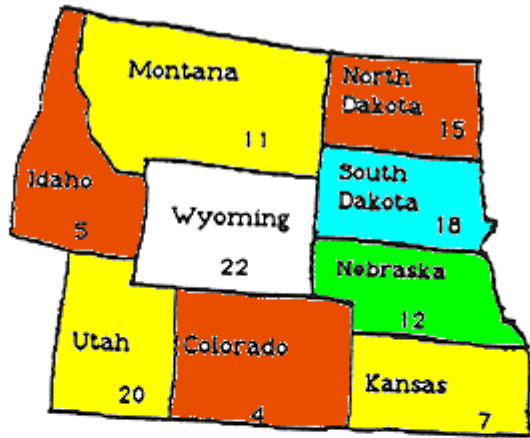


We color Colorado through
Utah without a problem.

Colorado: red
Idaho: red
Kansas: yellow
Montana: yellow
Nebraska: green
North Dakota: red
South Dakota: blue

Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.

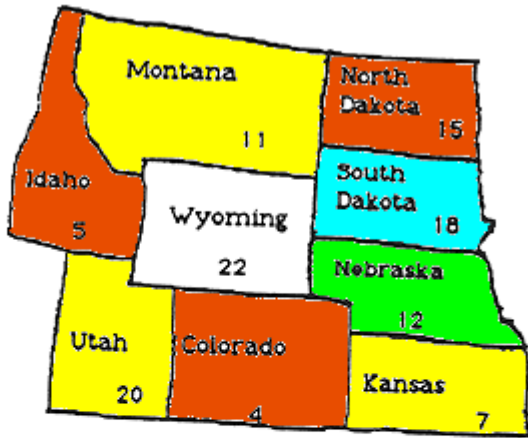


We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue
- Utah: yellow

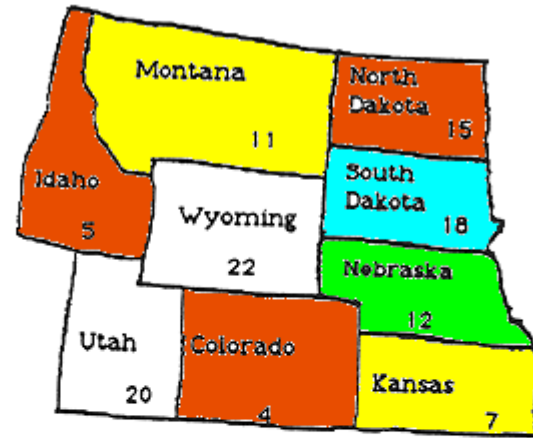
Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

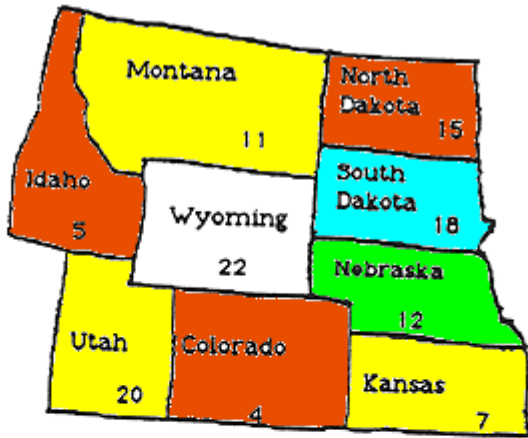
- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue
- Utah: yellow



No color works for Wyoming, so we backtrack...

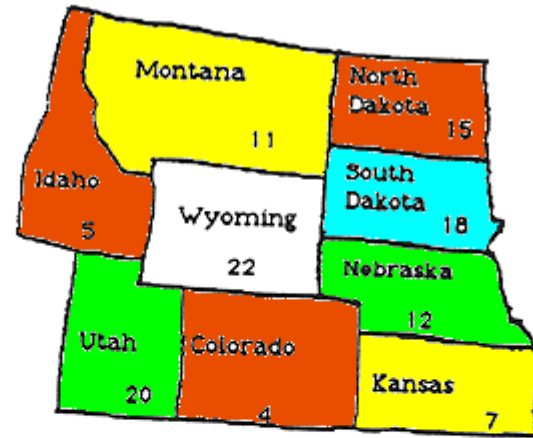
Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue
- Utah: yellow

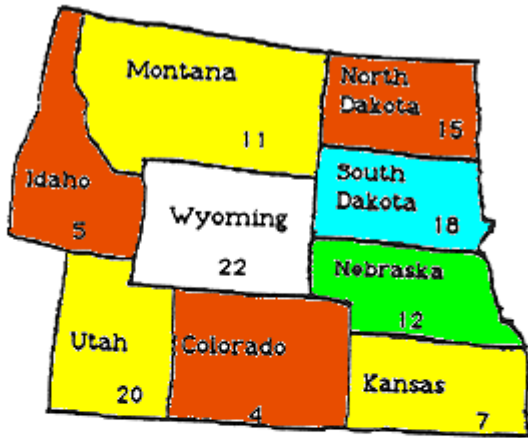


No color works for Wyoming, so we backtrack...

Color Utah green.

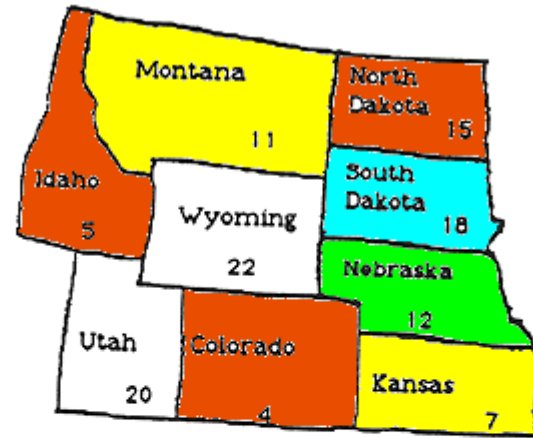
Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue
- Utah: yellow

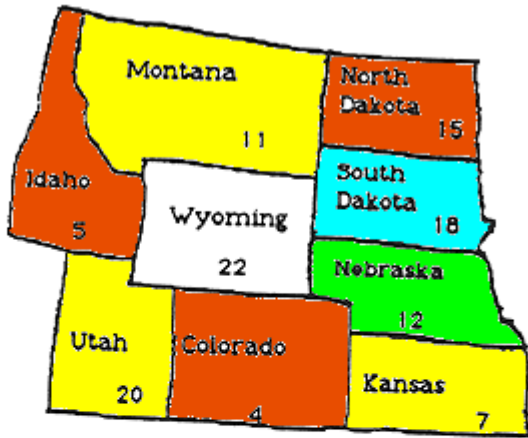


No color works for Wyoming, so we backtrack...

- Color Utah green.
- No color works for Wyoming.
- Backtrack to Utah.

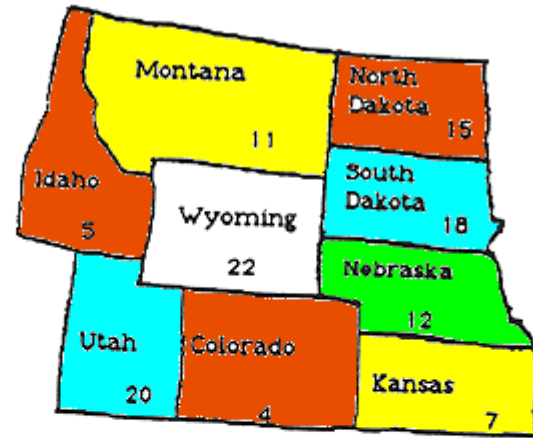
Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

Colorado: red
Idaho: red
Kansas: yellow
Montana: yellow
Nebraska: green
North Dakota: red
South Dakota: blue
Utah: yellow

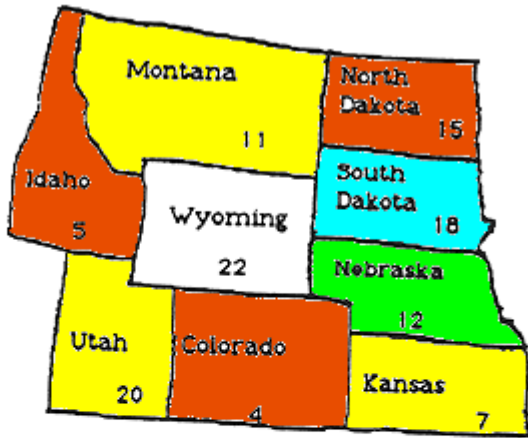


No color works for Wyoming, so we backtrack...

Color Utah green.
No color works for Wyoming.
Backtrack to Utah.
Color Utah blue.

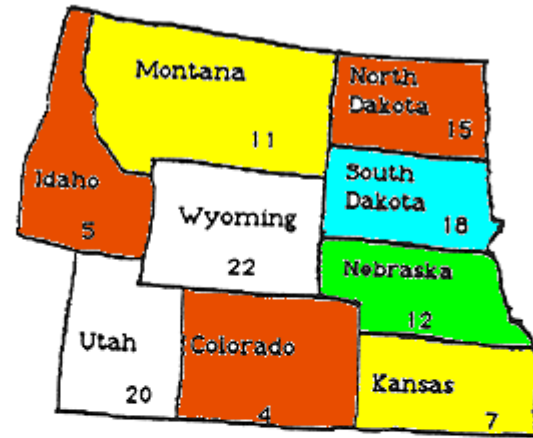
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consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

Colorado: red
Idaho: red
Kansas: yellow
Montana: yellow
Nebraska: green
North Dakota: red
South Dakota: blue
Utah: yellow

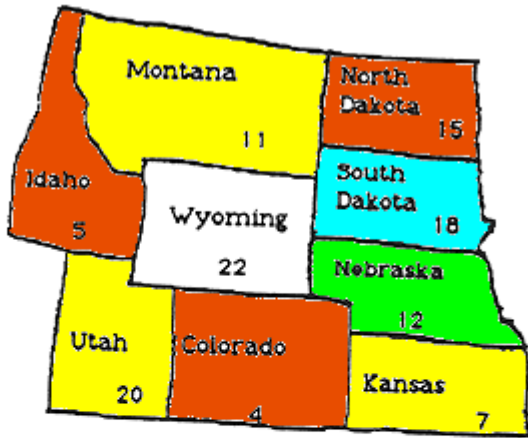


No color works for Wyoming, so we backtrack...

Color Utah green.
No color works for Wyoming.
Backtrack to Utah.
Color Utah blue.
No color works for Wyoming.
Backtrack to Utah.

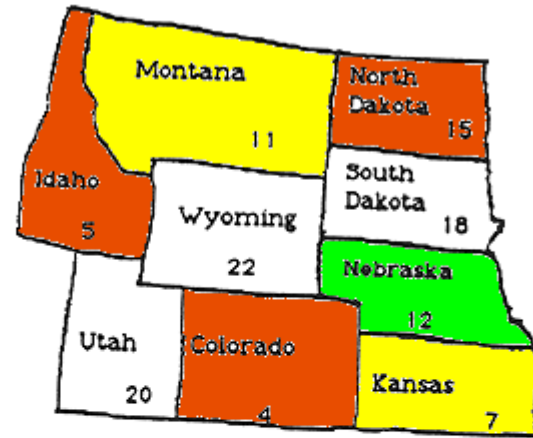
Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue
- Utah: yellow

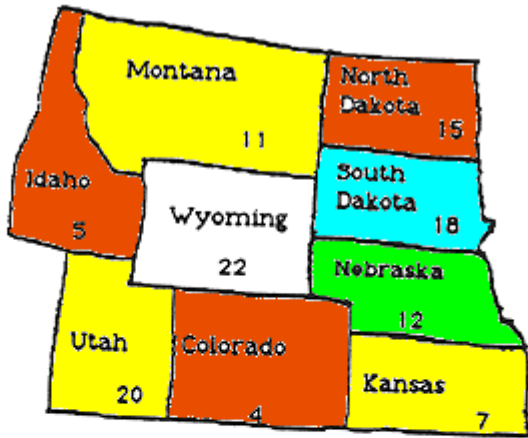


No color works for Wyoming, so we backtrack...

- Color Utah green.
- No color works for Wyoming.
- Backtrack to Utah.
- Color Utah blue.
- No color works for Wyoming.
- Backtrack to Utah.
- No colors left to try for Utah.
- Backtrack to South Dakota.

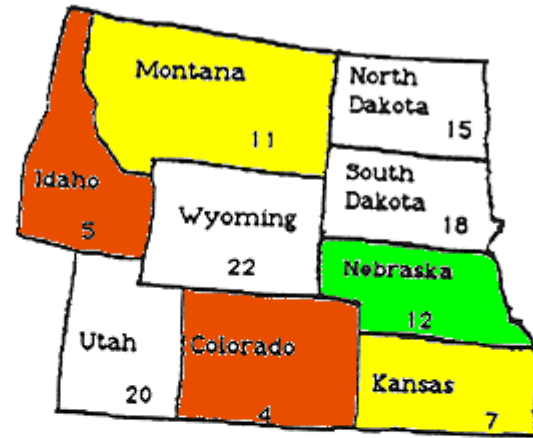
Map Coloring Example

consider the states in alphabetical order. colors = { red, yellow, green, blue }.



We color Colorado through Utah without a problem.

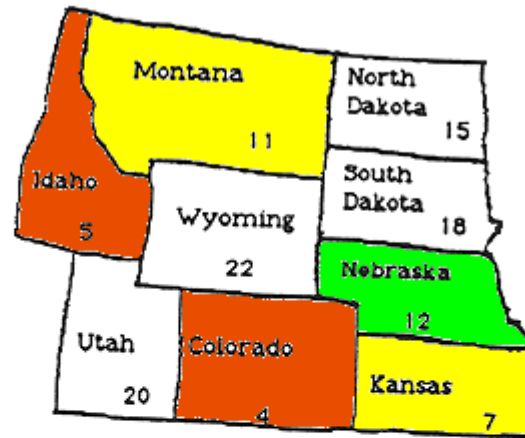
- Colorado: red
- Idaho: red
- Kansas: yellow
- Montana: yellow
- Nebraska: green
- North Dakota: red
- South Dakota: blue
- Utah: yellow



No color works for Wyoming, so we backtrack...

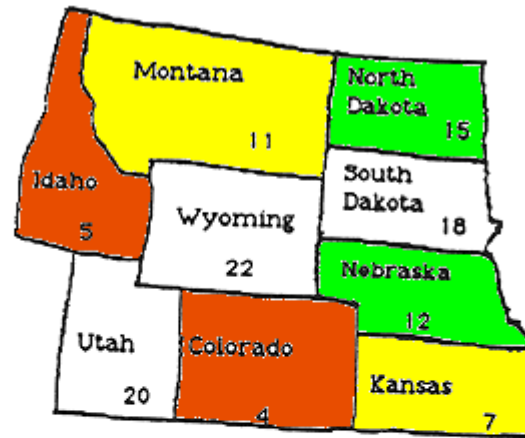
- Color Utah green.
- No color works for Wyoming. Backtrack to Utah.
- Color Utah blue.
- No color works for Wyoming. Backtrack to Utah.
- No colors left to try for Utah. Backtrack to South Dakota.
- No colors left to try for SD. Backtrack to North Dakota.

Map Coloring Example (cont.)



Now we can complete
the coloring:

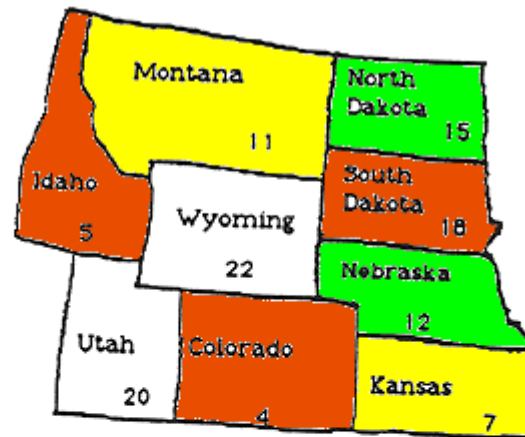
Map Coloring Example (cont.)



Now we can complete
the coloring:

North Dakota: green

Map Coloring Example (cont.)

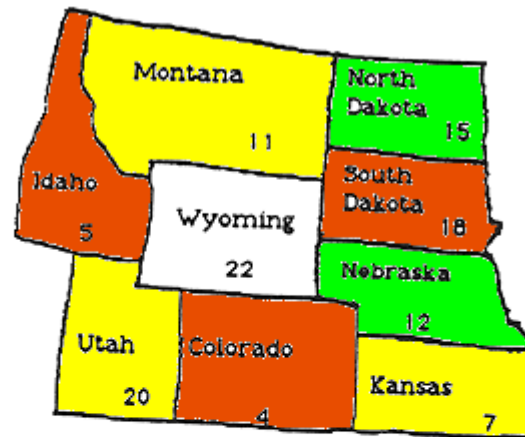


Now we can complete
the coloring:

North Dakota: green

South Dakota: red

Map Coloring Example (cont.)



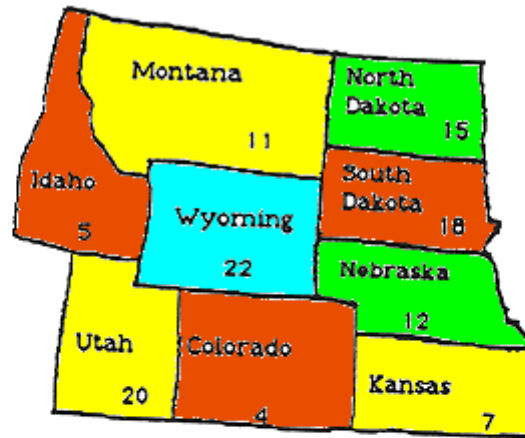
Now we can complete
the coloring:

North Dakota: green

South Dakota: red

Utah: yellow

Map Coloring Example (cont.)



Now we can complete
the coloring:

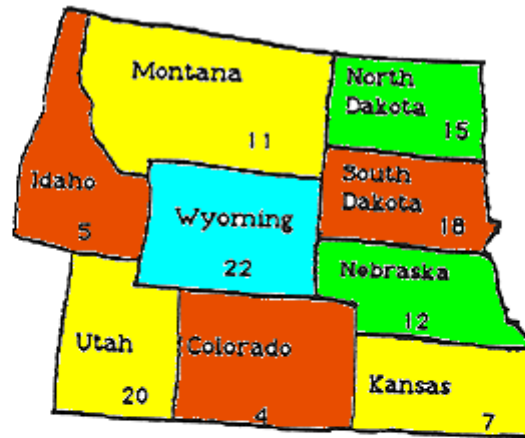
North Dakota: green

South Dakota: red

Utah: yellow

Wyoming: blue

Map Coloring Example (cont.)



Now we can complete
the coloring:

North Dakota: green

South Dakota: red

Utah: yellow

Wyoming: blue

done!