CS585: Image and Video Computing

HW3 Solution

February 24, 2021

Exercise 1

1.1 $H(S_1, S_2)$

By definition, we have

$$h(S1, S2) = \max\{d(A, D), d(B, E), d(C, F)\} = d(C, F) = |\overline{CF}| = 2\sqrt{2}$$

and

$$h(S2, S1) = \max\{d(D, A), d(E, B), d(F, B), d(G, C)\} = d(G, C) = |\overline{GC}| = \sqrt{13}$$

Thus the Hausdorff distance between S_1 and S_2 is

$$h(S1, S2) = \max\{h(S1, S2), h(S2, S1)\} = \sqrt{13} \approx 3.6056$$

1.2 $H(T, R)$

Let T and R be the set of all points in the triangle and rectangle, respectively.

By definition, we have $H(T, R) = max\{h(T, R), h(R, T)\}$.

For $h(T, R)$, one could prove that for any point in the triangle other than point C, the smallest distance from that point to any point in R must be less that D_1, the distance from point C to the segment GF. Thus, we have $h(T, R) = D_1 = 2$.

Similarly, to obtain $h(R, T)$, one could prove that for any point in the rectangle other than point G, the smallest distance from that point to any point in T must be less that D_2, the distance from point G to the segment AC. Thus, we have $h(R, T) = D2 = \frac{|-9-1+3|}{\sqrt{3^2+1^2}} = \frac{7\sqrt{10}}{10} \approx 2.2136$.

Finally, we have $H(T, R) = max\{h(T, R), h(R, T)\} = \frac{7\sqrt{10}}{10} \approx 2.2136$.

Exercise 2

By definition, peakiness

$$p(i, j, k) = \frac{\min\{H(g_i), H(g_j)\}}{H(g_k)}$$

where g_i, g_k are gray values of local maxima at some minimum distance apart and g_k is the gray value of the minimum between g_i and g_j, for any (i, j, k) in the grayscale histogram of the image.

We can calculate the highest peakiness by maximizing $p(i, j, k)$ over all possible (i, j, k)

$$\max_{(i,j,k)} p(i, j, k) = p(103, 231, 157) = \frac{\min\{H(103), H(231)\}}{H(157)} = \frac{1300}{190} \approx 6.8421$$

Therefore the highest peakiness is 6.8421 and the corresponding threshold is 157.