Exploring the Benefits of CDMA in Optical Networks

Teletraffic Capacity and Other Ideas

Sharon Goldberg

Prof. Paul Prucnal

General Exam Research Seminar

Department of Electrical Engineering, Princeton University

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Talk Overview

Teletraffic Capacity of Optical CDMA:

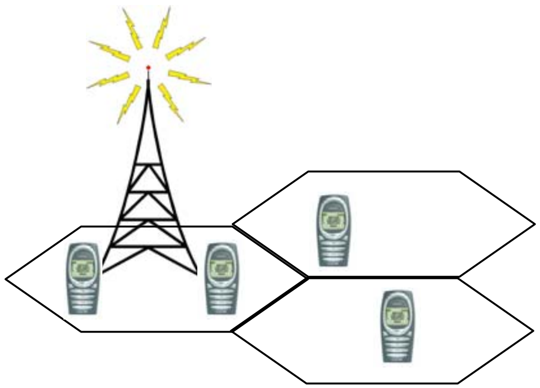
- Background + Preview
- Teletraffic Models:
  - Optical CDMA model
  - Wavelength Routed Network Model
- Results

Other OCDMA Projects:

- Call Admission Control
- Source-Matched Channel Coding
- OCDMA selective speedup for Queue Management
- Security of Coherent Spectral-Phase OCDMA

Code Division Multiple Access (CDMA)

Wireless CDMA



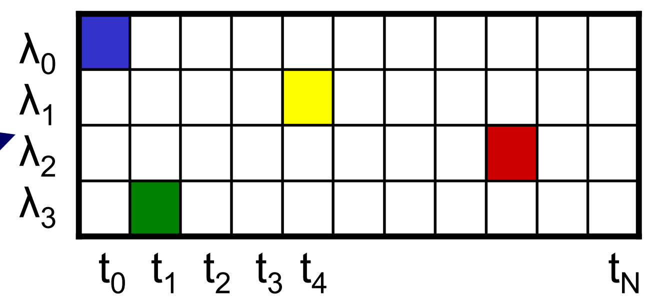
Cellular Systems

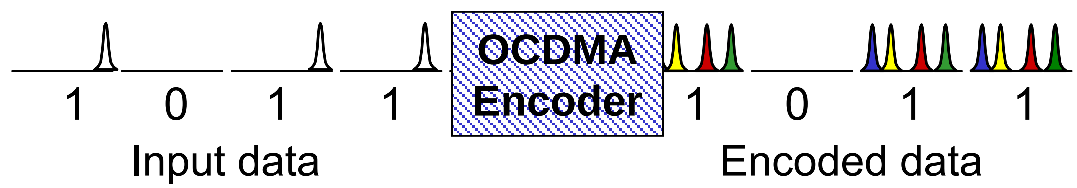
- Used in: (for example)
  - Wireless Cellular Systems
  - Unlicensed Spectrum Systems (2.4 GHz, 5.8 GHz)
- Allows:
  - Asynchronous multiple access
  - Frequency reuse
- Provides:
  - Immunity to interference
  - **Soft bounds on capacity (Soft Blocking)**

Optical CDMA

- Apply concept of wireless CDMA to optical domain
- Incoherent Coding
  - Time-Amplitude
  - Spectral-Amplitude
  - **Wavelength-Time**
- Coherent Coding
  - Temporal-Phase
  - Spectral-Phase

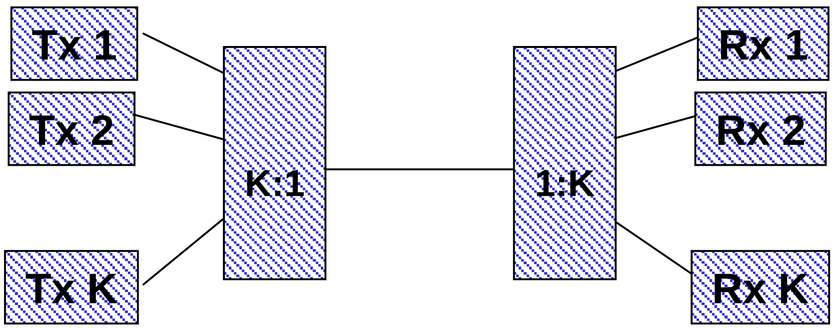
Sample Code Sequence



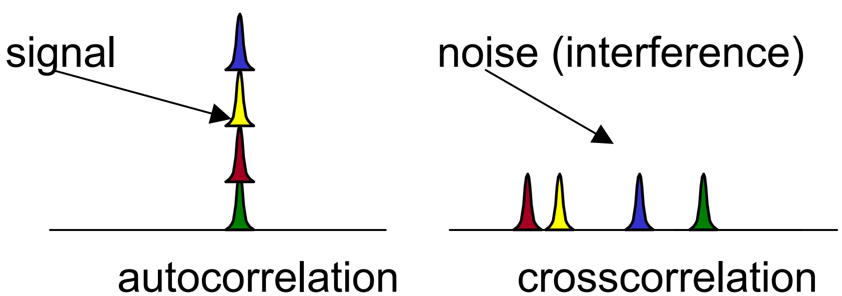


Optical CDMA Broadcast and Select Network

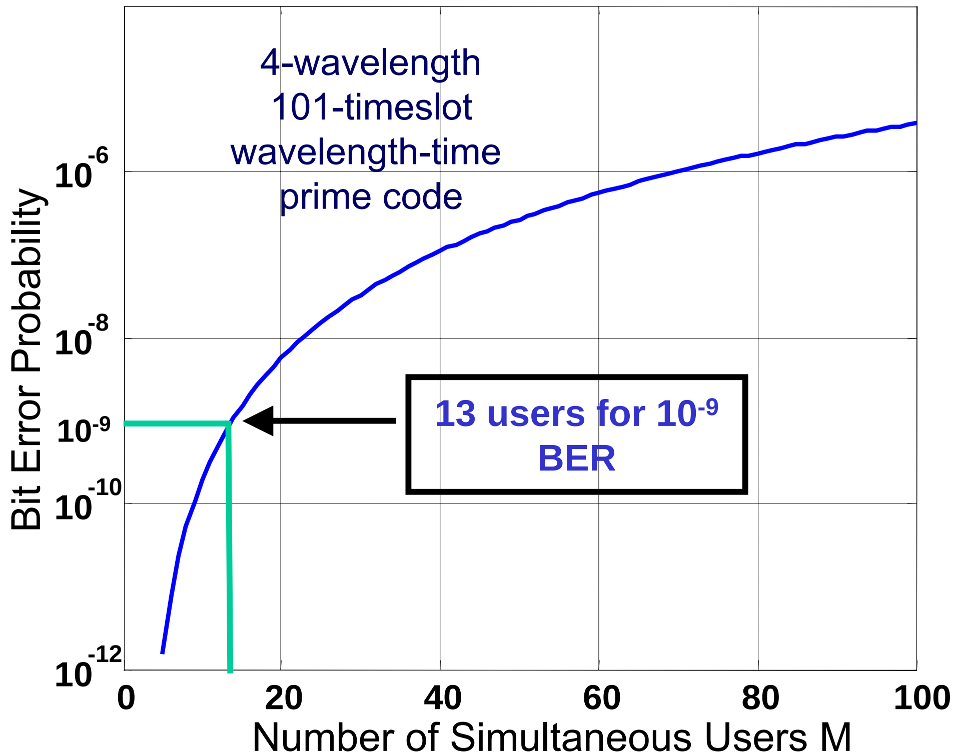
Broadcast-and-Select Network



Data is selected via correlation:



BER vs Number of Simultaneous Users



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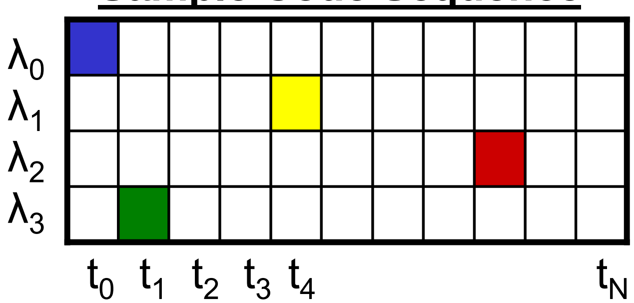


As the number of users increases, the interference increases.

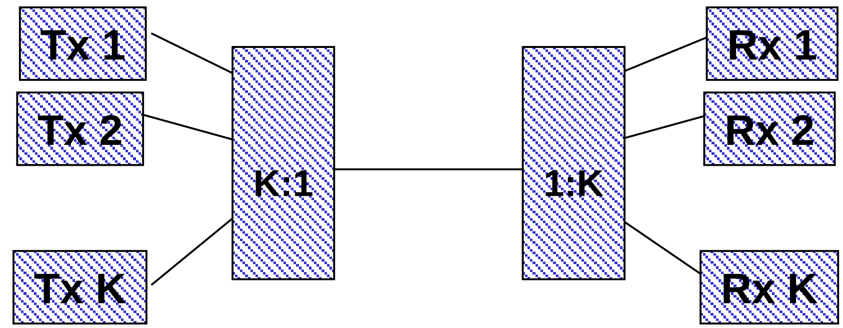
And this happens for most flavours of OCDMA!

Preview: Understanding Optical CDMA Capacity

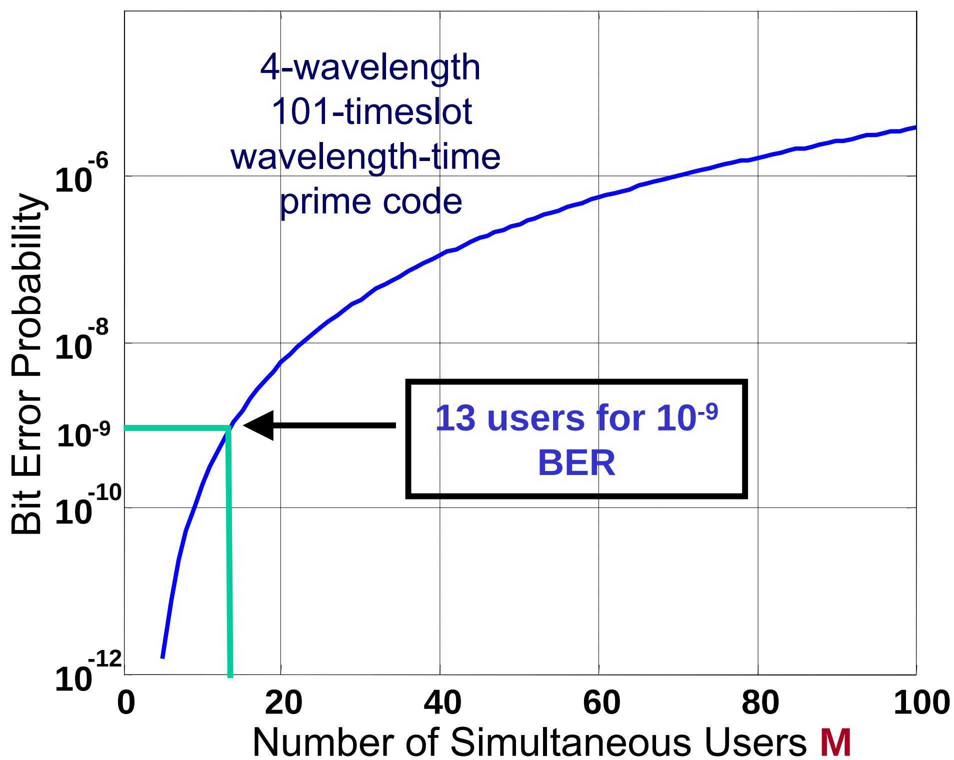
Traditional view of OCDMA capacity Γ:

Number of users accommodated by the system for a given bit error rate threshold.

$\Rightarrow$ **There is no hard limit on capacity!**



BER vs Number of Simultaneous Users



Motivation:

Model a network with stochastic utilization!

- Calls connected circuit-by-circuit
- Each circuit is active with probability ***p***
- More than Γ subscribers

Eg. Set a max BER threshold of **10^{-9}**

$\Rightarrow$ System admits **Γ = 13** simultaneous transmissions

$\Rightarrow$ When **M > 13**, BER degrades causing an *outage*.

$\Rightarrow$ Ensure that outages occur with probability **$P_{outage} < 10^{-3}$**

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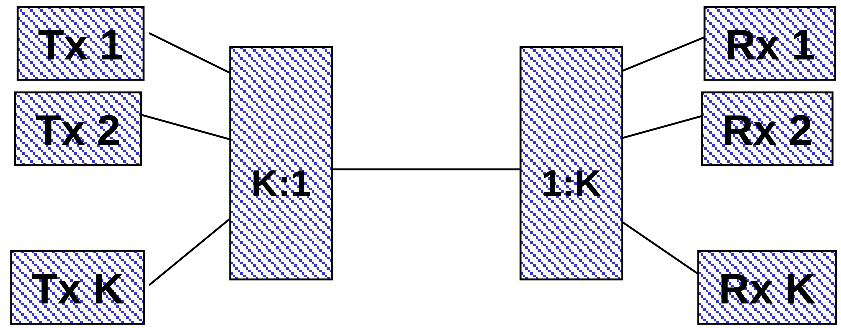
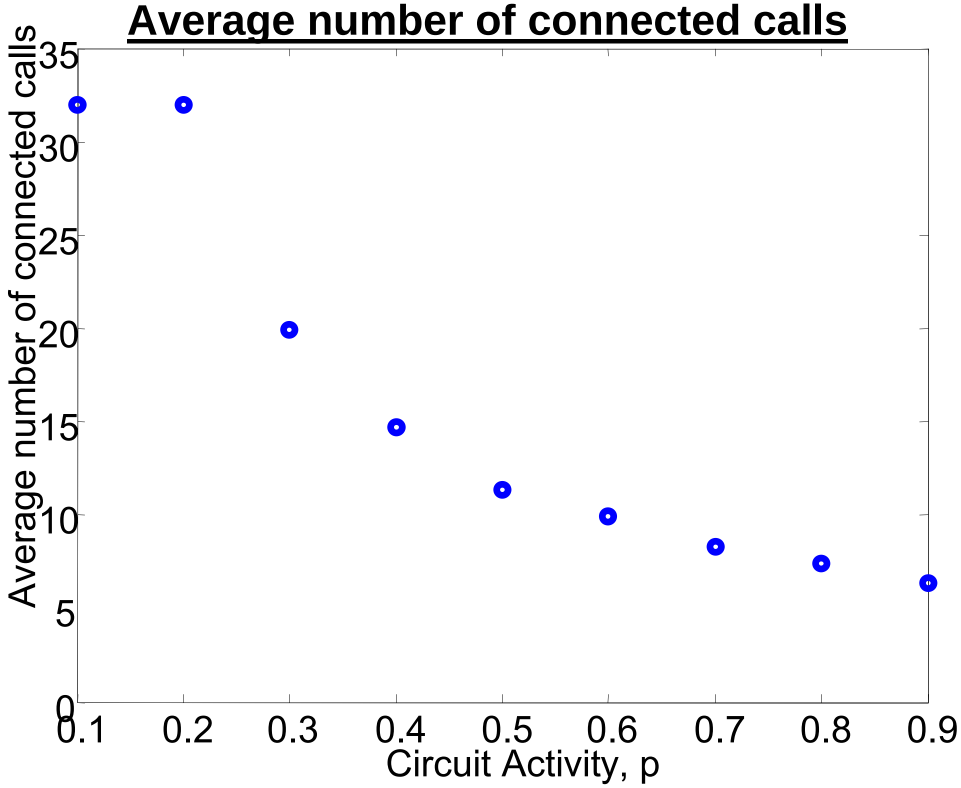


Capacity of OCDMA :
Average number of connected calls



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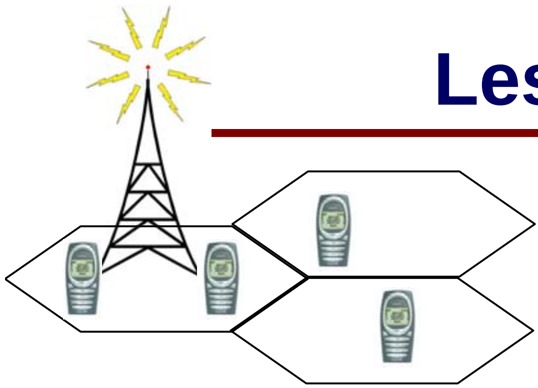
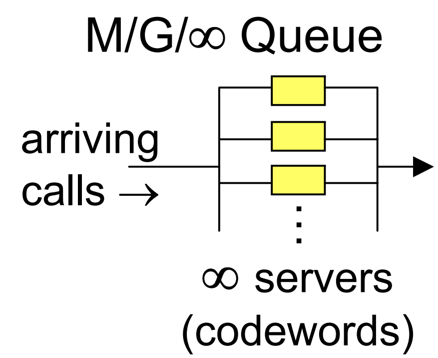
Lessons from Wireless CDMA



Analysis of the capacity of the mobile-to-base station link in a single cell of a wireless time-spreading CDMA cellular network. [(Viterbi)^2 1993], [Evans & Everitt 1999]



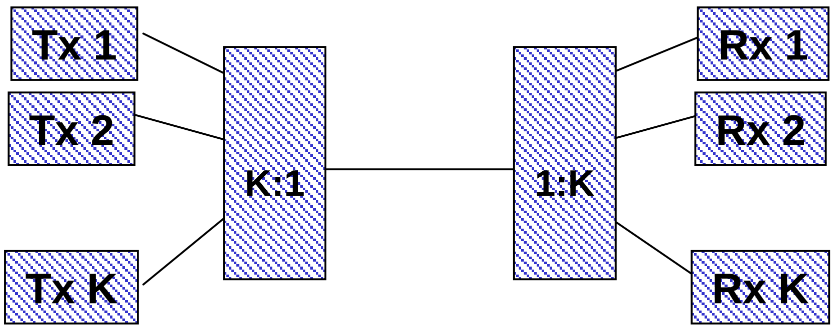
- Model cell as M/G/∞ queue:
  - Spreading code has large cardinality($\Rightarrow \infty$ servers)
  - No blocking! (New calls always connected)
  - Number of connected calls, ***N***, is a Poisson random variable
- Call utilization is intermittent (talking OR listening)
  - call activity, ***v***, is a Bernoulli random variable
- Each call is received with signal power ε
  - ε is a random variable that models fading, multipath, mobility, etc.
- An *outage* occurs when the interference increases beyond a threshold Γ
  - such that the performance of all the users in a cell is degraded

$$P_{outage} = P\left[\sum_{i=0}^{N} v_i \cdot \varepsilon_i > \Gamma\right]$$

Apply this idea to OCDMA!

Teletraffic Modelling Assumptions

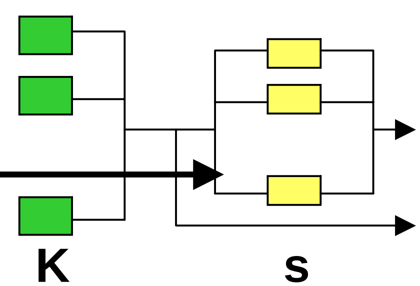
Broadcast-and-Select Network



We compare OCDMA and WRN (wavelength routed network) broadcast-and-select networks with ***K*** subscribers.

- Calls connected on a circuit-by-circuit basis:
  - Subscribers generate call requests whose interarrival times have arbitrary distribution with mean $1/\nu$ [hours]
  - The holding time of each call has an arbitrary distribution with mean $1/\mu$ [hours]
  - Thus, the offered load per idle source is $r = \nu/\mu$ [Erlang]
- Calls carry bursty data: (talking or listening/waiting)
  - call activity is a Bernoulli random variable with mean ***p***
- Number of connected calls is ***N(t)***
- Number of actively transmitting calls is ***M(t)***
- Study an interval where ***N(t)***, ***M(t)*** stationary – write as a random variables ***N***, ***M***

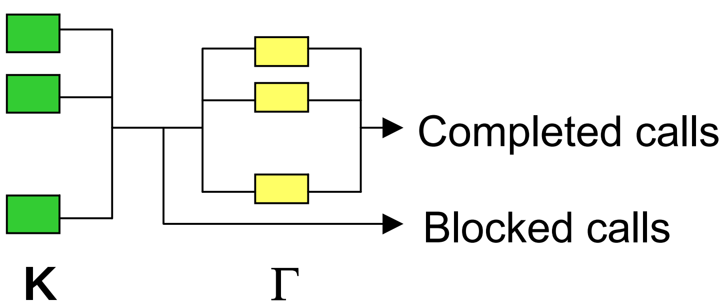
G(K)/G/s(0) Model



Wavelength Routed Network Model

- We model a WRN with ***K*** subscribers and Γ wavelengths
  - Tunable transmitters/receiver are used so that each subscriber can transmit on any wavelength
  - We are interested in physical layer capacity only
  - Thus, assume each wavelength can carry only one circuit at a time
- Incoming traffic is *blocked* when there are no free wavelengths (*i.e.* ***N* = Γ**)
- Model ***N*** using the generalized Engset **G(K)/G/ Γ(0)** loss model

G(K)/G/Γ(0) Model



Carried Load:

$$E[N] = Kr\sum_{n=0}^{\Gamma-1}\binom{K-1}{n}r^n\left[\sum_{i=0}^{\Gamma}\binom{K}{i}r^i\right]^{-1}$$

Distribution of connected calls:

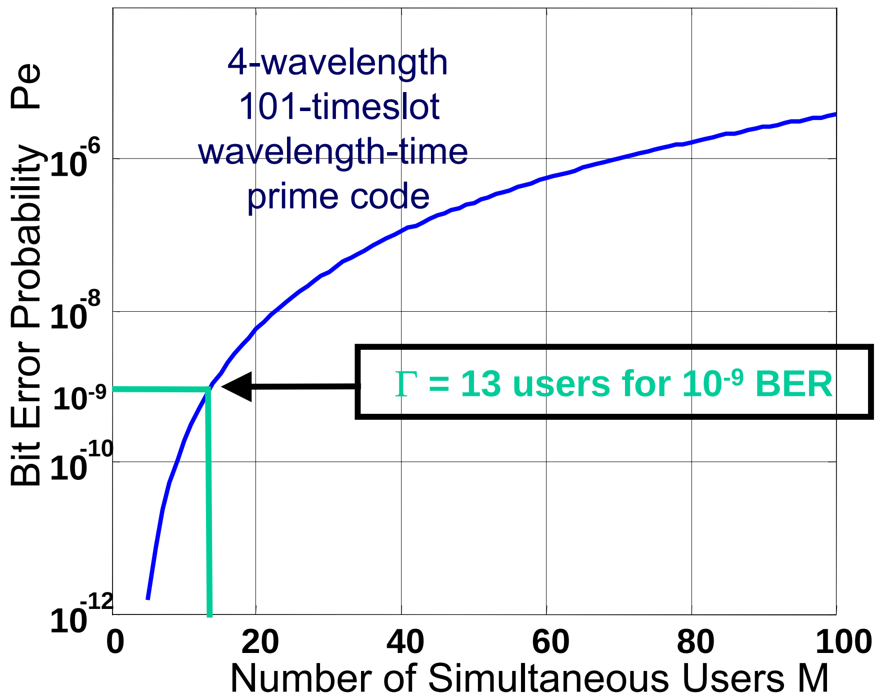
$$P[N=n] = \binom{K}{n}r^n\left[\sum_{i=0}^{\Gamma}\binom{K}{i}r^i\right]^{-1}$$

Call Congestion: P[Arriving Call sees N= Γ]

Blocking Probability:

$$P_{block} = \binom{K-1}{\Gamma}r^\Gamma\left[\sum_{i=0}^{\Gamma}\binom{K-1}{i}r^i\right]^{-1}$$

Optical CDMA Model (1)

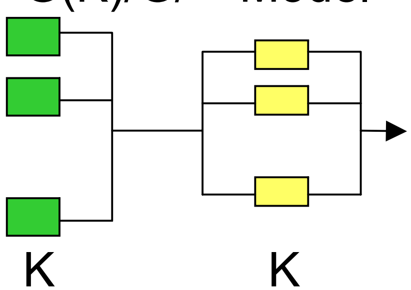


We model an OCDMA network with ***K*** subscribers and Γ maximum simultaneous users

- Each circuit is received with equal power
- The cardinality of the spreading code is large (> ***K***)
- Every incoming call is carried by the network (no blocking !)

Model ***N*** using the generalized finite-K source, infinite server **G(K)/G/∞** model

G(K)/G/∞ Model



Distribution of connected calls:

$$P[N=n]=\binom{K}{n}r^n(1+r)^{-K}$$

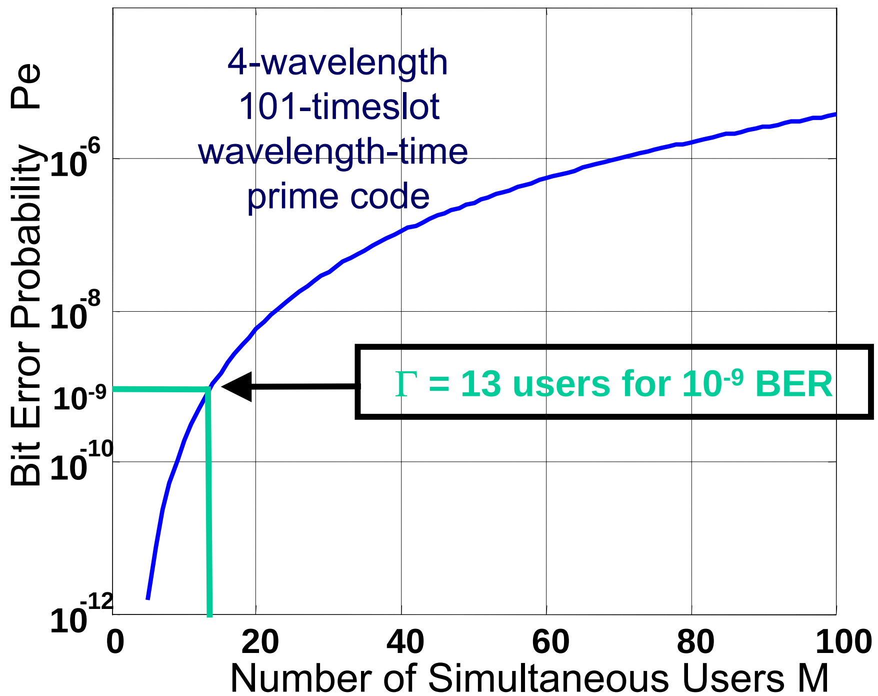
Carried Load:

$$E[N]=K\frac{r}{1+r}$$

Blocking Probability:

$$P_{block}=0$$

Optical CDMA Model (2)



We model an OCDMA network with ***K*** subscribers and Γ maximum simultaneous users

An *outage* occurs when the number of active calls exceeds the maximum Γ, *i.e.*

$$M > \Gamma$$

so that the performance of all circuits degrades beyond the max BER threshold.

Distribution of active calls:

$$
\begin{aligned}
P[M=m] &= \sum_{n=m}^{K} P[M=m \mid N=n] \cdot P[N=n] \\
&= \sum_{n=m}^{K} \binom{n}{m} p^m (1-p)^{n-m} \cdot \binom{K}{n} r^n (1+r)^{-K} \\
&= \binom{K}{m} (pr)^m (1+(1-p)r)^{K-m}
\end{aligned}
$$

P[BER > 10^-9]

Outage probability: $P[\,M > \Gamma\,]$

$$P_{outage} = \sum_{m=\Gamma+1}^{K} \binom{K}{m} (pr)^m (1+(1-p)r)^{K-m}$$

Blocking Probability:

$$P_{block} = 0$$

And now to define the teletraffic capacity…

The teletraffic capacity is the maximum value of E[N] satisfying:

1. Outage Constraint: $P_{outage} < P_{outage}^{max}$
2. Blocking Constraint: $P_{block} < P_{block}^{max}$

We compare teletraffic capacities of:

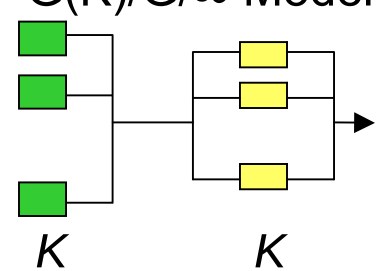
- OCDMA with Γ maximum simultaneous users and ***K*** subscribers
- WRN with Γ wavelengths and ***K*** subscribers.

OCDMA

$$E[N] = K\frac{r}{1+r}$$

$$P_{outage} = \sum_{m=\Gamma+1}^{K}\binom{K}{m}(pr)^{m}(1+(1-p)r)^{K-m}$$

G(K)/G/∞ Model

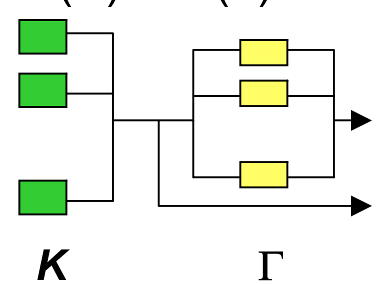


WRN

$$E[N] = Kr\frac{\sum_{n=0}^{\Gamma-1}\binom{K-1}{n}r^{n}}{\sum_{i=0}^{\Gamma}\binom{K}{i}r^{i}}$$

$$P_{blocking} = \frac{\binom{K-1}{\Gamma}r^{\Gamma}}{\sum_{i=0}^{\Gamma}\binom{K}{i}r^{i}}$$

G(K)/G/Γ(0) Model



OCDMA vs Wavelength Routed Network

K=48 subscribers

Γ=32 OCDMA maximum simultaneous users

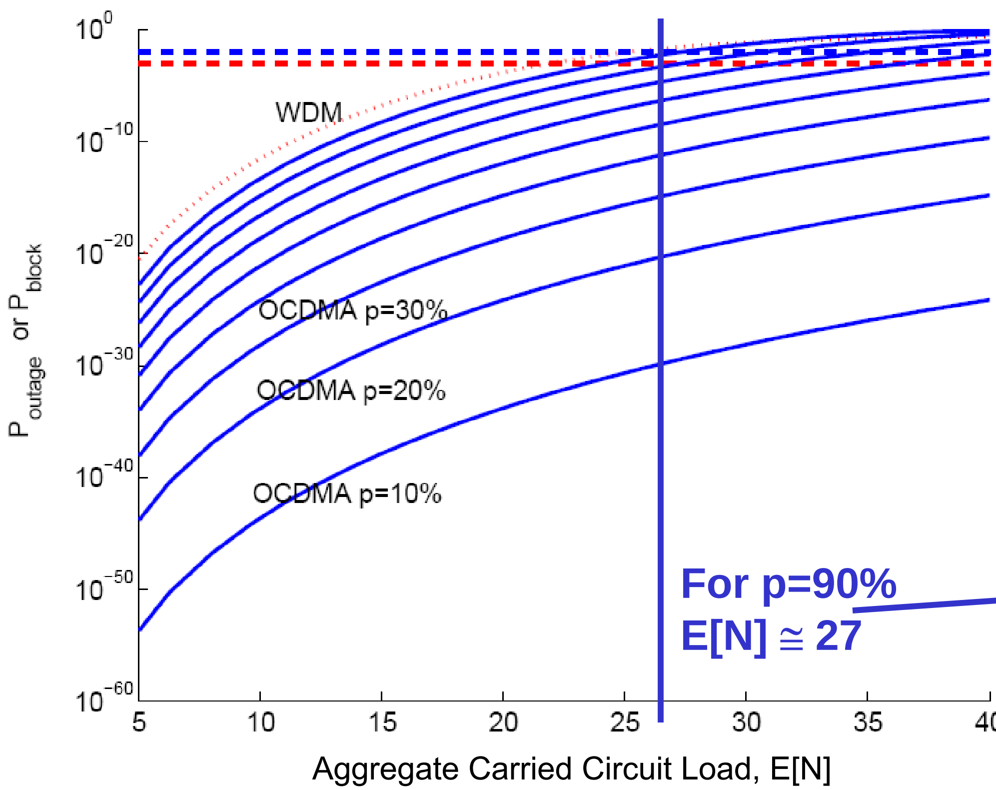
Γ=32 WRN wavelengths

Constraints: $P_{outage}^{max}=10^{-2}$ and $P_{block}^{max}=10^{-3}$

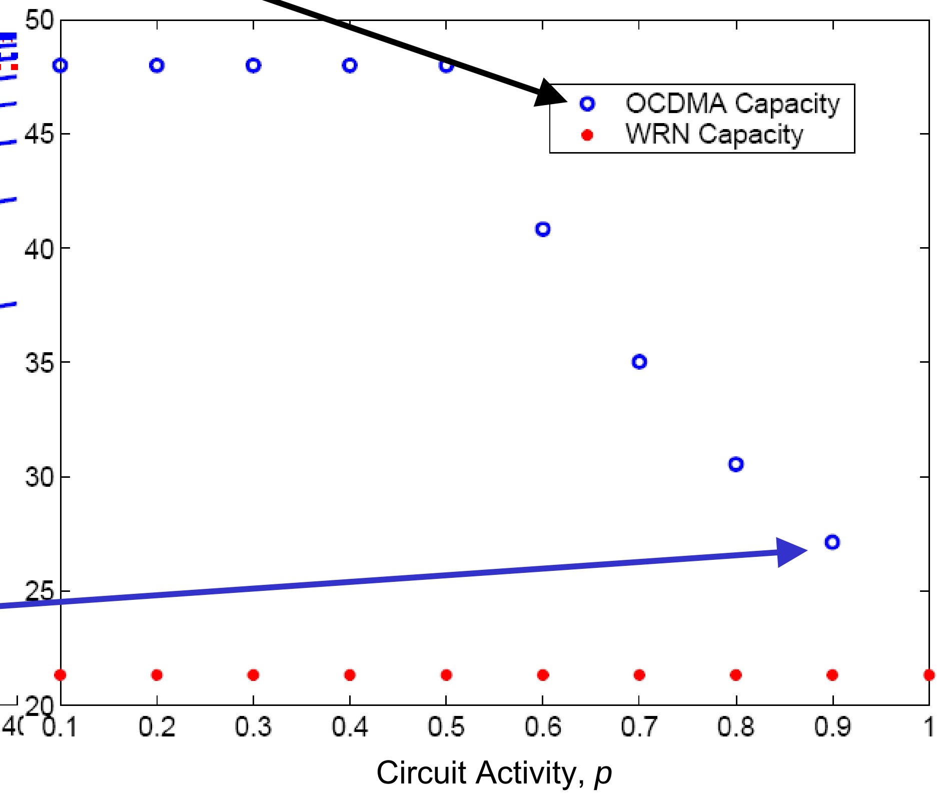
More important to accommodate new circuits than to ensure service availability of existing circuits

Capacity increases due to statistical multiplexing!

Outage Probability (OCDMA)
Blocking Probability (WRN) vs E[N]



Teletraffic Capacity of OCDMA and WRN



OCDMA vs Wavelength Routed Network

K=64 subscribers

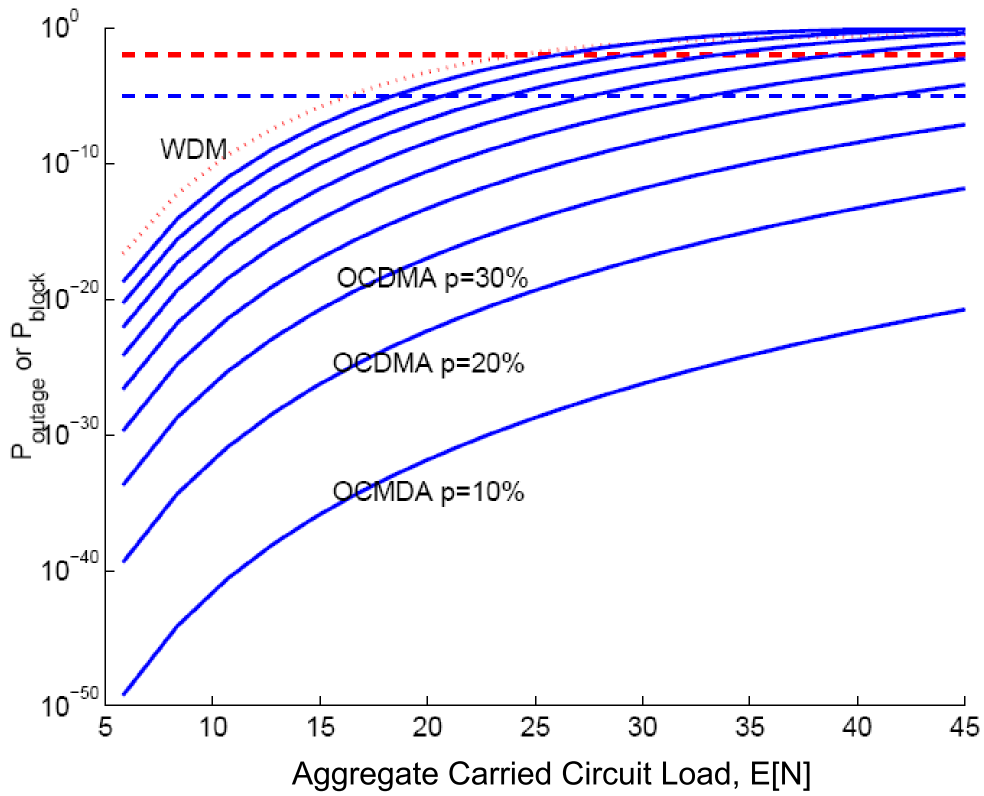
Γ=32 OCDMA maximum simultaneous users

Γ=32 WRN wavelengths

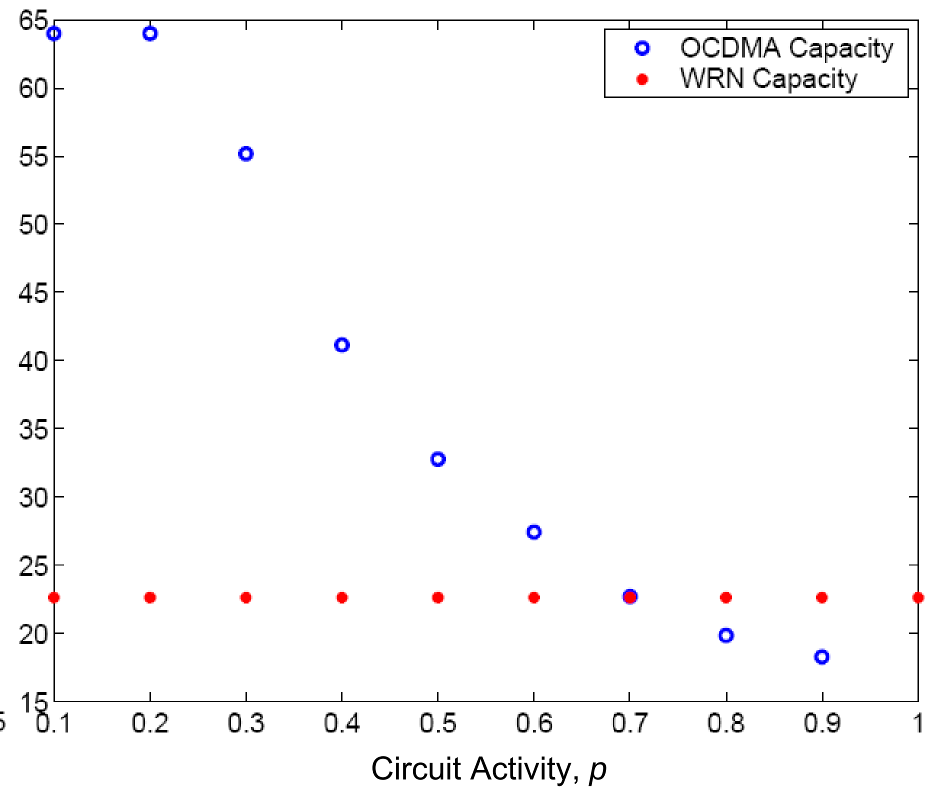
Constraints: $P_{outage}^{max}=10^{-5}$ and $P_{block}^{max}=10^{-2}$

More important to ensure service availability of existing circuits than to accommodate new circuits

Outage Probability (OCDMA)
Blocking Probability (WRN) vs Offered Load



Teletraffic Capacity of OCDMA and WRN



On the Teletraffic Capacity of OCDMA*: Conclusions

- We developed a framework for understanding capacity under stochastic utilization (teletraffic capacity)
- We found that
  - OCDMA capacity exceeds that of WRN
  - …except when activity $p \rightarrow 1$ and $P_{block}^{max} << P_{outage}^{max}$
  - To remedy this, we will show how OCDMA with **call admission control** can match or exceed WRN capacity (stay tuned!)
- Our framework can be extended to model more complex systems
  - users have different values of p, μ, v
  - multicode systems

* S. Goldberg, P.R. Prucnal, "On the teletraffic capacity of OCDMA", accepted for publication in IEEE Transactions on Communications

Talk Overview

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- Teletraffic Models:
  - Optical CDMA model
  - Wavelength Routed Network Model
- Results

Other OCDMA Projects:

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- Source-Matched Channel Coding
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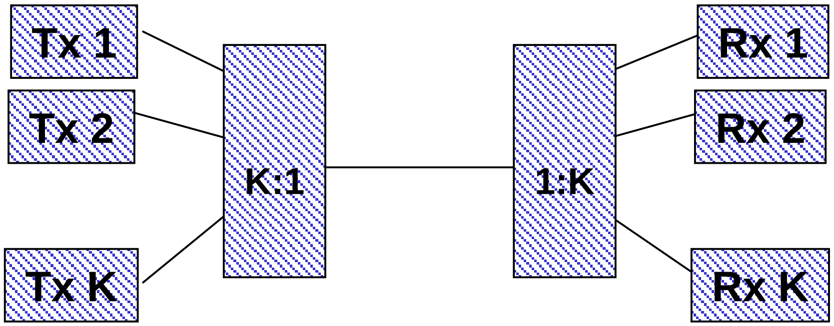
Call Admission Control (CAC) for OCDMA



With CAC the network blocks some new circuit requests to reduce interference and thus reduce occurrence of outages.

We now have both outage and blocking!

- For perfect service availability ($P_{outage}^{max}=0$)
  - Block new calls when $N=\Gamma$
  - No statistical multiplexing $\Rightarrow$ OCDMA network operates like a WRN

But we can do better if we allow $P_{outage}^{max} > 0$!

- We propose two CAC protocols:
  - Complete Sharing CAC: Block new calls when $N=B$
  - Check Interference on Call Arrival CAC: Block new calls when $M \geq B$
- Design the CAC protocols (i.e. find optimal blocking threshold B^*) that maximizes the teletraffic capacity E[N] while satisfying:
  1. Outage Constraint: $P_{outage} < P_{outage}^{max}$
  2. Blocking Constraint: $P_{block} < P_{block}^{max}$

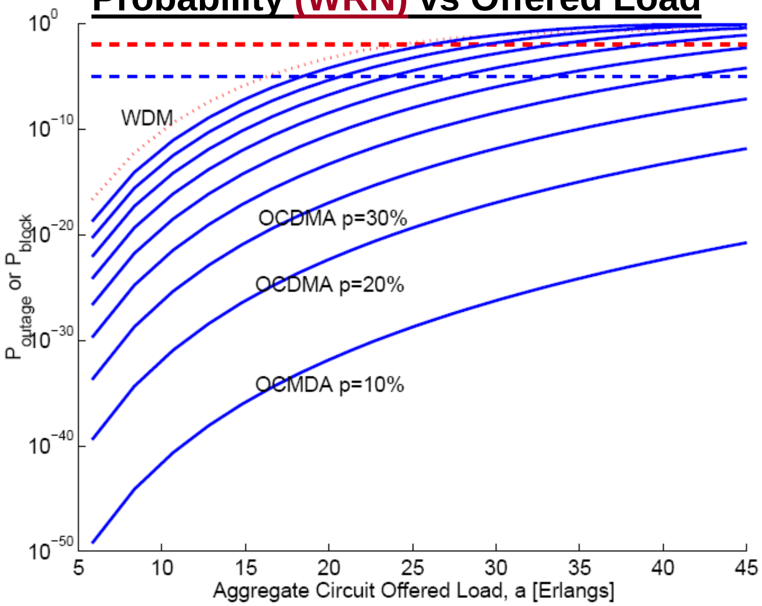
Call Admission Control Results: K=64, Γ=32, p=75%

P_{outage}^{max}	P_{block}^{max}	OCDMA	CS CAC	CIUCA CAC	WRN
10^{-2}	10^{-5}	31.2	31.2	31.2	16.2
10^{-2}	10^{-4}	31.2	31.2	31.2	18.0
10^{-2}	10^{-3}	31.2	31.2	31.2	20.4
10^{-2}	10^{-2}	31.2	32.0	31.8	23.6
10^{-3}	10^{-2}	27.4	28.7	28.3	23.6
10^{-4}	10^{-2}	24.5	26.6	25.8	23.6
10^{-5}	**10^{-2}**	**22.0**	**25.9**	**24.3**	**23.6**

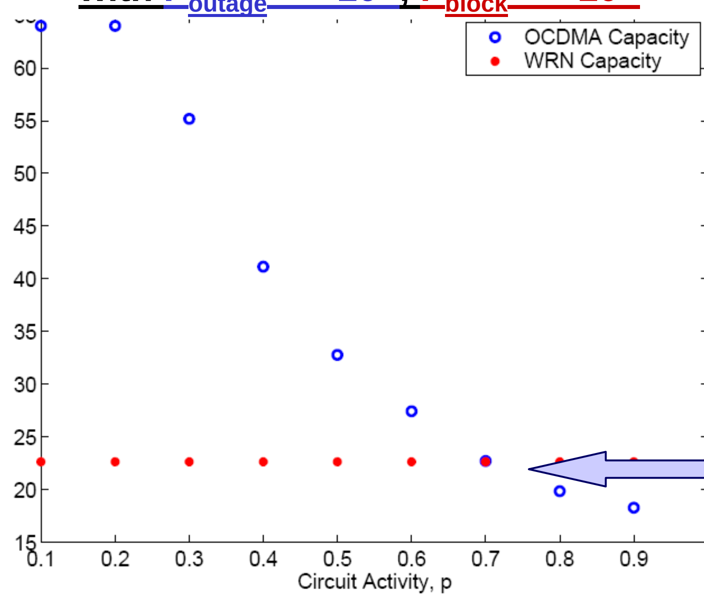
The blocking constraint is binding so $B^*=K$ (no CAC)

CS: $B^*=34$
17% capacity increase
CIUCA: $B^*=27$
10% capacity increase

Outage Probability (OCDMA) Blocking Probability (WRN) vs Offered Load

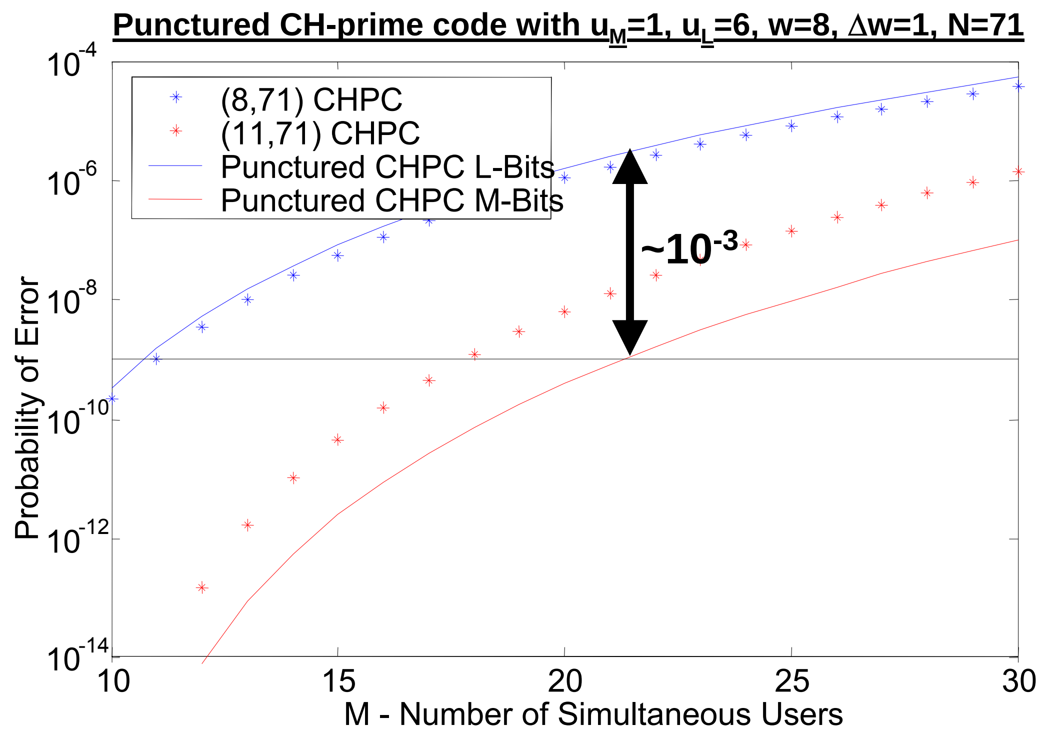


Teletraffic Capacity of OCDMA/ WRN with $P_{outage}^{max}=10^{-5}$, $P_{block}^{max}=10^{-2}$



Source-Matched OCDMA Spreading Codes

When an analog signal is source-encoded (i.e. quantized and compressed) redundancies in encoding mean that the datastream contains bits of varying importance.



Use a priority scheme:
- to maximize system capacity
- to send important bits with reliably.

⇒ Data arranged in blocks:
- u_M more important bits (M-bits)
- u_L less important bits (L-bits).

e.g. Message sent via punctured CH-prime code, $u_M = 1$, $u_L = 4$, $w = 3$, $\Delta w = 2$, $N=5$.

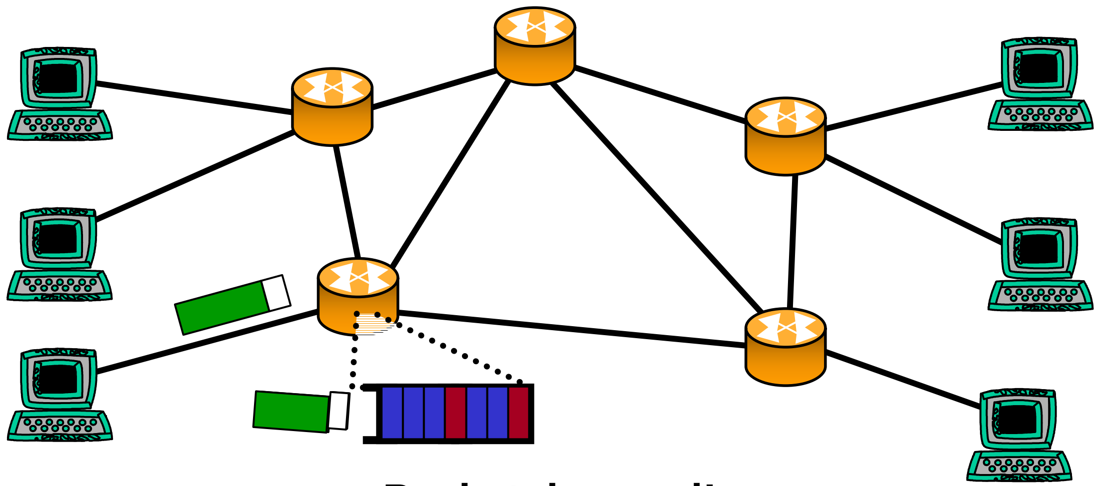
0	0	1	0	0	0	0	1	0	0	0	0
0	1	0	0	0	0	1	0	0	0	0	1
1	0	0	0	0	1	0	0	0	0	1	0
		0	0	1	0	0					
		0	1	0	0	0					

1	0	0	0	0	1	0	0	0	0	1	0
0	0	0	0	1	0	0	0	0	1	0	0
0	0	0	1	0	0	0	0	1	0	0	0
										0	0
										0	1

* S. Goldberg, V. Baby, T. Wang, P.R. Prucnal, "Source matched spreading codes for optical CDMA", accepted for publication in IEEE Transactions on Communications

OCDMA Selective Speedup for Queue Management

Active Queue Management

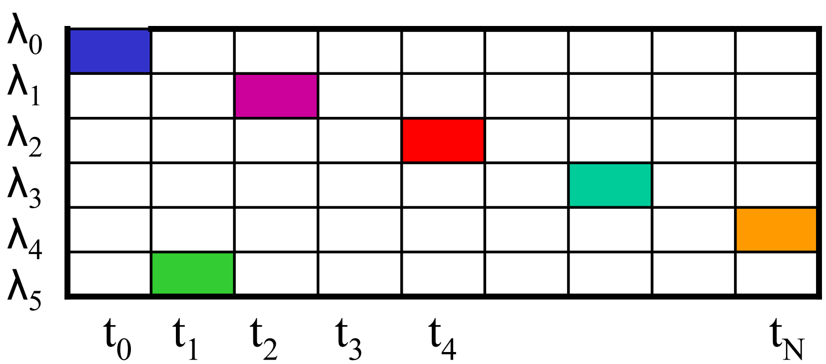


Packet dropped!

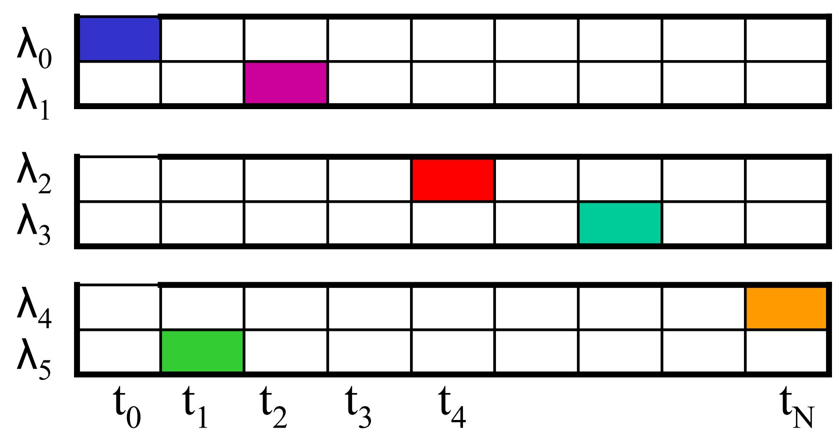
Instead of dropping packets during congestion, send some with selective speedup!

Selective speedup decreases packet dropping probability, queue length and delay.

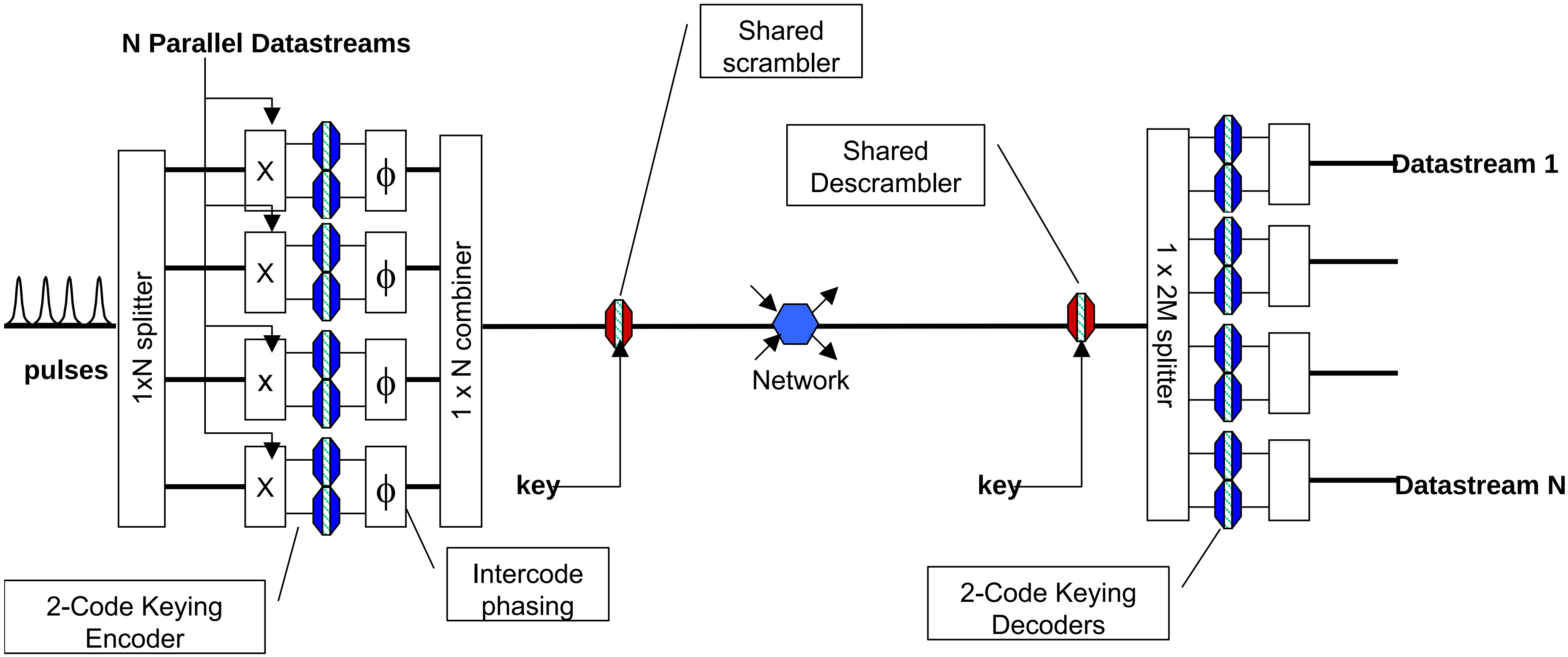
Regular packets sent one at a time with weight **w** codewords and high reliability (low BER)



Speedup packets sent **three** at a time with weight **w/3** codewords and lower reliability (higher BER)



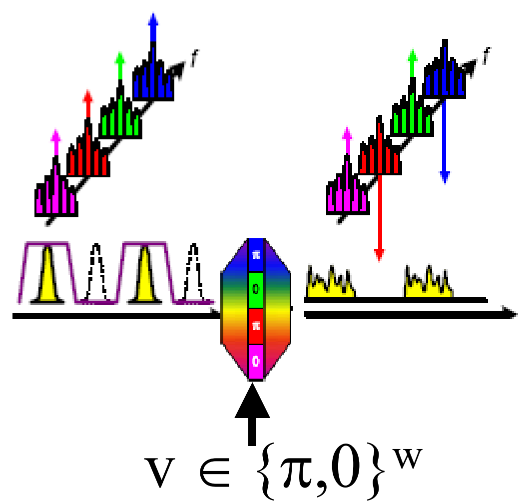
Security of Coherent Spectral-Phase OCDMA



Goal: To study the security of the system against an eavesdropper in a chosen plaintext attack.

Determine system security as a function of

- **N** – Number of users
- c^w – number of scrambler states
- Randomness of intercode phasing



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Prof. B. Barak

Dr. R. Menendez

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Teletraffic Capacity and Other Ideas

BACK UP SLIDES

Selected References

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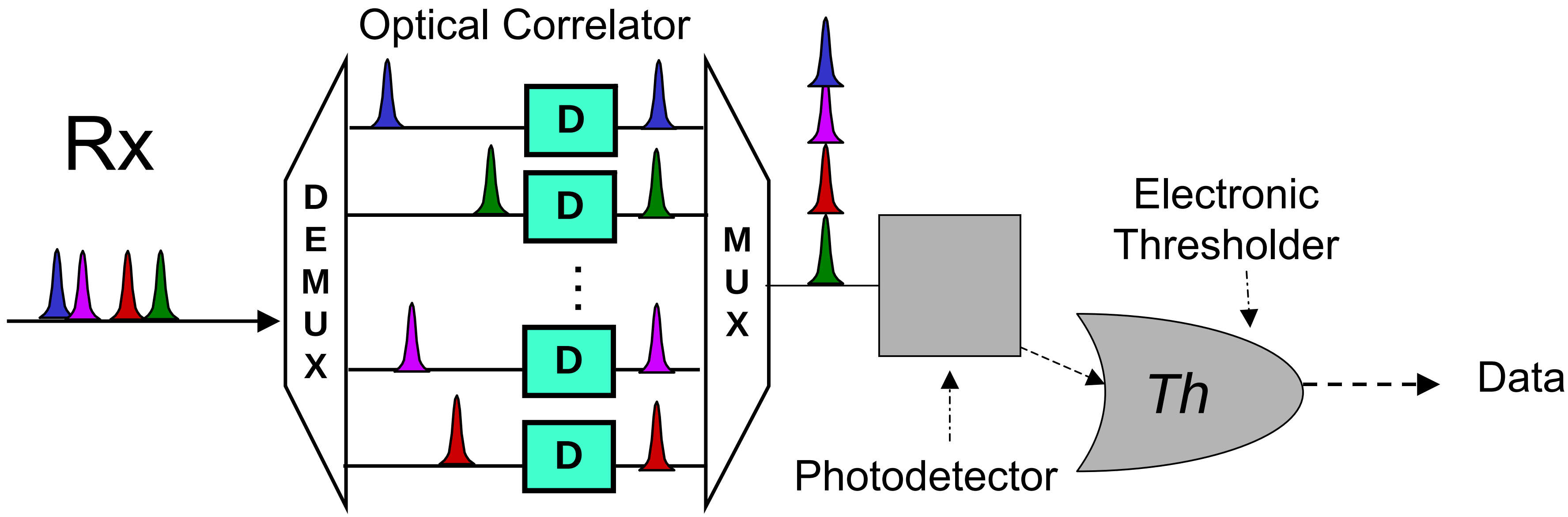
- H. Kobayashi and B. L. Mark, “Product-form loss networks," in *Frontiers in Queueing*, J. H. Dshalalow, Ed. Boca Raton, FL: CRC Press, Inc., 1997, ch. 6, pp. 147{165.

Obtaining Γ: Performance Analysis

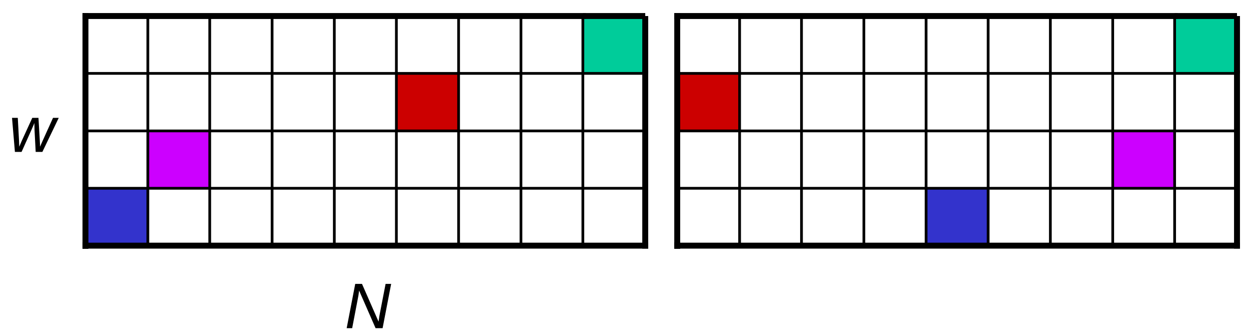


Hit probability:

$$q = \frac{w}{2N}$$

Bit Error Rate:

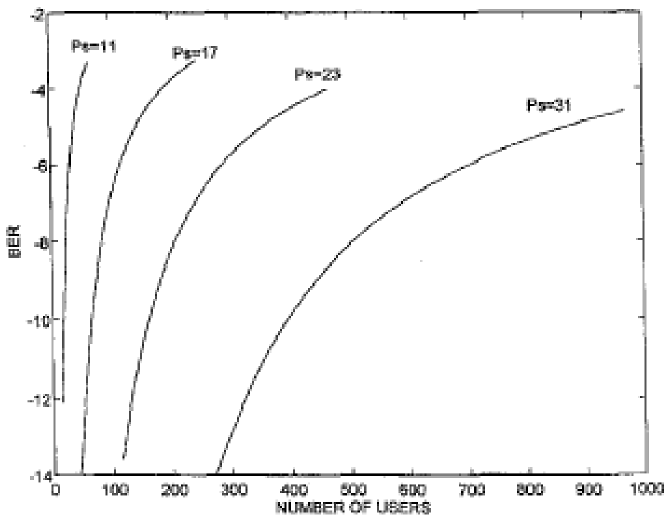
$$P_{err} = \sum_{i=Th}^{M-1} \binom{M-1}{i} q^i (1-q)^{M-i-1}$$



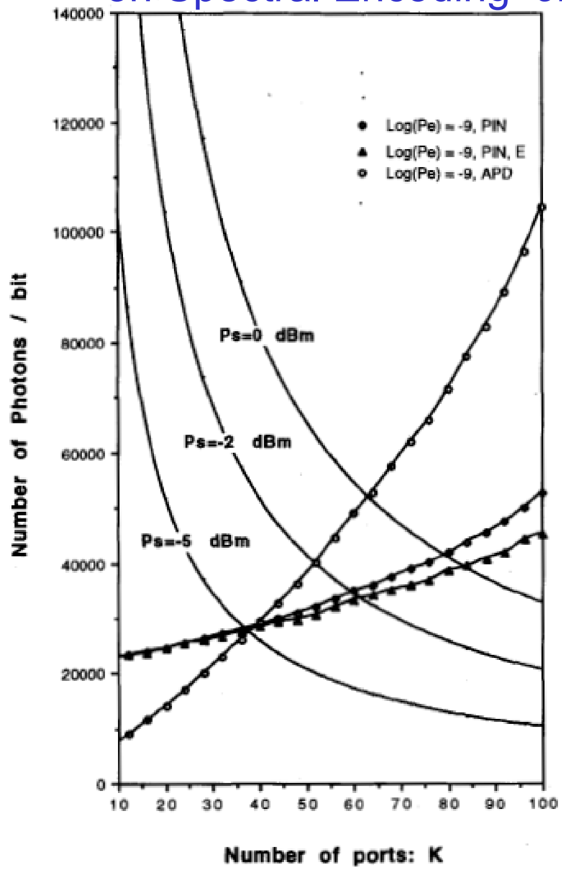
Usually $Th = w$

BER vs M Plots – Different Flavours

Tancevski, Andonovic, “Wavelength Hopping/Time Spreading”, *JLT,* 1996



Havehrad, Zaccarin “Optical CDMA Systems based on Spectral Encoding” *JLT,* 1995.



ig. 10. Number of photons needed for $\mathrm{Log}(P_e) = -9$, $N = 127$ M-sequences.

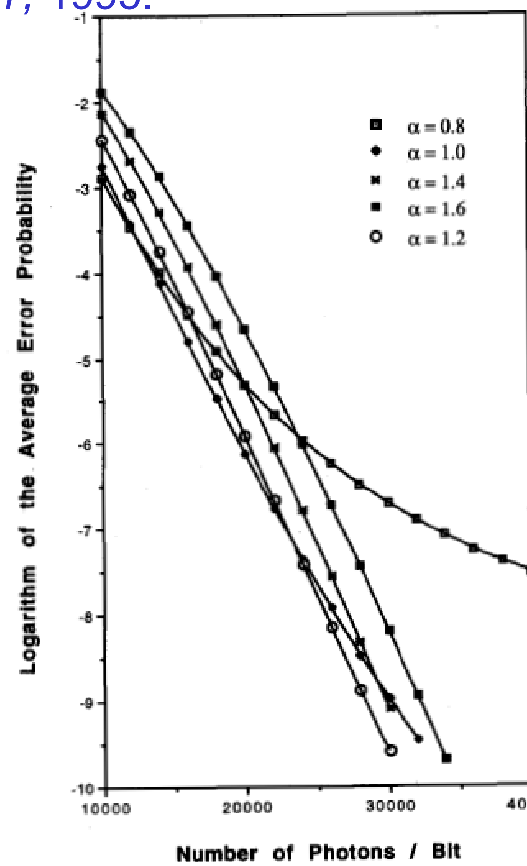
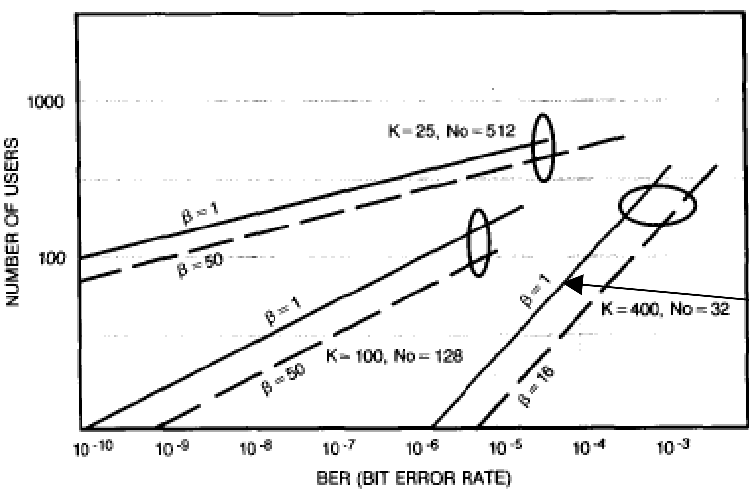


Fig. 11. Performance as the encoded bandwidth varies; $N = 127$ M-sequences, $K = 40$.

Salehi, Weiner, Heritage, “Coherent Ultrashort Light Pulse CDMA Communication Systems”, *JLT,* 1990.



β=1 (Ideal detection- perfect time gating),
K=400 timeslots,
N=32 code elements

More on Call Admission Control

CS CAC & CIUCA CAC Models

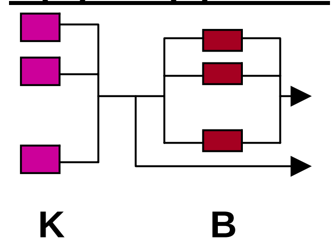
CS CAC

Block when N = B **Outage when M > Γ**

$$E[N] = Kr\sum_{n=0}^{B-1}\binom{K-1}{n} r^n \left[\sum_{i=0}^{B}\binom{K}{i} r^i\right]^{-1} \qquad P_{block} = \binom{K-1}{B} r^B \left[\sum_{i=0}^{B}\binom{K-1}{i} r^i\right]^{-1}$$

$$P_{outage} = P[M > \Gamma] = \sum_{m=\Gamma+1}^{B}\sum_{n=M}^{B}\binom{n}{m}\binom{K}{n}(pr)^m(1+(1-p)r)^{n-m}\left[\sum_{i=0}^{B}\binom{K}{j} r^j\right]^{-1}$$

G(K)/G/B(0) Model



CUICA CAC

Block when M $\geq$ B **Outage when M > Γ**

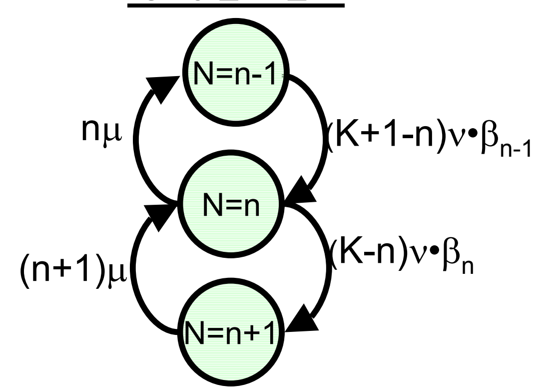
$$\beta_n = \begin{cases} 1 & 0 \leq n < B \\ \sum_{m=0}^{B-1}\binom{n}{m} p^m (1-p)^{n-m} & B \leq n < K \end{cases}$$

$$E[N] = Kr\sum_{n=0}^{K-1}\binom{K-1}{n} r^n \prod_{j=0}^{n}\beta_j \left[\sum_{i=0}^{K}\binom{K}{i} r^i \prod_{j=0}^{i-1}\beta_j\right]^{-1}$$

$$P_{block} = 1-\sum_{n=0}^{K-1}\binom{K-1}{n} r^n \prod_{j=0}^{n}\beta_j \left[\sum_{i=0}^{K-1}\binom{K-1}{i} r^i \prod_{j=0}^{i-1}\beta_j\right]^{-1}$$

$$P_{outage} = \sum_{m=\Gamma+1}^{K}\binom{K}{m}(pr)^m \sum_{\ell=0}^{K-m}\binom{K-m}{\ell}((1-p)r)^{\ell}\prod_{j=0}^{\ell+m-1}\beta_j \left[\sum_{i=0}^{K}\binom{K}{i} r^i \prod_{j=0}^{i-1}\beta_j\right]^{-1}$$

Markov Chain Model for 0 $\leq$ N $\leq$ K



OCDMA Complete Sharing CAC Model

Admit calls only when the total number of ***connected*** circuits is less than a threshold ***B***.

Block when *N* = *B*. **Outage when *M* > Γ.**

- Model ***N*** using the **G(K)/G/ B(0)** loss model with $\Gamma \leq \mathbf{B} \leq \mathbf{K}$
  - Incoming traffic is *blocked* when ***N = B***

$$P_{block} = \binom{K-1}{B} r^B \left[\sum_{i=0}^{B} \binom{K-1}{i} r^i \right]^{-1}$$

  - Carried circuit load

$$E[N] = Kr \sum_{n=0}^{B-1} \binom{K-1}{n} r^n \left[\sum_{i=0}^{B} \binom{K}{i} r^i \right]^{-1}$$

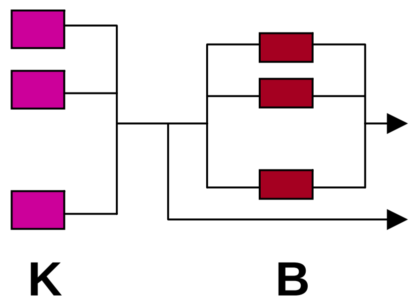
  - As before, we find the distribution of ***M*** (active circuits) from

$$P[M=m] = \sum_{n=m}^{B} P[M=m \mid N=n] \cdot P[N=n]$$

  - So that the outage probability is

$$P_{outage} = P[M > \Gamma] = \sum_{m=\Gamma+1}^{B} \sum_{n=M}^{B} \binom{n}{m} \binom{K}{n} (pr)^m ((1-p)r)^{n-m} \left[\sum_{i=0}^{B} \binom{K}{j} r^j \right]^{-1}$$

G(K)/G/B(0) Model



Check Interference on Call Arrival CAC Model (1)

Admit calls only when the total number of ***active*** circuits is less than a threshold ***B***.

Block when $M \geq B$. **Outage when $M > \Gamma$.**

- Model ***N*** using a Markov chain for $0 \leq N \leq K$
  - Introduce state dependant blocking probability: $1-\beta_n = P$[New call blocked $|N=n$]

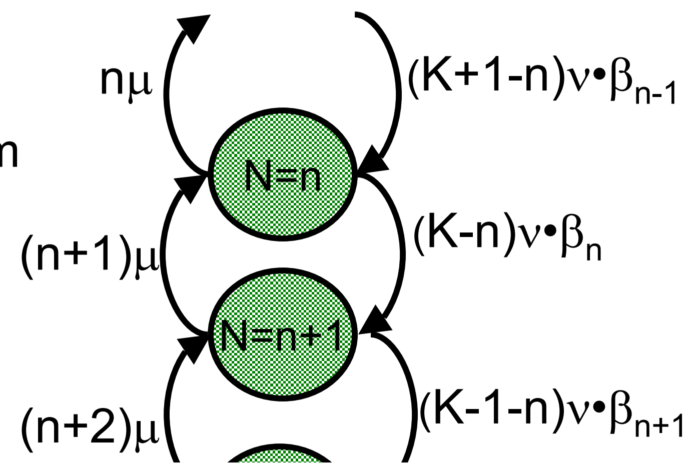
$$\beta_n = P[M < B \mid N = n] = \begin{cases} 1 & 0 \leq n < B \\ \sum_{m=0}^{B-1} \binom{n}{m} p^m (1-p)^{n-m} & B \leq n < K \end{cases}$$

  - From the Markov chain, the distribution of connected calls ***N*** is

$$P[N = n] = \binom{K-1}{n} r^n \prod_{j=0}^{n-1} \beta_j \left[\sum_{i=0}^{K} \binom{K}{i} r^i \prod_{j=0}^{i-1} \beta_j \right]^{-1}$$

  - As usual, find the distribution of active calls ***M*** from

$$\begin{aligned} P[M = m] &= \sum_{n=m}^{K} P[M = m \mid N = n] \cdot P[N = n] \\ &= \sum_{n=m}^{K} \binom{n}{m} p^m (1-p)^{n-m} \cdot P[N = n] \end{aligned}$$



Check Interference on Call Arrival CAC Model (2)

Admit calls only when the total number of ***active*** circuits is less than a threshold ***B***.

Block when *M* $\geq$ *B*. Outage when *M* > Γ.

- Then the outage probability can be found from the distribution of ***M***

$$P_{outage} = P[M > \Gamma] = \sum_{m=\Gamma+1}^{K} \binom{K}{m} (pr)^m \sum_{\ell=0}^{K-m} \binom{K-m}{\ell} ((1-p)r)^{\ell} \prod_{j=0}^{\ell+m-1} \beta_j \left[\sum_{i=0}^{K} \binom{K}{i} r^i \prod_{j=0}^{i-1} \beta_j \right]^{-1}$$

- To find the blocking probability we start with

$$\begin{aligned} P[M \geq B] &= \sum_{n=0}^{K} P[M \geq B \mid N = n] \cdot P[N = n] \\ &= \sum_{n=m}^{K} (1-\beta_n) \cdot P[N = n] \\ &= 1 - \sum_{n=0}^{K} \binom{K}{n} r^n \prod_{j=0}^{n} \beta_j \left[\sum_{i=0}^{K} \binom{K}{i} r^i \prod_{j=0}^{i-1} \beta_j \right]^{-1} \end{aligned}$$

- and using the arrival theorem, we can find

$$P_{block} = 1 - \sum_{n=0}^{K-1} \binom{K-1}{n} r^n \prod_{j=0}^{n} \beta_j \left[\sum_{i=0}^{K-1} \binom{K-1}{i} r^i \prod_{j=0}^{i-1} \beta_j \right]^{-1}$$

Designing the CAC Protocols

- Design the CAC protocols (*i.e.* find B^*) such that the teletraffic capacity E[N] is maximized while satisfying both constraints:
  1. Outage Constraint: $P_{outage} < P_{outage}^{max}$
  2. Blocking Constraint: $P_{block} < P_{block}^{max}$

- To find B^* we used an exhaustive search:
  1. Check if CAC is required: If $P_{outage} < P_{outage}^{max}$ for all r then no CAC required.
  2. If CAC is required:
     – For $B = 1 \ldots K$ compute the maximum offered load per subscriber r_B^{max} satisfying the constraints. Use r_B^{max} to compute the teletraffic capacity $E[N]$ for this B
     – Choose the blocking threshold B^* that maximizes the teletraffic capacity

Comparison of CAC Protocols

K=64, Γ=32, $P_{outage}^{max}=10^{-5}$, $P_{block}^{max}=10^{-2}$

Activity *p*	OCDMA	CS CAC			CIUCA CAC		
	Capacity	B^*	Optimal Capacity	Capacity B=34	B^*	Optimal Capacity	Capacity B=27
90%	18.3	32	23.6	18.9	28	21.7	20.6
80%	20.5	33	24.6	22.4	27	23.1	23.1
75%	**22.0**	**34**	**25.9**	**25.9**	**27**	**24.3**	**24.3**
70%	23.5	35	26.6	25.9	27	25.1	25.1
60%	27.4	38	29.6	25.9	26	28.7	28.0
50%	33.1	43	34.6	25.9	26	33.6	33.2
40%	41.2	50	42.3	25.9	26	41.5	41.4

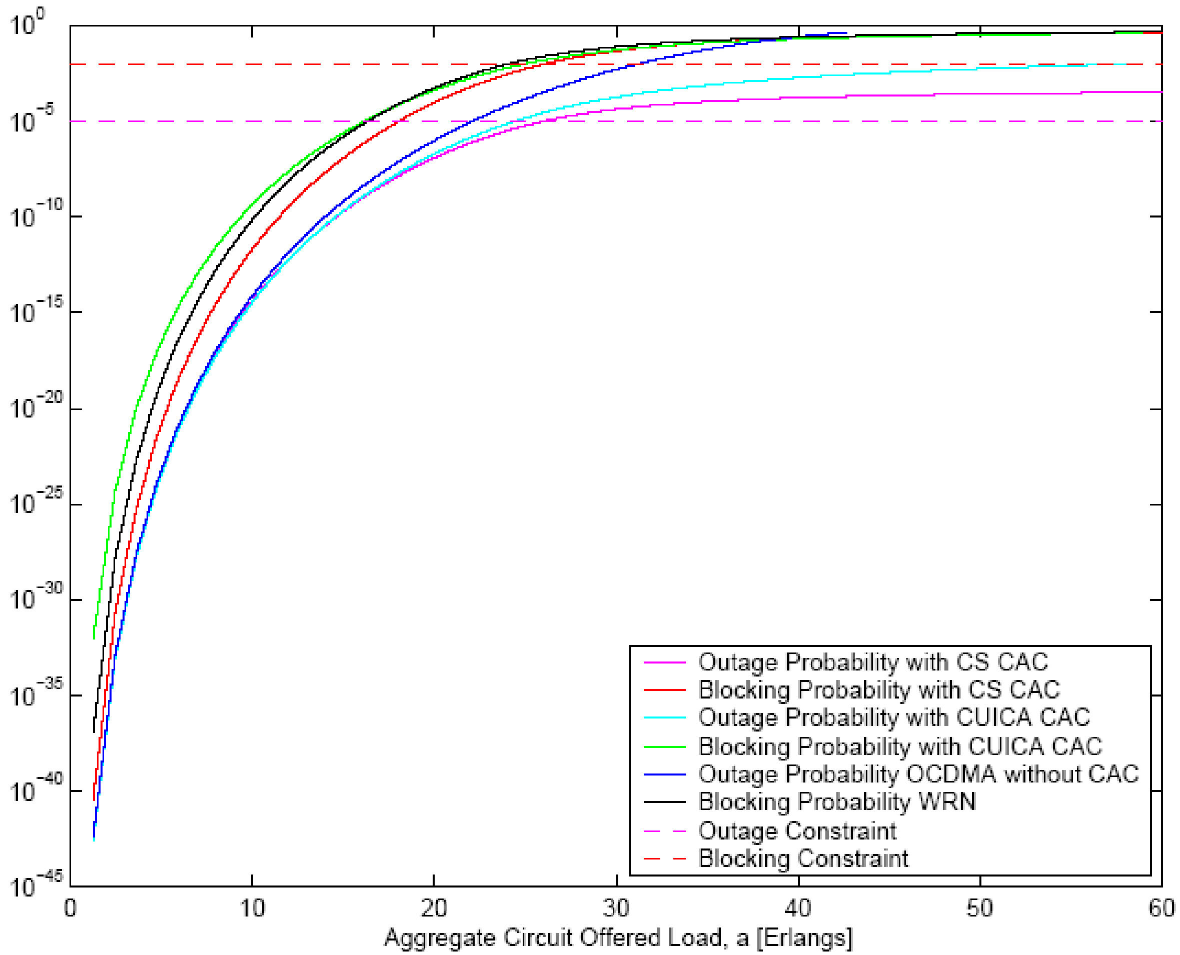
Protocol optimized for p = 40%

Protocol optimized for p = 75%

- CIUCA-CAC is more robust than CS-CAC to changes in activity ***p***
- CS-CAC provides higher capacity increases than CIUCA-CAC.

Additional Comparison of CAC Protocols

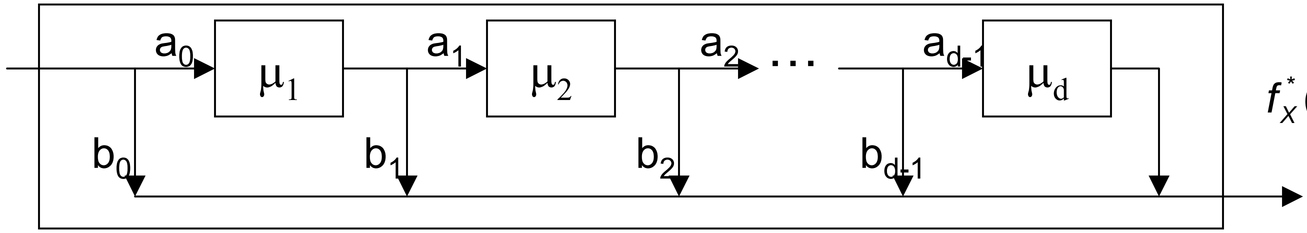
K=64, Γ=32, p=.75, $P_{outage}^{max}=10^{-5}$, $P_{block}^{max}=10^{-2}$



- CS-CAC reduces outage probability more effectively than CIUCA-CAC

Insensitivity of Engset G(K)/G/m(0) model

- For the **M/G/∞** and **M/G/m(0)** we model the general service time distribution **X** using a Cox representation (a series of memoryless servers).



$$f_X^*(s) = b_0 + \sum_{j=1}^{d} a_0 \ldots a_{j-1} b_j \prod_{i=1}^{j} \frac{\mu_i}{s+\mu_i}$$

Laplace Transform

- Unlike in the **M/M/∞** and **M/G/m(0)** the variable **N(t)** for each model is no longer Markov. Instead we define a new Markov state variable

S(t)={ $\underline{N}$(t),f(t) } = { N_1(t), N_2(t), …, N_d(t), f(t) }

- Now the system dynamics have the following differential equation for

P(χ,$\underline{s}$,t)=P[χ departures in (0,t) and $\underline{N}$(t) = $\underline{s}$]

$$P(\chi,\underline{s},t) = \sum_{j=1}^{d} P(\chi-1,\underline{s}+1_j,t)(n_j+1)\mu_j b_j$$ Departures from server from $s+1_j$

$$+\sum_{j=2}^{d} P(\chi,\underline{s}+1_{j-1}-1_j,t)(n_{j-1}+1)\mu_{j-1}a_{j-1}$$ Customers moving through server from $s+1_{j-1}-1_{j-1}$

$$+\lambda a_0 P(\chi,\underline{s}-1_1,t)$$ Arrival to server from $s-1_1$

$$-P(\chi,\underline{s},t)\left(\lambda+\sum_{j=1}^{d} n_j\mu_j\right)$$ Transitions out of s

Which has solution:

$$P(\chi,\underline{s},t) = \frac{(\lambda t)^{\chi}}{\chi!}e^{-\lambda t}\cdot\prod_{j=1}^{d}\frac{\left(\frac{\lambda}{\mu_j}\right)^{n_j}}{n_j!}$$