

## ON THE NOTION OF A RANDOM SEQUENCE

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In [1] A. N. Kolmogorov offered a definition of a random object. The necessity for introducing such a notion is connected with a number of difficulties in justifying probability theory as a natural science theory. In the indicated paper the quantity  $k(x)$ , the algorithmic complexity of an object  $x$ , was introduced, and those objects were considered random for which  $k(x)$  differed little from the logarithm of the probability of  $x$ . (Their difference is called randomness deficiency.)

However, this approach in its original form was suitable only for a finite number of equiprobable objects. Passing to the nonequiprobable case (which is always unavoidable when considering a random variable with an arbitrary integral value or a countable sequence of random variables) resulted in difficulties. To overcome these difficulties P. Martin-Lof gave up the introduction of an invariant (independent of a probability distribution) quantity of the type  $k(x)$  and introduced a separate criterion (test) of randomness (cf. [2] for each computable distribution. However, the randomness deficiency defined by Martin-Lof was not expressed in a natural way by a probability distribution and some invariant quantity. Kolmogorov's original definition has an advantage in this respect which it would be a pity to lose. We shall show in particular how this definition can be improved so that it is suitable for the most general case.

Of the already well-known results on this subject mention should be made of Schnorr's result (cf. [4]), which gives a criterion of weak randomness in terms of the usual Kolmogorov complexity, and of P. Gač's results [6].

We shall consider sequences of natural numbers, finite (corteges) and infinite. We call a sequence in which only the numbers 0 and 1 appear, *binary*. We call two sequences  $x$  and  $y$ , one of which is the beginning of the other ( $x \subset y$  or  $y \subset x$ ), *coordinated*. We recall that a denumerable set  $A$  of pairs of corteges  $(x, y)$  such that, if  $(x, y) \in A$  and  $(x', y') \in A$  and  $x$  is coordinated with  $x'$ , then  $y$  is coordinated with  $y'$ , is called a *computable operator* (cf. [1]).

Let  $\alpha$  be a finite or infinite sequence. Then all corteges  $y$  such that for some  $x \subset \alpha$  the pairs  $(x, y) \in A$  are the beginnings of a sequence  $\beta$  (finite or infinite), which is called the *image of the sequence  $\alpha$*  ( $\beta = A(\alpha)$ ).<sup>(1)</sup>

**Definition.** The minimal length of a binary cortege  $x$  such that  $A(x) \supset y$  is called the *monotone complexity of the cortege  $y$*  with respect to the operator  $A$  ( $km_A(y)$ ).

**Theorem 1.** Among all computable operators an "optimal one" exists with respect

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<sup>(1)</sup> The definition of a computable operator was encountered in this form by Ju. T. Medvedev.



