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Assumptions of randomness in cosmology models

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ABSTRACT

Non-compact symmetries cannot be fully broken by randomness since non-compact groups have no invariant probability distributions. In particular, this makes trickier the “Copernican” random choice of the place of the observer in infinite cosmology models.

This problem may be circumvented with what topologists call *pointed spaces*. Then randomness will be used only in building (infinite) models around the pre-designated “observance point”, that thus would not need to be randomly chosen.

Additional complications come from the original randomness possibly being hidden. P. Gacs and A. Kucera proved that every sequence can be algorithmically generated from a random one. But Vladimir V'yugin discovered that randomized algorithms can with positive probability generate uncomputable sequences not algorithmically equivalent to any random ones.

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Physics laws allow all sorts of events and histories. In infinite cosmology models each one of them rolls out infinitely often: weirdest miracles are daily routine in some places. To understand our boring world one must assume our place is rather generic. So, theories assume some events happen at random, others are derived from them by physical equations or other algorithms. In fact, [1,2] prove that any sequence is computable from an algorithmically random one.¹ However, the theories of this type can only be complete if they specify which exactly are the random events and what is their probability distribution. These events need not be directly observable, rather just be a part of the model from which the observations are derived. By [7], resulting observables, unlike random ones, could be very diverse in algorithmically invariant properties.

But the specification of this set of “external” events (not meant to be derived in the model, but rather assumed random) is often glossed over. Someone like Laplace, or another classical deist, might assume this to be the set of positions and speeds of all particles at the initial state of the Universe. The Creator would set them up, then retire, leaving the rest to the equations of physics. But this choice is quite arbitrary. The Creator could as easily set up the final state of the Universe at random. (As all equations known to Laplace were time-reversible.) Then the initial state would be very non-random! This may just flip the timeline, but even more elegant would be to set externally both initial *and final* positions of particles (and those on borders, if any), but no speeds. After all, the positions of a thrown stone at two moments of time, determine its trajectory in-between. Different choices of events designated as external and random, lead to quite different predictions.

The model also needs to describe what is a legitimate set of observations, which includes specifying the observer (say, the humanity with its followers). Some models have the Universe as a whole in a pure state, any perceived entropy is due

URL: <https://www.cs.bu.edu/fac/Lnd>.

¹ Algorithmic randomness (see: [3,4]) of a digital sequence ω does not quite assure it obeys all probability 1 math laws (only computable ones). Yet, it assures (see: [5]) that ω either obeys all such laws or has an infinite information about some mathematically definable object. The latter case would be really weird. There are no ways for such ω to be generated; see this informational version of Church-Turing Thesis discussed at the end of [6].

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to the whole being unobservable: each specific observer can see only a part,² which is in a mixed state. (In QM, unlike Classical Physics, parts can have higher entropy than the whole.) Then the choice of the observer is actually the source of the randomness. While the theory may be largely indifferent to that choice, it may be impossible to formulate without one: remote observers cannot communicate to combine experiences. The choice of the observer is as much a part of a complete model, as is the state of the rest of the universe.

A technical difficulty comes up here. While the observable part of the world is finite, many models include it in a greater infinite realm, beyond our reach. How to choose our (observer's) place in this realm at random is puzzling. Say, in a most primitive setting, how to choose a random point in a pattern on an infinite Euclidean space? It carries no uniform probability distribution. (In other models, where ours is one of the infinitely many "bubbles," it may be even much trickier.) A way around this technicality can lie in the reverse order of choices. Instead of first building an infinite universe and then choosing the place of the observer there, we can first designate the observer, then build an infinite universe around her.³ This circumvents the lack of uniform distributions in infinite domains (and tickles our ego, besides :-). Such structures are called *pointed spaces* in topology. Among the challenges is the need to expand the universe not only to the observer's present and all future times, but also to her whole past (if there are any causal relations with the past).

This brings up a question, what is an observer? Its many meanings depend on what symmetries are meant to be broken by choosing this sort of a "gauge". In quantum physics observer can mean a macroscopic system brought into an entanglement with the observed microscopic variables. "Macroscopic" is a vague term with unclear relevance. Sometimes it is put in relation with the Planck mass (e.g., by R. Penrose). But it is unclear why a microscopic bacteria, much lighter than Planck mass, cannot perfectly serve as an observer. (It could transmit its observed data to its descendants, which may include members of Royal Society. :-)

What seems relevant here, is that the observed data, unlike generic quantum states, can be reliably copied, preserved, transmitted, etc. For this an observer needs some sophistication, access to mechanisms for error-correction, self-preservation, etc. (This requires free energy flow: so gravity, its ultimate source, may play a role for observers, even if not via Planck mass. To enter a mixed state, observers need to discard parts of the system. Thus they consume free energy and expel heat to some horizon: cosmological or black hole's, or some such. This generates entropy, both of the observer and of the horizon if taken separately, though commonly used entropy formulas for those horizons are somewhat speculative.⁴) Such features are readily present in life.

Our world is in a state that harbors life, however mysterious is life's origin. The present living creatures developed by evolution. But for evolution even to start, it needs systems capable of copying themselves, along with accumulated random mutations. The first such systems could not be produced by evolution, so must appear spontaneously. The minimal complexity C of such systems may be significant, and the chance P of their spontaneous generation too tiny: $1/P$, exponential in C , exceeding cosmological scales.

If so, how to deal with these tiny probabilities? It may be that the laws of physics are fine-tuned to boost them. But it may be that this fine-tuning requires high complexity of such fine-tuned laws, which just transfers the tiny probability issue from live systems to laws of physics.

(I avoid the popular but unclear term "Anthropic principle". As was said "To discover a land means to look upon it with the eyes of a European, preferably British." Is fluent English required for being fully Anthropic? (-) I assume for observers only the seemingly relevant abovementioned "copyright" for observed data. I even ignore another frequent vague assumption – the free will.⁵)

² The unitary evolution of the whole Universe preserves its pure state. Observers can be separated by horizons but even if not, the heat escaping to the horizon leaves the remaining observable subsystem in a mixed high entropy state. If the Universe stops expanding and recollapses, the escaped heat returns, and being entangled with the left behind matter, might restore its pure state. However, imagining the mechanism for this is tricky, which provokes switching to more complicated infinite models that face problems, including those discussed below.

³ One may argue that computing probability of any *finite* event requires only placing the observer in a finite region, which can be done at random. However this does not provide a complete model of the infinite universe. Some global properties do not manifest in any finite region. And a compact event X is the intersection of a monotone family of clopen (finite) events X_i . Its probability is $\inf_i \text{Prob}(X_i)$. But X_i s must all agree on the observer's surroundings. So that would anyway amount to setting the observer first and then using randomness to generate the universe around.

⁴ When a max mass neutron star absorbs a little more gas and collapses into a black hole, its mass changes little, radius shrinks just twice or so, but its entropy grows by 20 orders of magnitude, approaching the combined entropy of all neutron stars in the universe. This sounds strange. Ordinary matter systems cannot have entropy S in volume less than about S^2 in Planck units: by Pauli exclusion for fermions, Stefan-Boltzmann energy density for photons, Schwarzschild radius limit, etc. But black holes of such volume S^2 get $S^{4/3}$ Bekenstein-Hawking entropy (= area), the bound inspired by a GR theorem that the total black holes surface areas cannot shrink. I do not know if GR has counterexamples to a stronger statement that the sum of square roots of black holes volumes never shrinks either, or if LIGO can find colliding black holes that radiate away over 20% of their combined mass.

⁵ Stretching the relevance, let me at least explain how I understand this "free will" notion. Physics equations generate the world from some random variables. Observations change their original probability distribution, conditioning it on the observed knowledge. (Observations reveal only some complicated derivative restrictions on the original, not directly observed, random variables. This makes tricky guessing our environment in some useful ways: we use "Occam Razor," analogies, etc., instead of computing probabilities directly.) We model some systems describing their behavior or a probability distribution under which that behavior is random. An impossibility of having an adequate model is expressed as system's free will. A modeling entity needs to exceed the modeled one in complexity, so cannot model itself, must accept its own free will. And free will is "contagious": extends to anything interacting with a free-willed entity.

As an example of free will subtlety let me bring Hillel's Golden Rule (expect yourself to be treated the way you treat others). It can be promoted by effective systems of justice (expensive and tricky to design) that reveal, judge, and reciprocate what we do to each other. It can also be promoted by

However, the mystery seems to soften with the abovementioned priority of choosing the observer. If models start with designating an observer (and then choosing her details and building a universe around) then the worlds with no observers are excluded before computing the probabilities.

Such settings may also help with another issue. Some fundamental laws of physics seem to yield paradoxes or even conflict with each other. But it may suffice for our needs if the laws are only *approximately* sound. For instance, the observer cannot be absolutely reliable. It may spontaneously tunnel into something entirely different, albeit with an exponentially small probability. A theory may be clear if such effects are ignored, assuming the observer behaves “as advertised”. And it may be O.K. for the theory to become incomprehensible if negligible likelihoods, such as, e.g., observer’s drastic tunneling, must be accounted for.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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a PR-induced faith that such a system exists, working in some mysterious way. But PR and mysteries, overused for corrupt purposes, meet widespread skepticism nowadays which impairs that approach.

The meaning of free will can clarify another approach: Observing my own **unpredictable** actions, I learn more about myself. I interact with fellow humans who share with me origins, environment, and many qualities. Thus learning about myself, I indirectly learn about them, too. This knowledge changes the probability distribution on my environment. For instance, my kind or wicked acts, besides their immediate effects, reveal to me more aspects of human nature, of which mine is the most intimately visible example. This subtle “two-way street” effect of our free will (on human nature we get to expect) sheds some light on the faith in the Golden Rule.