

# On Distributed Virtual Network Embedding with Guarantees

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**Abstract**—To provide wide-area network services, resources from different infrastructure providers are needed. Leveraging the consensus-based resource allocation literature, we propose a general distributed auction mechanism for the (NP-hard) virtual network (VNET) embedding problem. Under reasonable assumptions on the bidding scheme, the proposed mechanism is proven to converge, and it is shown that the solutions guarantee a worst-case efficiency of  $(1 - \frac{1}{e})$  relative to the optimal node embedding, or VNET embedding if virtual links are mapped to exactly one physical link. This bound is optimal, that is, no better polynomial-time approximation algorithm exists, unless  $P = NP$ .

Using extensive simulations, we confirm superior convergence properties and resource utilization when compared with existing distributed VNET embedding solutions, and we show how by appropriate policy design, our mechanism can be instantiated to accommodate the embedding goals of different service and infrastructure providers, resulting in an attractive and flexible resource allocation solution.

**Index Terms**—Network virtualization, approximation algorithms, consensus algorithms, resource allocation, virtual network embedding.

## I. INTRODUCTION

THE challenge of deploying wide-area virtualization based network services recently spurred interest in both the business and the research communities: from a research perspective, this enables the networking communities to concurrently experiment with new Internet architectures and protocols, each running on an isolated instance of the physical network. From a market perspective, this paradigm is appealing as it enables multiple infrastructure and service providers (InPs and SPs) to experiment with new business models that range from leasing their infrastructure to hosting multiple concurrent network services.

A virtual network (VNET) is a set of virtual instances spanning a set of physical resources, *e.g.* processes and physical links, and by network service we mean the commodity supplied by the VNET, *e.g.* an online game or the access to a distributed virtual network testbed. Examples of service providers are content delivery networks, high-performance computing systems such as cluster-on-demand, or large-scale distributed testbed platforms (*e.g.* Emulab [38], GENI [2]). InPs may cooperate or compete to provide such services themselves, or they could lease their resources to an SP. We consider a model in which a set of InPs receive a VNET request from

an SP (or an intermediary “connectivity” provider [42]), and they then attempt to embed it in a distributed fashion: each InP independently submits a set of bids, and a distributed process is used to reach consensus.

The virtual network embedding problem<sup>1</sup> consists of three tasks: (1) resource discovery, which involves monitoring the state of the physical resources, (2) virtual network mapping, which involves matching VNET requests to the available resources, and (3) allocation, which involves assigning the resources that match the VNET requests. These three tasks are tightly coupled, and although there exists a wide spectrum of solutions that solve a particular task, at most two tasks along with their interactions have been considered (see Section II or [15] for a complete survey).

Distributed virtual network mapping solutions that allow different InPs to collectively embed a virtual network already exist [7], [21], [40]: some of them focus on the desirable property of letting InPs use their own (embedding) policies [7], while others rely on truthfulness of a virtual resource auction [40]. Although they have systematic logic behind their design, such distributed solutions are still restricted to a subset of the three virtual network embedding tasks, they have performance (*e.g.* convergence speed or resource utilization) tightly determined by the chosen heuristic, and they are limited to a single distribution model — the type and amount of information propagated to embed a VNET.

Existing embedding solutions are also restrictive with respect to VNET’s arrival rate and duration: the lifetime of a VNET can range from few seconds or minutes (in the case of cluster-on-demand services) to several months and years (in the case of a VNET hosting a content distribution service similar to Akamai [34], or a GENI [2] VNET hosting a novel architecture looking for new adopters/users to opt-in). For instance, in wide-area testbed applications, virtual networks are provided in a best-effort manner, and the inter-arrival time between VNET requests and the lifetime of VNETs are typically much longer than the virtual network embedding time, so existing solutions assume complete knowledge of the network state, and ignore the overhead of resource discovery and the VNET embedding time. In applications with higher churns, *e.g.*, cluster-on-demand such as financial modeling, anomaly analysis, or heavy image processing, where VNET providers have rigid Service Level Objectives (SLO) — the technical requirements within a Service Level Agreement (SLA) — or where VNETs have short lifetime and expect short response time, it is desirable that solutions attempt to reduce the VNET

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<sup>1</sup>Two alternative terms for this problem are “slice embedding” coined in [18], and “virtual network provisioning” [20].

embedding time, and employ limited resource discovery to reduce overhead.

In summary, due to the wide range of providers' goals and allocation models (*e.g.*, best effort or SLO), a flexible solution that is adaptable to different provider goals and tackles the distributed VNET embedding with its three phases does not yet exist. Moreover, none of the previously proposed solutions give guarantees on both the convergence of the VNET embedding process, and on allocation performance — ratio of the number of VNETs successfully allocated on the physical infrastructure to the total requested.

**Our Contributions.** To this end, leveraging properties from the consensus literature [30], we propose a general Consensus-based Auction mechanism for Distributed virtual network embedding (CAD). The mechanism is general as it supports a large spectrum of applications and providers' objectives along with their distribution models by tuning its policies. CAD iterates over a bidding and an agreement (or consensus) phase to embed virtual nodes, before a third phase embeds virtual links. By only exchanging bids and few other policy-driven information with their neighbors, physical nodes discover available resources, find a mapping solution and agree on a VNET assignment.

To demonstrate its flexibility, we compare and analyze the tradeoffs between two different policy configurations of CAD: the first, that we call *Single Allocation Distributed* embedding (SAD), allows bidding on a single virtual node per auction round. The second, called *Multiple Allocation Distributed* embedding (MAD), allows bidder physical nodes to win multiple virtual nodes simultaneously and therefore leads to faster VNET embedding (convergence) time. Using extensive simulations, we show the counter-intuitive result that despite full knowledge of the entire VNET to be allocated before bidding, MAD may yield lower allocation efficiency. This is surprising as typically (online) algorithms that operate without the full knowledge of the input perform no better than their offline counterpart. Moreover, we show that SAD better balances the load and often has shorter response time — time to identify whether a VNET can be embedded — independently from the virtual network topology. Furthermore, we investigate the effects of a path auction (PAD) policy, that can be instantiated to avoid intermediate non-bidding (relay) physical nodes when embedding a virtual link on loop-free physical paths. The PAD policy simultaneously embeds virtual nodes and links.

It is known that distributed auctions converge to a solution if the bidding function has a property called *Diminishing Marginal Gain (DMG)*, that is, the gain in adding another item to the auction bundle cannot marginally increase [6]. We were able to improve the bound on optimality by relaxing the DMG assumption of [6], assuming that our bids on virtual nodes are obtained using a sub-modular function [26].<sup>2</sup> Moreover, we obtain the same convergence result relaxing the sub-modularity assumption and using the notion of *pseudo-submodularity*

<sup>2</sup>This DMG condition is a subset of the sub-modularity condition: both DMG and sub-modular functions lead to a notion of non-increasing marginal gain when adding an item to a set, but there is no notion of ordering in the items of a sub-modular set while a DMG function is defined on ordered sets.

of the utility function that physical nodes use to bid, that is, each physical node is free to use *any* private bidding function for each auction round, provided it communicates its bids in a way so as they *appear* to be obtained from a sub-modular function. We show that independently from the bidding policy that InPs decide to adopt, CAD has a worst-case convergence time of  $D \cdot |V_H|$ , where  $D$  is the diameter of the physical network and  $|V_H|$  is the size of the virtual network  $H$  to be embedded; as a direct consequence, we show bounds on the CAD communication overhead as well. Under the same assumptions, we also show that CAD has a minimum performance guarantee of  $(1 - e^{-1})$  relative to the optimal solution, and that this bound is optimal, that is, no better approximation exists unless  $P = NP$ .

**Paper Organization.** The rest of the paper is organized as follows: in Section II we discuss related centralized and distributed virtual network embedding solutions, as well as related work that uses auctions to guarantee some bounds in other allocation problems. In Section III we describe the Consensus Based Auction (CAD) mechanisms, as well as the notion of pseudo sub-modular utility functions. Section IV contains our theoretical results on convergence, and similar resource allocation approaches that provide guarantees with respect to similar network utility maximization problems. Section V discusses the evaluation via our simulation campaign, and we conclude our work in Section VI.

## II. RELATED WORK

**Centralized virtual network embedding:** existing virtual network embedding solutions either solve a specific task of the VNET embedding problem, or are hybrids of two tasks. Some solutions jointly consider resource discovery and virtual network mapping [22], [35], or discovery and allocation [3] (mapping single virtual machines); others only focus on the mapping phase [10], [25], [41], or on the interaction between virtual network mapping and allocation [39], yet others consider solely the allocation step [4], [8], [9], [17], [27]. Moreover, there are solutions that assume the virtual network mapping task is solved, and only consider the interaction between the resource discovery and allocation [37]. In addition to considering one [4], [41] or more tasks [35], [39], solutions also depend on whether their objective is to maximize the utility of VNET requesters [35] or infrastructure providers [4], [29], [41]. CAD simultaneously considers discovery, mapping and assignment, and its policies can be oriented towards the goals of either users or providers.

**Distributed virtual network embedding:** to avoid restricting services within a limited single provider's domain, distributed solutions to the VNET embedding have been proposed. Some solutions rely on a centralized authority that partitions the VNET and orchestrates the mapping [20], [42], while others do not require such orchestration and hence we classify them as fully distributed [21]. The only (to the best of our knowledge) fully distributed embedding solution existing today [21] has discouraging discovery overhead as each mapping information is flooded to all physical nodes. The resource discovery phase is different in PolyViNE [7], where an SP sends the entire VNET to a subset of trusted InPs, which can eventually

map the VNET partially, and forwards the residual virtual subgraph to another set of trusted InPs. The process continues and the VNET is rejected if a threshold number of hops is reached before its mapping is complete. The SP does the final allocation, based on the best price among the multiple candidate mapping solutions returned by different sets of InPs. The mapping and the allocation depend on the discovery, that is, on the sequence of visited InPs and therefore the proposed heuristic in practice lead to heavy sub-optimality or to significant overhead (in case the residual virtual network is flooded to all remaining InPs).

Our mechanism also supports VNET splitting and centralized embedding orchestration, but its bidding mechanism (thanks to the max-consensus strategy) provides a complete resource discovery relying on low overhead nearest-neighbor communications, and furthermore mapping/assignment is concurrently done.

**Auctions and guarantees:** the idea of using auctions for a distributed VNET allocation has been floated before: V-Mart [40] ensures a fair market but its auction winner determination algorithm does not provide any guarantees with respect to the sum of provider utilities. Auction algorithms and their optimality performance have also been theoretically studied in several application domains [5]. In electronic commerce for example [11], truthful auction strategies are sought when multiple items are released by a centralized auctioneer, and guarantees on an equilibrium are proven to exist [28]. Our approach does not need a centralized auctioneer, and we also prove bounds on the number of iterations to reach an equilibrium (convergence to an embedding), as a function of the physical network diameter, and the size of the VNET to allocate. Moreover, in our settings truthful strategies may not work as there is uncertainty on whether more VNETs, or even more virtual nodes in the same VNET, are to be assigned in the future; bidders may have incentives to preserve resources for stronger future bids.

In different settings, Choi *et al.* [6] present a decentralized auction that greedily assigns tasks to a fleet of robots. Not only our problem formulation allocates virtual nodes and links, and physical nodes do not move as robots do, but most importantly, in [6], the notion of Diminishing Marginal Gain has been used, a stronger notion than sub-modularity that yields a bound of 0.5 w.r.t. the optimal utility. We show how we can obtain a bound of  $1 - 1/e$  (circa 0.632) using the known sub-modularity property, but without using sub-modular functions (pseudo-submodularity is enough.) We also show that no better approximation exists using a novel reduction from the maximum  $k$ -coverage problem (see Appendix B).

### III. CONSENSUS-BASED AUCTIONS FOR DISTRIBUTED VIRTUAL NETWORK EMBEDDING

**Problem statement.** Given a virtual network  $H = (V_H, E_H, C_H)$  and a physical network  $G = (V_G, E_G, C_G)$ , where  $V$  is a set of nodes,  $E$  is a set of links, and each node or link  $e \in V \cup E$  is associated with a capacity

TABLE I  
SYMBOLS AND NOTATIONS

Virtual Network Embedding Model	
SP	$\triangleq$ Service Provider
InP	$\triangleq$ Infrastructure Provider
$H$	$\triangleq$ Virtual Network to be embedded
$G$	$\triangleq$ Hosting Physical Network of InP processes
$D$	$\triangleq$ diameter of the physical network $G$ .
VN	$\triangleq$ Virtual Node
VL	$\triangleq$ Virtual Link
VNET	$\triangleq$ Virtual Network
$V_H (V_G)$	$\triangleq$ Node set in the virtual (physical) network $H (G)$
$E_H (E_G)$	$\triangleq$ Edges set in the virtual (physical) network $H (G)$
$E_H (E_G)$	$\triangleq$ Edges set in the virtual (physical) network $H (G)$
$\mathcal{P}$	$\triangleq$ Set of loop-free physical paths in $G$
$l^H \in E_H$	$\triangleq$ $(s^H, r^H)$ virtual link starting at at virtual node source $s^H$ and ending at virtual destination $r^H$
$p \in \mathcal{P}$	$\triangleq$ $(s^G, \dots, r^G)$ physical loop-free path starting at physical node source $s^G$ and ending at $r^G$
$C(e)$	$\triangleq$ Constraint associated to element $e \in V \cup E$ of the (virtual or physical) network
$\mathcal{M}$	$\triangleq$ $H \rightarrow (V_G, \mathcal{P})$ mapping function
$U_i$	$\triangleq$ $\mathbb{R}_+^{ V_H } \rightarrow \mathbb{R}_+$ utility function of physical node $i$
$\mathcal{N}_i$	$\triangleq$ set of neighbors for physical node $i$

CAD Mechanism Definitions	
$\mathbf{b}_i \in \mathbb{R}_+^{ V_H }$	$\triangleq$ vector of highest bids $b_{ij}$ known so far by physical node $i$ on virtual node $j$
$T_i$	$\triangleq$ size of the bundle vector at physical node $i$
$\mathbb{T}_i$	$\triangleq$ target virtual node and link capacity at physical node $i$
$S_{ij}$	$\triangleq$ stress on physical node $i$ , including virtual node $j$ , whose bidding is being evaluated
$\mathbf{m}_i \in \mathbb{V}_H^{T_i}$	$\triangleq$ vector (bundle) of size $T_i$ containing virtual node IDs currently mapped on physical node $i$
$\mathbf{a}_i \in V_G^{ V_H }$	$\triangleq$ vector of size $ V_H $ containing the latest information on the assignment of all virtual nodes as known by physical node $i$
$\mathbf{w}_i \in V_G^{ V_H }$	$\triangleq$ vector of size $ V_H $ containing the latest information on physical node IDs currently hosting all virtual nodes as known by physical node $i$
$\mathbf{x}_i \in V_G^{ V_H }$	$\triangleq$ binary vector of size $ V_H $ containing the latest information on the assignment of all virtual nodes as known by physical node $i$
$h_{ij}$	$= \begin{cases} 1, & \text{if } U_{ij} > b_{ij} \\ 0, & \text{otherwise} \end{cases}$
$W_{ij}(U_{ij}, \mathbf{b}_i)$	$\triangleq$ $\mathbb{R}_+^{ V_H } \rightarrow \mathbb{R}_+$ warping function of physical node $i$ , with bid vector $\mathbf{b}_i$ when bidding on virtual node $j$ using utility $U_{ij}$

constraint  $C(e)$ ,<sup>3</sup> a virtual network mapping (or embedding) is a mapping of  $H$  onto a subset of  $G$ , such that each virtual node is mapped onto exactly one physical node, and each virtual link is mapped onto a loop-free physical path  $p$ . Formally, the mapping is a function  $\mathcal{M} : H \rightarrow (V_G, \mathcal{P})$  where  $\mathcal{P}$  denotes the set of all loop-free paths in  $G$ .  $\mathcal{M}$  is called a *valid mapping* if all constraints of  $H$  are satisfied, and for each  $l^H = (s^H, r^H) \in E_H$ ,  $\exists$  at least one physical loop-free

<sup>3</sup>Each  $C(e)$  could be a vector  $(C_1(e), \dots, C_\gamma(e))$  containing different types of constraints, *e.g.* physical geo-location, delay, or jitter. Such constraints can be expressed as Service Level Objectives (SLOs) with an object specification language, such as JSON or Google Protocol Buffers, as described in our technical report [13].

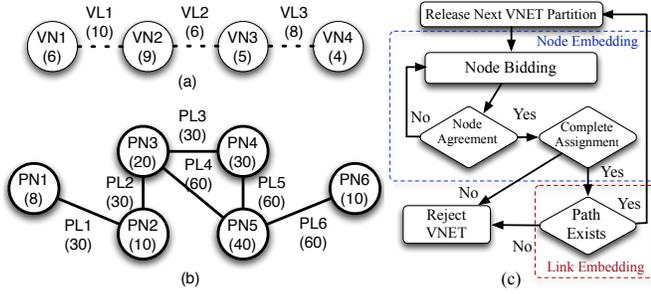


Fig. 1. (a) Virtual network with capacity constraints to be embedded. (b) Each physical node (PN) can be owned by a different InP, and can have a different capacity. Virtual Nodes (VNs) connected by Virtual Links (VLs) are embedded on Physical Nodes (PNs) connected by Physical Links (PLs). (c) CAD workflow: a virtual link embedding phase follows the virtual node bidding and agreement phases.

path  $p : (s^G, \dots, r^G) \in \mathcal{P}$  where  $s^H$  is mapped to  $s^G$  and  $r^H$  is mapped to  $r^G$ . All the symbols and notations used in this paper are summarized in Table I.

**Objective:** multiple valid mappings of  $H$  over  $G$  may exist; each physical node  $i$  has a utility function  $U_i : \mathbb{R}_+^{|V_H|} \rightarrow \mathbb{R}_+$ . We are interested in finding in a distributed fashion the embedding solution that maximizes the sum of the utilities of all providers  $\sum_{i \in V_G} U_i$ , *e.g.*, by letting InPs instantiate policies according to their goals and run the auction. A natural objective for an embedding algorithm is to maximize *revenue*. The revenue can be defined in various ways according to economic models. As in [39], we use the notion of a higher economic benefit (reward) from accepting a VNET or virtual request that requires more resources (*e.g.*, bandwidth, CPU) from the physical network, assuming that such revenue is proportional to the amount of requested resource. This means that physical nodes bid first on virtual nodes with the highest capacity, in a descending order, to maximize their potential “revenue”.

**CAD mechanism:** consider a VNET embedding request by an SP (Figure 1a) on a physical network (Figure 1b) where each physical node (PN) belongs to a different InP. The SP sends to (a subset of) all physical nodes a request with (a subset of) the virtual elements (nodes and links), *e.g.* virtual nodes VN1 and VN2 connected by virtual link VL1. Each physical node  $i$ , where  $i \in V_G$ , uses a private utility function  $U_i \in \mathbb{R}_+^{|V_H|}$  to bid on the virtual nodes, knowing that it could be the winner of a subset (for example VN1 or VN2 or both), and stores its bids in a vector  $\mathbf{b}_i \in \mathbb{R}_+^{|V_H|}$ . Each entry  $b_{ij} \in \mathbf{b}_i$  is a positive real number representing the highest bid known so far on virtual node  $j \in V_H$ . Also, physical nodes store the identifiers of the virtual nodes on which they are bidding in a list (bundle vector)  $\mathbf{m}_i \in V_H^{T_i}$ , where  $T_i$  is a target number of virtual nodes mappable on  $i$ . After the private bidding phase, each physical node exchanges the bids with its neighbors, updating an assignment vector  $\mathbf{a}_i \in V_G^{|V_H|}$  with the latest information on the current assignment of all virtual nodes, for a distributed auction winner determination.

The winner physical nodes communicate the mapping to the SP which, if possible, releases the next VNET(s) or the next

VNET partition if any (*e.g.* VN3, VN4, VL3 in Figure 1a)<sup>4</sup>. Once the physical nodes have reached consensus on who is the winner for all the virtual nodes of the (partial or full) VNET released for the auction, a distributed link embedding phase is run to embed each virtual link on a set of (one or many) loop-free physical paths (Figure 1c.) The mechanism iterates over multiple node bidding and agreement (consensus) phases synchronously, that is, the second bidding phase does not start until the first agreement phase terminates. Physical nodes act upon messages received at different times during each bidding phase and each consensus phase; therefore, each individual phase is asynchronous. In the rest of the paper, we denote such *rounds or iterations* of node bidding followed by consensus with the letter  $t$  and we omit  $t$  when it is clear from the context.

Adapting the definition of *max-consensus* from the consensus literature [30] to the VNET embedding problem we have:

**Definition 1.** (*max-consensus.*) Given a physical network  $G$ , an initial bid vector of physical nodes  $\mathbf{b}(0) \triangleq (\mathbf{b}_1(0), \dots, \mathbf{b}_{|V_G|}(0))^T$ , a set of neighbors  $\mathcal{N}_i \forall i \in V_G$ , and the consensus algorithm for the communication instance  $t+1$ :

$$\mathbf{b}_i(t+1) = \max_{j \in \mathcal{N}_i \cup \{i\}} \{\mathbf{b}_j(t)\} \quad \forall i \in V_G, \quad (1)$$

Max-consensus on the bids among the physical nodes is said to be achieved with convergence time  $\tau$ , if  $\exists \tau \in \mathbb{N}$  such that  $\forall t \geq \tau$  and  $\forall i, i' \in V_G$ ,

$$\mathbf{b}_i(t) = \mathbf{b}_{i'}(t) = \max\{\mathbf{b}_1(0), \dots, \mathbf{b}_{|V_G|}(0)\}, \quad (2)$$

where  $\max\{\cdot\}$  is the component-wise maximum.

**Assumptions:** we assume that physical nodes are aware of the physical outgoing link capacity to reach each of its first-hop neighbors to propagate the highest bids, the routing table for the path embedding phase, and the diameter  $D$  of the physical network, useful as a termination condition: if a physical node has received more than  $D$  messages the auction phase terminates.<sup>5</sup>

**CAD Policies:** one of the design goals of CAD is its flexibility — ability to create customizable VNET embedding algorithms to satisfy desired policies, rules, and conditions. We describe here such policies, and later in this section we show a few examples of how CAD can be instantiated to satisfy other goals. A straightforward example of policy is the (normalized) utility function  $\mathbf{U}$  that InPs use to bid on virtual resources (nodes). In our evaluation (Section V), the bid value of physical node  $i$  on virtual node  $j$  is equivalent to  $U_{ij}$ , where:

$$\mathbb{T}_i = C_i + \sum_{k \in \mathcal{N}_i} C_{ik}, \quad U_{ij} = \frac{\mathbb{T}_i - S_{ij}}{\mathbb{T}_i} \quad (3)$$

where  $\mathbb{T}_i$  is the target virtual (node and links) capacity that is allocatable on  $i$ , and  $S_{ij}$  is the stress on physical node  $i$ ,

<sup>4</sup>The VNET partitioning problem has been shown to be NP-hard, *e.g.* in [20] and it is outside the scope of this paper. In this work, we consider the VNET partitioning as a separate mechanism, necessary for the VNET embedding problem, and we let SP decide the objective, the type, and the size of each VNET partition.

<sup>5</sup>Algorithms to compute the diameter of a network in a distributed way are well known [30], and they are outside the scope of this paper.

namely, the sum of the virtual node capacity already allocated on  $i$ , including virtual node  $j$  on which  $i$  is bidding, plus the aggregate (allocated and requested) virtual link capacity attached to the virtual node under bidding. Note that we do not need any link embedding information to compute  $S_{ij}$  as its value is obtained based on the residual node and adjacent link capacities known from the previous embedding round. After a VNET is successfully embedded, the residual physical node and link capacity are updated with a new final value. Note that, due to the max consensus definition, the bid  $b_{ij}$  at physical node  $i$  on virtual node  $j$  is the maximum utility value seen so far. The normalization factor  $\frac{1}{\mathbb{T}_i}$  ensures that such bids are comparable across physical nodes.

We have seen from related work, *e.g.* [21], [42], how embedding protocols may require SPs to split the VNET request. CAD is able to express this requirement by enforcing a limit on the length of the bid vector  $\mathbf{b}_i$ , so that physical nodes bid only on the released VNET partition. Each InP can also enforce a load target on its resources by limiting its target allocatable capacity  $\mathbb{T}_i$ , which, in turn, limits its bundle size  $T_i$ .

Another auction policy is the *assignment vector*  $\mathbf{a}_i$ , that is, a vector that keeps track of the current assignment of virtual nodes.  $\mathbf{a}_i$  may assume two forms: *least* and *most informative*. In its least informative form,  $\mathbf{a}_i \equiv \mathbf{x}_i$  is a binary vector where  $x_{ij}$  is equal to one if physical node  $i$  hosts virtual node  $j$  and 0 otherwise. In its most informative form,  $\mathbf{a}_i \equiv \mathbf{w}_i$  is a vector of physical nodes that are so far winning the hosting of virtual nodes;  $w_{ij}$  represents the identifier of the physical node that made the highest bid so far to host virtual node  $j$ . Note that when  $\mathbf{a}_i \equiv \mathbf{w}_i$  the assignment vector reveals information on which physical nodes are so far the winners of the auction, whereas if  $\mathbf{a}_i \equiv \mathbf{x}_i$  physical node  $i$  only knows if it is winning each virtual node or not. As a direct consequence of the max-consensus, this implies that when the assignment (allocation) vector is in its least informative form, each physical node only knows the value of the maximum bid so far without knowing the identity of the bidder. We also leave as a policy whether the *assignment vector* is exchanged with the neighbors or not. In case all physical nodes know about the assignment vector of the virtual nodes, such information may be used to allocate virtual links in a distributed fashion. Instead, if  $\mathbf{a}_i \equiv \mathbf{x}_i$ , to avoid physical nodes flooding their assignment information,  $i$  asks the SP about the identity of the physical node hosting the other end of the virtual link and then attempts to allocate at least one loop-free physical path.

#### A. Phase 1: CAD Bidding (Auction) Phase

Consider Procedure 1 (locally executed by the physical nodes): after the initialization of the assignment vector  $\mathbf{a}_i$ , the bid vector  $\mathbf{b}_i$  and the bundle vector  $\mathbf{m}_i$  for the current iteration  $t$  (line 3)<sup>6</sup>, each physical node checks if another bidding phase is needed (line 4), for example because there is enough capacity or because the auction policy allows another bidding, or else terminates. If a physical node can bid, but cannot outbid any virtual node, the bidding phase terminates. If instead there is at least one biddable virtual node  $j$ , *i.e.*

<sup>6</sup>We elaborate on the need to reset  $\mathbf{m}_i$  at the end of Remark 2, Section III-C.

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#### Procedure 1 CAD biddingPhase for physical node $i$ at iteration $t$

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1: Input:  $\mathbf{a}_i(t-1)$ ,  $\mathbf{b}_i(t-1)$ 
2: Output:  $\mathbf{a}_i(t)$ ,  $\mathbf{b}_i(t)$ ,  $\mathbf{m}_i(t)$ 
3:  $\mathbf{a}_i(t) = \mathbf{a}_i(t-1)$ ,  $\mathbf{b}_i(t) = \mathbf{b}_i(t-1)$ ,  $\mathbf{m}_i(t) = \emptyset$ 
4: if biddingIsNeeded( $\mathbf{a}_i(t)$ ,  $\mathbb{T}_i$ ) then
5:   if  $\exists j : h_{ij} = \mathbb{I}(U_{ij}(t) > b_{ij}(t))$  then
6:      $\eta = \operatorname{argmax}_{j \in V_H} \{h_{ij} \cdot U_{ij}\}$ 
7:      $\mathbf{m}_i(t) = \mathbf{m}_i(t) \oplus \eta$  // append  $\eta$  to bundle
8:      $b_{i\eta}(t) = U_{i\eta}(t)$ 
9:     update( $\eta$ ,  $\mathbf{a}_i(t)$ )
10:    Send / Receive  $\mathbf{b}_i$  to / from  $k \ \forall k \in \mathcal{N}_i$ 
11:    if  $\mathbf{a}_i \equiv \mathbf{w}_i$  then
12:      Send / Receive  $\mathbf{w}_i$  to / from  $k \ \forall k \in \mathcal{N}_i$ 
13:    end if
14:  end if
15: end if

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if  $U_{ij}(t) > b_{ij}$  (line 5),<sup>7</sup> physical node  $i$  registers in its bid vector the bid with the highest reward  $\eta = \operatorname{argmax}_{j \in V_H} \{h_{ij} \cdot U_{ij}\}$  (line 6) and updates the state vectors (lines 7–9.) At the end of the bidding phase, the current winning bid vector (line 10) and if the auction policy allows it (lines 11–13), the assignment vector  $\mathbf{a}_i$  is exchanged with each neighbor. Depending on the configured policies, the functions biddingIsNeeded() and update() of Procedure 1 may behave differently.

**SAD configuration:** in particular, let us consider a scenario in which InPs (1) wish to reveal the least possible information to other (competitor) InPs, and (2) they are interested in the quickest possible response time for a VNET request. To accommodate these goals, we set the assignment vector policy to its least informative form, the partition size to two (so that a VNET is rejected as soon as one of the two virtual nodes or their adjacent virtual link is not allocatable), and the bundle vector size to one, so that the auction is on a single item. As we are forcing physical nodes to bid on a *single* virtual node per auction round, we refer in the rest of the paper to this policy configuration as Single Allocation for Distributed embedding (SAD).

**SAD bidding:** given such policy configuration, the biddingIsNeeded() function can be implemented by only verifying if  $A(t) = \sum_{j \in V_H} x_{ij}(t) = 0$ , knowing that bidders are only allowed to win one virtual node per round “ $t$ ”, that is,  $A(t) \leq 1$ . Given the SAD policy configuration, the update() function implementation simply changes the assignment vector from  $x_{i\eta}(t) = 0$  to  $x_{i\eta}(t) = 1$ .

**Example 1. (SAD bidding).** Consider Figure 1: virtual nodes VN1 and VN2 are released by the SP. Assuming that all nodes use as utility their residual node capacity, PN1, PN3 and PN5’s initial bidding vectors are  $\mathbf{b}_{PN1}(0) = (8, 0)$ ,  $\mathbf{b}_{PN3}(0) = (0, 20)$ , and  $\mathbf{b}_{PN5}(0) = (0, 40)$ . Note that the first bid of each physical node is its initial capacity, and PN1 could not bid on VN2 since VN2 requires 9 capacity units whereas PN1’s capacity is 8. Also we assume that a physical node, whenever feasible, bids on the virtual node with highest residual capacity as this brings higher reward

<sup>7</sup> $\mathbb{I}(\cdot)$  is an indicator function, unitary if the argument is true and 0 otherwise.

(revenue.) In their first bidding phase, physical nodes assign themselves as winners for the virtual nodes as they do not know yet each other's bids, and so  $\mathbf{x}_{PN1} = (1, 0)$  and  $\mathbf{x}_{PN3} = \mathbf{x}_{PN5} = (0, 1)$ .

**MAD configuration:** let us now consider a scenario in which embedding VNETs with the least possible auction iterations (convergence time) is more desirable than hiding information from other physical nodes. To this end, we remove the limit on the number of biddable virtual nodes within the same auction round, and we do not partition the VNET request so that each physical node has an offline knowledge of the entire VNET request (as opposed to SAD that releases the VNET components in an online fashion, *i.e.* the VNET embedding algorithm runs without a complete knowledge of the input.) Moreover, we set the assignment vector policy to its most informative form, so that the consensus is run simultaneously on both the bid vector and on the assignment vector.

**MAD bidding:** under these settings, the function `biddingIsNeeded()` is implemented so that it returns true while there is still room for additional virtual resources. The amount of virtual resources that physical node  $i$  is attempting to host can be expressed either in terms of the total number of virtual nodes in its current bundle  $\mathbf{m}_i(\mathbf{t})$ , *i.e.*  $|\mathbf{m}_i(\mathbf{t})|$ , or in terms of the resulting virtual capacity stress on physical node  $i$  as in (3). Also under these settings, the `update()` function implementation updates the allocation vector with  $w_{i,\eta}(t) = i$  (not just with 1 or 0 but with the identifier of the winning physical node).

**Example 2. (MAD bidding.)** Let us consider Figure 1 and let us assume that the target allocated capacity of PN3 is 16 units, and that the requested virtual capacity is equivalent to the reward that a physical node gets if it wins the hosting of that virtual node. In this example, let us also assume that physical node bids are equivalent to their residual physical capacity, *e.g.*, a physical node with residual capacity 10 units bids 10 to attempt the hosting of a virtual node whose requested capacity is no higher than 10 units. Let us apply MAD to construct the bundle of PN3. First PN3 bids on VN2, as it is the virtual node with the highest requested capacity (reward) and so  $\mathbf{b}_{PN3} = (0, 20, 0, 0)$ . After filling its bundle with VN2, PN3 updates its residual capacity from 20 to 11 (as VN2 requested capacity is 9). The next virtual node to be inserted in the bundle is hence VN1, as it has the highest requested capacity among the remaining virtual nodes. PN3 bidding phase terminates with  $\mathbf{b}_{PN3} = (11, 20, 0, 0)$ ,  $\mathbf{w}_{PN3} = (PN3, PN3, -, -)$  and bundle  $\mathbf{m}_{PN3} = (VN2, VN1)$ , as embedding more virtual nodes would increase the allocated capacity beyond the target.

### B. Phase 2: CAD Agreement Phase

In this phase, physical nodes make use of a maximum consensus strategy to converge on the winning bids  $\bar{\mathbf{b}}$ , and to compute the allocation vector  $\bar{\mathbf{a}}$  (Procedure 2).

The consensus, for example on the bid vector  $\mathbf{b}_i$  after receiving the bids from each physical node  $k$  in  $i$ 's neighborhood  $\mathcal{N}_i$ , is performed by comparing the bid  $b_{ij}$  with  $b_{kj}$  for all  $k$  members of  $\mathcal{N}_i$ . This evaluation is performed by the function `IsUpdated()` (line 5.) In case the auction requires

---

### Procedure 2 CAD agreementPhase for physical node $i$ at iteration $t$

---

```

1: Input:  $\mathbf{a}_i(t), \mathbf{b}_i(t), \mathbf{m}_i(t)$ 
2: Output:  $\mathbf{a}_i(t), \mathbf{b}_i(t), \mathbf{m}_i(t)$ 
3: for all  $k \in \mathcal{N}_i$  do
4:   for all  $j \in V_H$  do
5:     if IsUpdated( $b_{kj}$ ) then
6:       update( $\mathbf{b}_i(t), \mathbf{a}_i(t), \mathbf{m}_i(t)$ )
7:     end if
8:   end for
9: end for

```

---

consensus only on a single virtual node at a time, *i.e.*  $|\mathbf{m}_i| = 1$  as in SAD, the function `IsUpdated()` merely checks if there is a higher bid, that is, if  $\exists k, j : b_{kj} > b_{ij}$ . This means that when a physical node  $i$  receives from a neighboring physical node  $k$  a higher bid for a virtual node  $j$ , the receiver  $i$  always updates its bid vector  $\mathbf{b}_i$  ( $b_{ij} \leftarrow b_{kj}$ ), no matter when the higher bid was generated. In general, *i.e.*, when  $|\mathbf{m}_i| > 1$ , physical nodes may receive higher bids that are out of date. We discuss the conflict resolution of CAD in Section III-C.

**Example 3. (SAD consensus.)** We have assumed that hosting higher capacity virtual nodes brings higher revenue, and so continuing Example 1, after exchanging its bid vector with PN5, PN3 updates  $\mathbf{b}_{PN3}$  from  $(0, 20)$  to  $(0, 40)$ , and  $\mathbf{x}_{PN3}$  from  $(0, 1)$  to  $(0, 0)$ . Having lost the auction for node VN2 (the most profitable virtual node) to PN5, PN3 bids on VN1, and so updates again its bid vector from  $\mathbf{b}_{PN3} = (0, 40)$  to  $(20, 40)$ , as all PN3's capacity can now be used for VN1 and PN5's bid on VN2 is recorded. PN3 also changes its allocation vector again from  $\mathbf{x}_{PN3} = (0, 0)$  to  $(1, 0)$ . Eventually, all physical nodes agree that PN5's bid is the highest for the most profitable virtual node VN2, while PN4 wins VN1 as it has the highest residual capacity after VN2 assignment.

When instead physical nodes are allowed to bid on multiple virtual nodes in the same auction round ( $|\mathbf{m}_i| > 1$ ) as in MAD, even if the received bid for a virtual node is higher than what is currently known, the information received may not be up-to-date. In other words, the standard max-consensus strategy may not work. Each physical node is required to evaluate the function `IsUpdated()`. In particular, `IsUpdated()` compares the time-stamps of the received bid vector, and updates the bundle, the bid and the assignment vector accordingly (Procedure 2, line 6.) Intuitively, a physical node loses its assignment on a virtual node  $j$  if it gets outbid by another physical node that has a more recent bid, or after realizing that its bid for  $j$  was subsequent to another previous bid that it had lost more recently.

More precisely, in CAD, bids from a physical node for the same virtual node are required to be lower if more virtual nodes are previously allocated. This is obvious in our examples, as to bid, a physical node uses its residual capacity that decreases as more virtual nodes are added to the bundle — as we show later, this monotonically non-increasing condition must hold for any other utility function. This means that if a physical node  $i$  is outbid on a virtual node  $j$ , all the subsequent virtual nodes  $\mathbf{m}_{ij'}$ , for all  $j'$  appended in the bundle  $\mathbf{m}_i$  after

virtual node  $j$ , were computed using an invalid value and therefore need to be released, that is,  $\mathbf{b}_{ij'} = 0$ .

### C. Conflicts resolution

When it receives a bid update, physical node  $i$  has three options: (i) ignore the received bid leaving its bid vector and its allocation vector as they are, (ii) update according to the information received, *i.e.*  $w_{ij} = w_{kj}$  and  $b_{ij} = b_{kj}$ , or (iii) reset, *i.e.*  $w_{ij} = \emptyset$  and  $b_{ij} = 0$ . When  $|\mathbf{m}_i| > 1$ , the bids alone are not enough to determine the auction winner as virtual nodes can be released, and a physical node  $i$  does not know if the bid received has been released or is outdated. We show the complete conflict resolution table in the appendix.

We conclude this subsection with two remarks that explore how such conflicts are resolved. In particular, we illustrate how bids should be ignored or reset if they are outdated, and how subsequent bids to a more recently lost bid should be released.

**Remark 1.** (*bids may be ignored or reset.*) *There are cases in which the bid values are not enough to resolve conflicts, and so the time-stamps at which the bid was generated are used to resolve conflicts. In particular, (1) if a sender physical node  $i$  thinks that a receiver  $k$  is the winner and  $k$  thinks the winner is  $n \notin \{i, k\}$ , or (2) when  $i$  thinks  $n$  is the winner and  $k$  thinks the winner is  $m \notin \{n, i, k\}$ , or when (3) both  $i$  and  $k$  think  $m$  is winning but with a different bid. In all these cases, knowing which bid is most recent allows  $k$  to either ignore or update its bid based on the bid from  $i$ . In other cases, even the time-stamps are not enough and  $i$  and  $k$  need to reset their bids. In particular, (4) when  $i$  thinks the winner is  $k$  and  $k$  thinks the winner is  $i$ . In this case, even if  $i$ 's bid were more recently generated, it might have been generated before  $k$ 's bid was received by  $i$ .*

**Remark 2.** (*releasing subsequent bids.*) *Given PN3's bidding phase in Example 2, and computing PN5's vectors we have:  $\mathbf{m}_{PN5} = (VN2, VN1, VN3, VN4)$ ,  $\mathbf{b}_{PN5} = (31, 40, 25, 20)$  and  $\mathbf{w}_{PN5} = (PN5, PN5, PN5, PN5)$ . After receiving the bids from PN5, PN3 realizes that its first bundle's entry is outbid ( $20 < 40$ ) and so it must release VN2. Therefore PN3 needs to also release the other subsequent node in its bundle VN1, as its bid value was a function of the bid on VN2, *i.e.*, the bid on VN1 assumed the residual capacity after VN2 is allocated on PN3 (see also Figure 2).*

Since CAD allows physical nodes to bid using their most updated residual capacity, releasing subsequent items from a bundle intuitively improves the sum of the utilities of the physical nodes and hence, when physical nodes cooperate, this improves the number of virtual networks allocated. Moreover, as we show in Section IV, such residual capacity utility guarantees convergence to a VNET embedding. Note also that, if the utility considers both node and link capacity as in equation (3), a change of assignment of any virtual node not present in a bundle may invalidate all bids due to the VNET topology constraints. Assume, for example (Figure 1), that PN5 is winning VN2 when PN3 bids on VN1. The utility (and so the bid) on VN1 may change if the connected VN2 is later hosted by another physical node *e.g.* PN4, given a variation

Utility	Iteration 1	Iteration 2	Iteration 3
Non Sub-modular	$b_1 = \{10, 30\}, m_1 = \{A, C\}$ $b_2 = \{10, 20\}, m_2 = \{C, A\}$	Both PNs outbid on first VN $b_1 = \{\}, m_1 = \{\}$ $b_2 = \{\}, m_2 = \{\}$	identical to iteration 1 $b_1 = \{10, 30\}, m_1 = \{A, C\}$ $b_2 = \{10, 20\}, m_2 = \{C, A\}$
Sub-modular	$b_1 = \{20, 10\}, m_1 = \{A, C\}$ $b_2 = \{20, 10\}, m_2 = \{C, A\}$	Agreement $b_1 = \{20\}, m_1 = \{A\}$ $b_2 = \{20\}, m_2 = \{C\}$	

Fig. 2. Releasing outbid items (see Remark 2) combined with non sub-modularity may lead to instability: with a non sub-modular utility, after the first round both physical nodes have been outbid on the first virtual node, and their bids on the second virtual node have been invalidated, so the bundle is empty. Bids subsequent to an outbid are released seeking a Pareto optimality.

of a link stress during the bidding process. In particular, the residual physical link capacity to connect physical nodes PN3 and PN4 may become bigger than the residual capacity of the physical link connecting PN3 and PN5, for example due to a release of a VNET previously co-hosted by PN4 between two bidding phases of the current VNET. In another example, the residual capacity of the physical link PN3-PN4 can become null, not allowing the embedding of the VNET at all. To avoid storing bids computed with an out-of-date utility value, physical nodes simply reset their own bundle at the beginning of every bidding phase, regardless from the embedding policy (Procedure 1, line 3).

### D. Pseudo sub-modular utility functions

As we will see in Section IV, our CAD mechanism guarantees convergence allowing InPs to use their own bidding policies, as long as the function appears to be sub-modular to other bidders [24]. Sub-modularity is a well studied concept in mathematics [33], and applied to the distributed VNET embedding problem, can be defined as follows:

**Definition 2.** (*sub-modular function.*) *The marginal utility function  $U(j, \mathbf{m})$  obtained by adding a virtual resource  $j$  to an existing bundle  $\mathbf{m}$ , is sub-modular if and only if*

$$U(j, \mathbf{m}') \geq U(j, \mathbf{m}) \quad \forall \mathbf{m}' \mid \mathbf{m}' \subset \mathbf{m}. \quad (4)$$

This means that if a physical node uses a sub-modular utility function, a value of a particular virtual resource  $j$  cannot increase because of the presence of other resources in the bundle. CAD relies on the notion of sub-modularity to work: releasing outdated bids without a sub-modular utility may break the convergence to a conflict-free VNET embedding (see *e.g.*, Figure 2).<sup>8</sup>

Although having sub-modular utility functions may be realistic in many resource allocation problems [26], in the distributed VNET embedding problem this assumption may be too restrictive, as the value of a virtual node may increase as new resources are added to the bundle, *e.g.* the cost of mapping a virtual link between two virtual nodes decreases if a physical node hosts both virtual source and destination.

To guarantee convergence without using a sub-modular utility function, as in [24], we let each physical node communicate

<sup>8</sup>Figure 2 shows a counterexample of how a non sub-modular utility function may lead to non convergence of our mechanism. However, given our policy of physical nodes bidding on virtual nodes in the same (decreasing) order of their resource requirements, a non sub-modular utility function will converge to a stable virtual node assignment.

its bid on virtual node  $j$  obtained from a bid warping function:

$$\mathcal{W}_{ij}(U_{ij}, \mathbf{b}_i) = \min_{k \in \{1, \dots, |\mathbf{b}_i|\}} \{U_{ij}, \overline{\mathcal{W}}_{ik}\} \quad (5)$$

where  $\overline{\mathcal{W}}_{ik}$  is the value of the warping function for the  $k^{\text{th}}$  element of  $\mathbf{b}_i$ . Note how by definition, applying the function  $\mathcal{W}$  to the bid before sending it is equivalent to communicating a bid that is never higher than any previously communicated bids. In other words, bids *appear* to other physical nodes to be obtained from a sub-modular utility function.

### E. Phase 3: Virtual Link Embedding

Similar to the bidding and agreement phases for virtual nodes, in the virtual link embedding phase, our CAD mechanisms allow applications and provider’s goals to tune the VNET embedding protocol behavior through policy instantiation.

This last phase is based on the observation that all virtual link embedding schemes have two commonalities: information known at each physical node about physical paths, and the algorithm for determining the best physical path(s) to allocate a virtual link. We hence define three CAD policies for virtual link embedding: (i) the type of information known at each physical node, for example the routing table or the available paths for any source-destination, (ii) the update frequency of such information, for example every hour or every time a new VNET is requested, and (iii) the selection of physical path(s) over which a virtual link is mapped. One example of such virtual link embedding scheme is a simple SP assisted auction, where, similarly to [40] and [23], an SP elicits bids from each InP, computes the “cheapest” loop-free physical path according to the bids, and then allocates the virtual link on that path. As shown in [39], another effective example is a  $k$ -shortest path algorithm with path splitting [12].

In our experiments we let physical nodes know the routing table, computed only once at the beginning of our experiments using Dijkstra’s algorithm, and we also use the  $k$ -shortest (hop distance) path algorithm with  $k = 3$ . This virtual link (path) embedding policy has the limitation of expecting intermediate physical nodes on a path to accept the allocation of a virtual link if they have capacity. Internet Service Providers (ISPs) today often have to rely on other ISPs to relay traffic in order to reach some of their customers (see, e.g. [31]). In Section V-C, we describe another (path auction) embedding policy that let InPs avoid the drawback of relaying external traffic, at a cost of a lower physical network utilization.

## IV. CONVERGENCE AND PERFORMANCE GUARANTEES

In this section we show results on the convergence properties of CAD. By convergence we mean that a valid mapping (Section III) is found in a finite number of steps (Definition 1). Moreover, leveraging well-known results on sub-modular functions [16], [33], we show that under the assumption of pseudo sub-modularity (Section III-D) of the utility function, CAD guarantees a  $(1 - \frac{1}{e})$  optimal approximation, that is, a better approximation does not exist unless  $P = NP$ .<sup>9</sup>

<sup>9</sup>Note that in this paper we use utility functions that optimize the allocation of virtual nodes and their first-hop links, but not virtual path allocations.

**Convergence Analysis.** All physical nodes need to be aware of the mapping, by exchanging their bids with only their first-hop neighbors, therefore a change of bid information needs to traverse all the physical network, which we assume has diameter  $D$ . The following proposition (Proposition IV.1) states that a propagation time of  $D$  hops is also a necessary and sufficient condition to reach max-consensus on a single virtual node allocation. Another interesting observation that follows from the result is that the number of steps for CAD to converge on the embedding of a VNET of  $|V_H|$  virtual nodes is always  $D \cdot |V_H|$  in the worst case, regardless of the size of the bundle vector. This means that the same worst-case convergence bound is achieved if CAD runs on a single or on multiple virtual nodes simultaneously. These claims are a corollary of Theorem 1 in [6], which deals with a distributed task allocation problem for a fleet of robots.

Let the tasks allocated by a robot represent the virtual nodes to be hosted by a physical node. Therefore, by induction on the size of the bundle the following result holds as a corollary of Theorem 1 in Choi *et al.* [6]:

**Proposition IV.1.** (*Convergence of CAD*). *Given a virtual network  $H$  with  $|V_H|$  virtual nodes to be embedded on a physical network with diameter  $D$ , the utility function of each physical node is pseudo sub-modular, and the communications occur over reliable channels, then the CAD mechanism converges in a number of iterations bounded above by  $D \cdot |V_H|$ .*

*Proof.* We use  $\mathcal{W}_{ij}(U_{ij}, \mathbf{b}_i)$  as a bid function (sub-modular by definition). From [6] we know that a consensus-based auction run by a fleet of  $N_u$  agents, each assigned at most  $L_t$  tasks, so as to allocate  $N_t$  tasks, converges in at most  $N_{min} \cdot D$  where  $N_{min} = \min\{N_t, N_u \cdot L_t\}$ . Note that the proof of Theorem 1 in [6] is independent of the utility function used by the agents as long as they are sub-modular, and of the constraints that need to be enforced on the tasks. Since for CAD to converge, every virtual node needs to be assigned, in the distributed VNET embedding problem,  $N_{min}$  is always equal to  $N_t \equiv |V_H|$ , and therefore we prove the claim.  $\square$

As a direct corollary of Proposition IV.1, we compute a bound on the number of messages that physical nodes have to exchange in order to reach an agreement on a VNET embedding. Because we only need to traverse the physical network once, the following result holds:

**Corollary IV.1.** (*Communication overhead*) *The number of messages exchanged to reach an agreement on the node assignment using the CAD mechanisms is at most  $D \cdot |E_G| \cdot |V_H|$ , where  $D$  is the diameter of the physical network,  $|E_G|$  is the number of directed edges in the physical network, and  $|V_H|$  is the virtual network size.*

**Performance Guarantees.** It is known that distributed auctions converge to a solution if the bidding function has a property called *Diminishing Marginal Gain (DMG)*, that is, the gain in adding another item to the auction bundle cannot marginally increase [6]. We improve such bound by relaxing the DMG assumption of [6], and assuming that our bids on virtual nodes are obtained using a pseudo sub-modular function [26].

**Theorem IV.1.** (*CAD Approximation*). *The CAD node consensus algorithm yields an  $(1 - \frac{1}{e})$ -approximation with respect to the optimal node assignment solution. If we restrict the embedding of virtual links to physical paths of length at most one hop, the same approximation holds for the entire VNET (both node and link embedding).*

*Proof.* The CAD node consensus algorithm assumes that each physical node  $i$  does not bid on a virtual node  $j$  unless it brings a positive utility, therefore  $U_{ij}$  and so  $\mathcal{W}_{ij}$  are positive. Moreover, if we append to the bid vector  $\mathbf{b}_i$  an additional set of virtual nodes  $\mathbf{v}$  resulting in bid vector  $\mathbf{b}'_i$ , we have:

$$\mathcal{W}_{ij}(U_{ij}, \mathbf{b}'_i) \leq \mathcal{W}_{ij}(U_{ij}, \mathbf{b}_i) \quad \forall \mathbf{v} \neq \emptyset \quad (6)$$

which means that  $\mathcal{W}_{ij}$  is monotonically non-increasing.

Since the sum of the utilities of each physical node, and since the bid warping function  $\mathcal{W}_{ij}(U_{ij}, \mathbf{b}_i)$  of CAD is a positive, monotone (non-increasing) and sub-modular function, all the axioms of Theorem 3.1 in Nemhauser *et al.* [33] on sub-modular functions are satisfied. Therefore the claim holds.  $\square$

Moreover, the following approximation bound holds:

**Theorem IV.2.** (*Approximation Bound*). *The CAD VNET embedding strategy cannot be approximated in polynomial time within a ratio of  $(1 - \frac{1}{e} - \epsilon)$   $\forall \epsilon > 0$ , unless  $P = NP$ .*

*Proof.* See Appendix B.  $\square$

## V. PERFORMANCE EVALUATION

To test the proposed distributed auction algorithms, we developed our own event-driven simulator, whose code and traces are publicly available at [1].

**Physical Network Model:** Using the BRITE topology generator [32], we obtain a physical topology. We use the generation model of BRITE to build a flat topology using either the Waxman model, or the Barabasi-Albert model with incremental growth and preferential connectivity. We tested our algorithms with physical network sizes varying  $n$  physical nodes with about  $5n$  physical links (as in [39]). Our simulations do not consider delay constraints, while link capacity constraints are discussed later in this section. The results are similar regardless of the topology generation model and the physical network size. In this paper we only show the results obtained for  $n = 50$  and a Barabasi-Albert physical topology.

**Virtual Network Model:** we use a real dataset of 8 years of Emulab [38] VNET requests [36]. For each simulation run we process 61968 requests; the average size of a request is 14 with standard deviation of 36 virtual nodes; 99% of the requests have less than 100 virtual nodes, and 85% have at most 20 virtual nodes. Excluding the 10% long-lived requests that cause the standard deviation of VNET lifetime to exceed 4-million seconds, the duration of the requests is on average 561 with 414 seconds of standard deviation (Figure 3a). As the dataset does not contain neither the number of virtual links nor the virtual network topology, we connect each pair of virtual nodes at random with different

average node degree (Figures 3b, c, g, and h). Moreover, we extend our evaluation comparing linear, star, tree, and fully connected virtual topologies (Figure 3f). All our simulation results show 95% confidence intervals; the randomness comes from both the virtual network topology to be embedded, and the virtual constraints, that is, virtual node and link capacity requirements. Similarly to previous work [39], we randomly assign physical link capacities between 1 and 100, then we assign the physical node capacity to be the sum of its outgoing physical link capacities. Then we assume the virtual link capacity to be randomly chosen between  $1/\mathcal{R}$  and  $100/\mathcal{R}$ , where  $\mathcal{R} = \{50, 100, 500\}$ , and the virtual node capacity is then assigned to be the sum of its outgoing virtual links. The results are similar and we only show plots for  $\mathcal{R} = 100$ .

**Compared Algorithms:** we compare our CAD mechanism, instantiated with the SAD and MAD configuration, with another policy based distributed virtual network embedding algorithm, PolyViNE [7], and with the first published distributed virtual network embedding algorithm [21], that we call Hub and Spoke due to the adopted heuristic.

**Evaluation Metrics:** our evaluation results quantify the benefits of our approach along two dimensions: embedding efficiency and time to find a solution. In particular, we evaluate the time to find a solution using two metrics: 1) *response time*: the number of steps measured in one-hop communication delays needed to realize a VNET can or cannot be embedded, and 2) *convergence time*: the number of steps until a *valid* embedding is found. The efficiency of an embedding is evaluated using three metrics: 3) *VNET allocation ratio*: the ratio between the number of virtual networks successfully embedded and requested, 4) *resource utilization*: the physical node and link capacity utilized to embed the VNET requests, and 5) *endurance* of the algorithm: the number of successfully allocated requests before the first VNET request is rejected. We also evaluate the effect of different utility functions.

### A. Allocation-based and Time-based Tradeoffs

We present here our event-driven simulation results summarizing the key observations.

(1) *MAD leads to larger VNET allocation ratio, as long as multiple physical paths are available for each virtual link.* When the virtual link allocation policy allows a virtual link to be allocated only on a single physical shortest path, SAD has a higher VNET allocation ratio (Figure 3b). This is because SAD, allowing a single virtual node allocation for each auction round, balances the load over physical resources more efficiently. When instead a physical node  $i$  is allowed to simultaneously win a bundle of virtual nodes  $\mathbf{m}_i$  as in MAD, the physical links adjacent to  $i$  quickly exhaust their capacity due to the VNET topology; all the outgoing virtual links adjacent to the virtual nodes in  $\mathbf{m}_i$  that are not mapped on  $i$  are in fact mapped onto a small set of physical paths emanating from physical node  $i$ . However, if the virtual link embedding policy uses a  $k$ -shortest path (with  $k \geq 3$ ), MAD is able to allocate more VNETs (Figure 3c). From this result we conclude that when fewer physical paths are available, InPs should consider (switching to) a SAD setting, otherwise MAD is more efficient. In the considered physical topologies, there

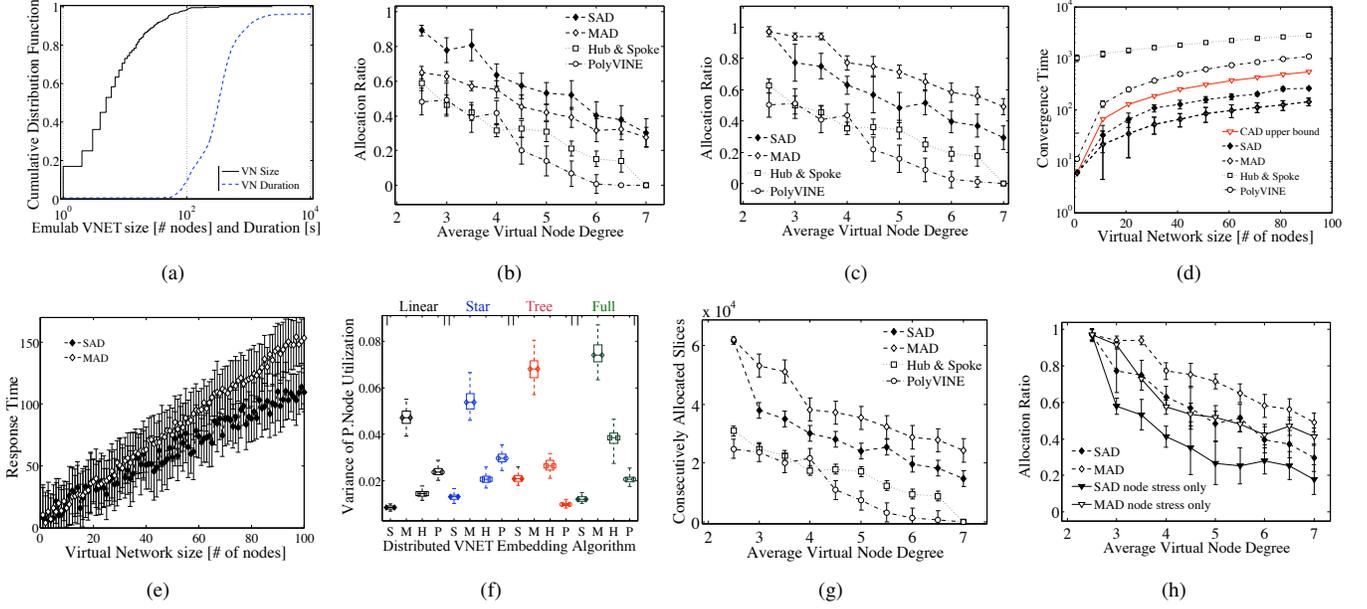


Fig. 3. (a) CDF of the size and lifetime of 8 years of Emulab VNET requests. (b) SAD allocates more VNETs when a single shortest path is available. (c) MAD allocates more VNETs when a  $k$ -shortest path link allocation policy (where  $k = 3$ ) is used. (d) MAD has shorter convergence time. (e) SAD has shorter response time. (f) SAD better balances the load on physical nodes ( $k = 3$ ). S, M, H and P indicate SAD, MAD, Hub and Spoke and PolyViNE, respectively. (g) MAD allocates more VNETs consecutively ( $k = 3$ ). (h) Considering simultaneously node and link stress in the utility improves the VNET allocation ratio.

are no more than 3 physical paths between any pair of physical nodes, and the confidence intervals overlap for SAD and MAD with  $k = 2$ .

(2) *MAD has faster convergence time.* Although we showed that MAD has the same worst-case convergence bound as SAD, simulation results show how MAD can in practice be faster (Figure 3d). In the best case, a single physical node has highest bids for all virtual nodes, and all the other bidders will converge on a VNET allocation in a single auction round.

(3) *SAD has faster response time.* Due to the VNET partitioning policy, that is, due to the fact that the SP releases only two virtual nodes at a time, SAD has a quicker response time as physical nodes immediately know if a virtual node or a link (and so the entire VNET) cannot be allocated (Figure 3e). We do not show the response time for the other algorithms in Figure 3e as they are similar to their convergence time.

(4) *SAD better balances the load independent of the VNET topology.* To verify our findings, we average over time the variance of the utilization across all nodes with 25% and 75% percentiles for each of the algorithms, and we repeat the experiment for linear, star, tree, and full virtual network topologies (Figure 3f). Note how SAD better balances the load, independent of the VNET topology. One exception is PolyViNE, that has lowest load variance for tree topologies, but at the expense of lowest VNET allocation ratio.

(5) *SAD allocates more VNETs before the first one is rejected.* As a direct consequence of a better VNET allocation ratio, we verify that SAD yields a larger number of VNETs allocated before the first one gets rejected in case the virtual link allocation policy allows only a single physical shortest path, while MAD allocates more requests if multiple physical loop-free paths are available (Figure 3g).

(6) *Considering link stress in the utility function improves*

*the VNET allocation ratio.* In this last experiment we show how different utility functions may lead to different VNET allocation efficiency. In particular, by comparing two different utilities, *i.e.*  $U'_{ij} = (\mathbb{T}_i - S'_{ij})$  where  $S'$  is only the stress on the physical nodes, and  $U_{ij}$  where the stress also includes adjacent physical links, we confirm the premise that considering nodes and links simultaneously in the VNET embedding problem leads to higher VNET allocation rate (Figure 3h). We leave the investigation of the best utility function given the goals of providers as an interesting research direction.

#### B. Physical Node Utilization: SAD to Balance the Load, MAD to Save Energy

In this set of experiments we aim to show the impact that different CAD policies have on the physical network load. In particular, by measuring the physical node utilization, we show how (7) *different CAD policies may result in a physical network with balanced load, confirming observation #4 described in Section V-A, or with multiple virtual nodes collocated on a small set of physical nodes.* The latter virtual node packing allows InPs to keep idle a higher number of physical nodes, therefore reducing the InP energy costs.

We compute the physical node utilization for two different CAD policies —MAD and SAD— as well as for our implementation of the PolyViNE and Hub&Spoke embedding heuristics, after embedding 100 virtual networks. Each virtual network has 50 virtual nodes and 0.5 probability of having an edge between any two virtual nodes; the physical network has 500 nodes, following a Barabasi-Albert connectivity model. When applying the SAD node embedding policy, all physical nodes have utilization lower than 35%, with over half of the physical nodes less than 20% utilized. When instead we instantiate CAD with the MAD node embedding policy, we

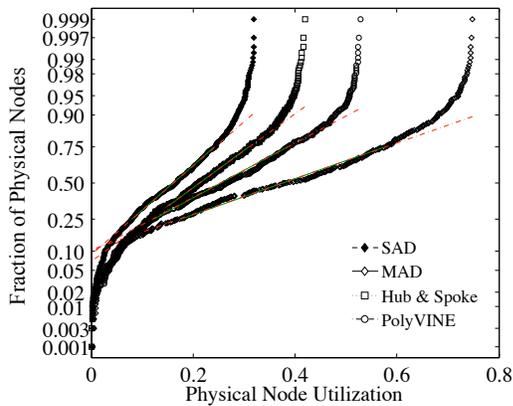


Fig. 4. Fraction of physical nodes utilization after embedding 100 virtual networks with 50 nodes and randomly connected topology, on a physical network with 500 nodes and a Barabasi-Albert connectivity model. Different CAD policies may be instantiated to balance the load on physical nodes, or to collocate multiple virtual nodes on a small set of physical nodes (allowing more physical nodes to remain idle).

obtained a higher physical node utilization: some physical nodes reached a 75% utilization (Figure 4). In this experiment, the available physical node and link capacities are enough to embed all the requested virtual networks. This means that a higher physical node utilization is a consequence of a higher number of idle physical nodes, since the (node and link) physical capacity necessary to embed the requests is equivalent across all embedding algorithms.

We were able to replicate this result across several physical network sizes and connectivity models. We only report a significant set of histograms that compare the smallest and the largest tested physical network sizes: 50 versus 500 nodes, with Barabasi-Albert and Waxman connectivity model (Figure 5). Independently from the physical network size and connectivity model, the SAD policy results in more physical nodes less utilized, and the MAD policy results in a few physical nodes hosting the majority of virtual nodes.

Note that our case study of load balancing versus bin packing behavior resulting from SAD and MAD, respectively, is an illustration of CAD’s flexibility, and a consequence of the monotone and sub-modular utility function used: the residual capacity. We do not exclude the existence of CAD settings that would enable bids on multiple resources (MAD-style), but with a utility function that rewards load balancing (to get the effect seen with SAD).

### C. Path Auctions

In this subsection we analyze the performance of a Path Auction for Distributed embedding (PAD), another CAD policy in which physical nodes attempt to host contiguous virtual paths. By contiguous virtual path we mean that neighboring virtual nodes are allocated to neighboring physical nodes. In other words, each virtual link is allocated on a single physical link, as opposed to being allocated on any (generally longer) loop-free physical path.

When the path auction policy is not applied, *e.g.*, for both SAD and MAD node allocation policies, virtual links may be established between non neighboring physical nodes, expecting intermediate physical nodes to relay data traffic. During the bidding phase, physical nodes applying the PAD

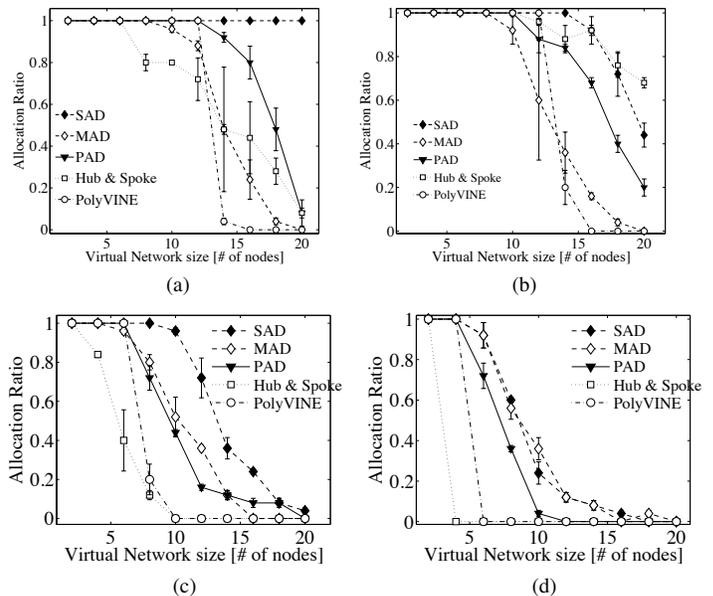


Fig. 6. Allocation ratio on a physical network of 100 nodes following a Barabasi-Albert connectivity model. (a – d) Virtual network allocation ratio with VNET topology: (a) linear, (b) hub & spoke, (c) random, and (d) full. The PAD policy performs better than the MAD policy but its performance degrades as we increase the number of virtual nodes and links.

embedding policy are allowed to attempt hosting a virtual node  $j$  only if the virtual nodes adjacent to  $j$  are currently won by the node itself, or by an adjacent physical node. By enforcing the PAD policy, a virtual path of length  $L > 0$  will be embedded on loop-free physical path of length at most  $L$ , hence avoiding physical node relays.

After implementing support for the PAD policy, we tested it against the other CAD embedding policies, as well as the other two heuristics —PolyViNE and Hub & Spoke. We found that (8) *the PAD policy has a higher VNET allocation ratio than the MAD policy but lower than the SAD policy, until there are “too many” virtual links to embed.* The advantage over the MAD policy vanishes as the number of virtual links grows, *i.e.* as we move from embedding a linear to a full virtual network topology (Figures 6a – d).

PAD limits the space of possible physical nodes that can participate in an embedding: during a PAD’s bidding phase, less physical nodes are permitted to bid, hence less physical node and link capacity is available to host the VNET. The PAD policy leads to an average physical path length in the range  $[0, 1]$  hops for embedding a virtual link (Figure 7a – d); a loop-free physical path has length 0 when the two end virtual nodes of a virtual link are both hosted by the same physical node, and length 1 when the two end virtual nodes of a virtual link are hosted by two neighboring physical nodes. Comparing Figure 6 and Figure 7 we observe how (9) *the physical link capacity provided by relay physical nodes helps improve the allocation ratio when embedding VNETs with higher number of virtual links, e.g. for a full VNET topology (Figure 7d).* Intuitively, the PAD policy is similar to the MAD policy except that fewer physical nodes are allowed to bid; when a smaller

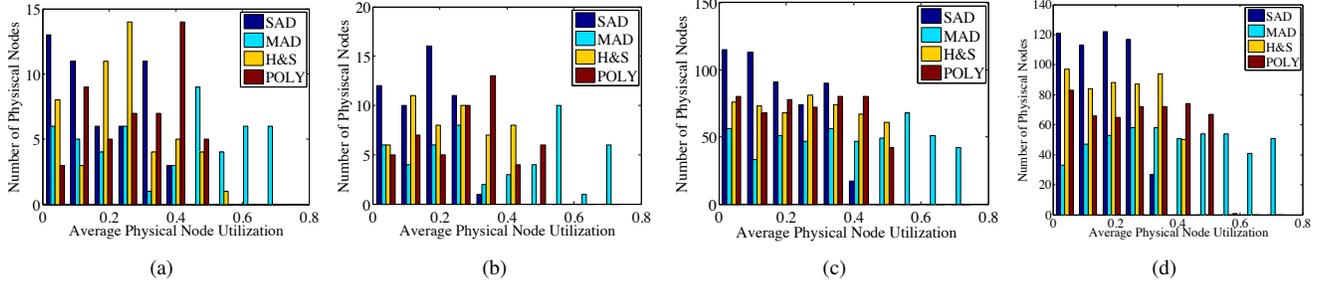


Fig. 5. Physical node utilization of a physical network. In (a) and (b) the network size is 50 physical nodes while in (c) and (d) 500 physical nodes. The physical network topology was obtained with the BRITE topology generator following a Waxman model (a) and (c) while the network topology follows a Barabasi-Albert connectivity model in (b) and (d). Independently from the physical network size and connectivity model, the SAD policy results in more physical nodes less utilized, and the MAD policy results in a few physical nodes hosting the majority of virtual nodes.

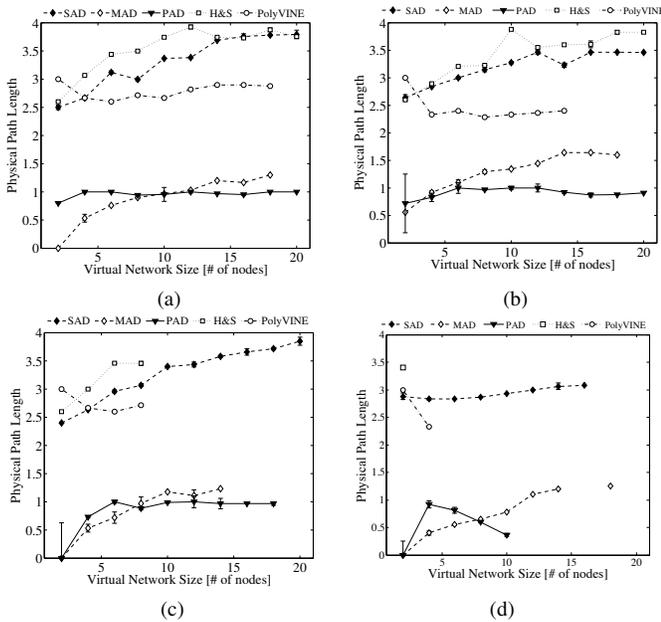


Fig. 7. Length of the physical loop-free paths hosting at least a virtual link for experiment described in Figure 6. The physical network has 100 nodes and follows a Barabasi-Albert connectivity model. The VNET topologies are: (a) linear, (b) hub & spoke, (c) random with 0.5 probability of having an edge between any two virtual nodes, and (d) full. The absent points correspond to zero allocation ratios in Figure 6.

set of physical nodes is allowed to embed a larger set of virtual (nodes and) links, the missing contribution of physical nodes that are not allowed to bid negatively impacts the allocation ratio. Note also that PAD falls under the category of multiple-item embedding policies, and so it has the same convergence time of MAD (results not shown).

#### D. Centralized Optimal versus Distributed Embedding

The VNET embedding problem has a history of centralized solutions, and a deep analysis of how CAD compares with such solutions is out of the scope of this work and it is outlined in our survey paper [15]. Even though our focus is on embedding involving multiple InPs that cooperate to provide a wide-area VNET service, we compare the impact of SAD and MAD with a centralized optimal virtual network embedding solution, solved with a CPLEX simulator using a *Branch and Bound* algorithm (Figure 8). We found that the

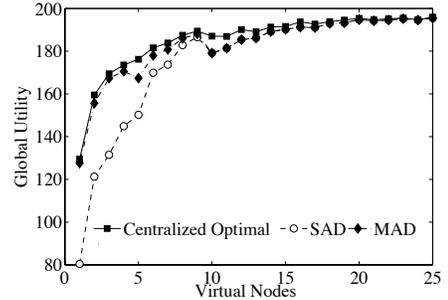


Fig. 8. In practice the values of Pareto (global) utility obtained with MAD and SAD are close to the centralized virtual network embedding optimal solution computed with Branch and Bound.

$a_{kj}$ for $k$ is	$a_{ij}$ for $i$ is	Receiver's action (default leave)
$k$	$i$	if $b_{kj} > b_{ij} \rightarrow$ update
	$k$	update
	$m \notin \{i, k\}$	if $s_{km} > s_{im}$ or $b_{kj} > b_{ij} \rightarrow$ update
	none	update
$i$	$i$	leave
	$k$	reset
	$m \notin \{i, k\}$	if $s_{km} > s_{im} \rightarrow$ reset
	none	leave
$m \notin \{i, k\}$	$i$	if $s_{km} > s_{im} \ \& \ b_{kj} > b_{ij} \rightarrow$ update
	$k$	$s_{km} > s_{im} \rightarrow$ update else $\rightarrow$ reset
	$n \notin \{i, k, m\}$	if $s_{km} > s_{im} \ \& \ s_{kn} > s_{in} \rightarrow$ update
		if $s_{km} > s_{im} \ \& \ b_{kj} > b_{ij} \rightarrow$ update
	none	if $s_{kn} > s_{in} \ \& \ s_{im} > s_{km} \rightarrow$ reset if $s_{km} > s_{im} \rightarrow$ update
none	$i$	leave
	$k$	update
	$m \notin \{i, k\}$	if $s_{km} > s_{im} \rightarrow$ update
	none	leave

TABLE II

RULES TABLE FOR CADE SYNCHRONOUS CONFLICT RESOLUTION. THE SENDER PHYSICAL NODE IS DENOTED WITH  $k$ , AND THE RECEIVER PHYSICAL NODE WITH  $i$ . THE TIME VECTOR  $\mathbf{s}$  REPRESENTS THE TIME STAMP OF THE LAST INFORMATION UPDATE FROM EACH OF THE OTHER PHYSICAL NODES.

Pareto optimality for a centralized embedding is not too far from the values obtained when applying the SAD or MAD policies in distributed settings. For larger scale, the separation between virtual node embedding and virtual link embedding into two phases may lead to suboptimal solutions. We study the problem of a distributed holistic VNET embedding using decomposition theory in [14].

## VI. CONCLUSIONS AND ONGOING WORK

In this work we proposed CAD, a general distributed approach to solve the virtual network embedding problem, consisting of three tightly coupled phases: discovery, virtual network mapping and allocation [15]. By leveraging the distributed task assignment literature, and well-known results on sub-modular function properties, we show how CAD has bounds on both convergence and performance. Using extensive event-driven simulations, we compare the performance of existing solutions with our mechanism, instantiated with different sets of policies, following different providers' goals.

To establish the practicality of our approach, we are currently augmenting a prototype of a policy-based virtual network embedding architecture that uses an asynchronous version of the CAD mechanism [13].

### ACKNOWLEDGMENT

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### APPENDIX

#### A. CAD Synchronous Agreement Rules

In this appendix we present the conflict resolution rules used in the agreement phase of the CAD protocol (Table II).

As defined in Section III, a virtual network is denoted by the graph  $H = (V_H, E_H)$  and a physical network by  $G = (V_G, E_G)$ , where  $V$  is a set of (physical or virtual) nodes, and  $E$  the set of (physical or virtual) edges.  $\mathbf{b}_i \in \mathbb{R}_+^{|V_H|}$  is the a vector of utility values. Each entry  $b_{ij} \in \mathbf{b}_i$  is a positive real number representing the highest utility value known so far on virtual node  $j \in V_H$ .  $\mathbf{a}_i \in V_G^{|V_H|}$  is the winner vector

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**Procedure 3** Physical Capacity Assignment
 

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- 1: **while** there are PNs with unassigned capacity **do**
  - 2:   consider subset  $i$  of the max  $k$ -coverage problem with the highest cardinality, and assign its cardinality as capacity of physical node  $i$ .
  - 3:   Remove subset  $i$  from the  $k$  subsets.
  - 4:   Remove  $i$  from set of PNs with unassigned capacity.
  - 5: **end while**
- 

—a vector containing the latest information on the current assignment of all virtual nodes, for a distributed auction winner determination.  $a_{ij} \in \mathbf{a}_i$  is the identity of the winner of virtual node  $j$ , as currently known by physical node  $i$ .  $\mathbf{s}_i \in \mathbb{R}_+^{|V_G|}$  is a vector of timestamps of the last information update from each of the other physical nodes, *i.e.*, the *message generation times*. There are three possible actions when a physical node  $i$  receives a bid message from a sender physical node  $k$ : (i) *update*, where both the utility vector and the allocation vector are updated according to the sender information; (ii) *reset*, where the utility value is set to zero, and the allocation vector to null, and (iii) *leave*, where both the utility vector and the allocation vector are left unchanged by the receiver physical node.

### B. Proof of Theorem IV.2

*Proof.* To prove the  $(1 - \frac{1}{e})$ -approximation, we use the fact that physical nodes greedily add virtual nodes and links in their bundle, using a non-negative, monotone non-increasing and sub-modular utility function. Therefore, all the axioms of Theorem 3.1 in Nemhauser *et al.* [33] on sub-modular functions are satisfied.

To prove the bound on the approximation, we reduce our problem from the maximum  $k$ -coverage problem. The maximum  $k$ -coverage [19] is the problem of allocating each set  $V_j$  to some set  $i$ , in order to maximize

$$\sum_{i=1}^k U_i(\bigcup_{j \in S_i} V_j) \quad (7)$$

where  $S_i$  are the indices of sets allocated to set  $i$ .

Given this instance of the maximum  $k$ -coverage, we define an instance of the VN embedding problem considering each virtual link forming the VN request as two virtual nodes, *i.e.*, the source and the destination. We know that to embed a VN the source and destination virtual nodes of each virtual link need to be allocated to some physical node  $i$ , in a way to maximize the sum of the utilities of each physical node. The set  $V_j$  is the set of (source and destination) virtual nodes that form the VN embedding request. In particular, we give next a polynomial algorithm that has as input an instance of a maximum  $k$ -coverage, and as output an instance of the virtual network embedding problem, *i.e.*, a tuple  $\langle H, G, U, \rangle$ , where  $H$  is the virtual network to embed,  $G$  the hosting physical network, and  $U$  the utility function of each physical node: The utilities  $U_{ij}$  are unitary. The virtual network  $H$  has as many virtual nodes as there are elements in the maximum  $k$ -coverage instance, and  $H$  is fully connected with unitary capacity. The physical network  $G$  has as many physical nodes as  $k$ , and it is fully connected, and each physical link capacity is set to  $|H|$ , *i.e.*, each link has enough capacity to host all virtual links. The capacities of each virtual node and link are set to 1. The capacities of the physical nodes are constructed as in Procedure 3.

Now, given a solution of the VN embedding, we get a solution of the maximum  $k$ -coverage instance by merely assigning as elements of the set  $V_j$ , the virtual nodes in the bundle  $m_i$  of the hosting physical node  $i$ .

Since we can compute a solution for maximum  $k$ -coverage with a solution of a VN embedding, we cannot approximate the VN embedding problem within a better ratio than the ratio used by maximum  $k$ -coverage. It was in fact proved by Feige [16] that  $\forall \epsilon > 0$  it is NP-hard to achieve an approximation better than  $(1 - \frac{\epsilon}{e})$  for the *maximum  $k$ -coverage* problem.  $\square$



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