CS480/CS680 Problem Set 1

Due in class Thursday, October 5 at the beginning of lecture.

Please prepare the answers to these questions, neatly written or typed, on separate paper.

1. (a) (3 points) Write a 4×4 homogeneous transform matrix $\mathbf{M}$ that when applied to a point $(x, y, z, 1)$ yields (x', y', z', w') where

$$\begin{aligned} x' &= -2x \\ y' &= -\frac{1}{\sqrt{2}}y - \frac{1}{\sqrt{2}}z + a \\ z' &= \frac{1}{\sqrt{2}}z - \frac{1}{\sqrt{2}}y + a \\ w' &= 1 \end{aligned}$$

   (b) (12 points)In words, what four basic computer graphics transforms occur when we apply $\mathbf{M}$ to a 3D point? Give a homogeneous transform matrix for each, and show the order in which they are multiplied.

2. (15 points) We have a unit cube centered at the point $\mathbf{c} = (1.5, 0.5, 2.5)$. Derive the homogeneous transformation matrix that will rotate the cube by angle θ around a vector in the direction **v** = (1,0,1). The pivot point for the rotation is the cube's center **c**.

3. Use quaternions in your answers to the following.

   - (5 points) Prove that in general two 3D rotations about different rotation axes do not commute. You can assume that both rotation axes pass through the origin.
   - (15 points) Derive the general conditions under which 3D rotations commute.

4. (20 points) Problem 5-3 in the Hearn and Baker text. Give all steps in the derivation of your answer, as well as multiply out the final transformation matrix.

5. (30 points) In 2D we specify a line by the equation $y = mx + b$. Derive a homogenous transformation matrix that can be used to reflect 2D points about this line. Extend your result to reflection about a plane in three dimensions.

6. (CS680 only)

   We can specify a 3D affine transformation by considering the location of a small set of points both before and after the points have been transformed.

   (a) (5 points) What is the minimum number of points N needed to determine the transform uniquely? Be sure to give a mathematical justification for your answer.

   (b) (5 points) Are there any special conditions that these points must satisfy? Be sure to give a mathematical justification for your answer.

   (c) (15 points) Assume that we are given the point positions before transformation $(\mathbf{p}_1, \mathbf{p}_2, \ldots \mathbf{p}_N)$ and their corresponding positions after the affine transformation $(\mathbf{p'}_1, \mathbf{p'}_2, \ldots \mathbf{p'}_N)$. Give the mathematical steps for estimating the affine transform given N points, where N is the minimum number required as determined in (a). Be sure to also include any mathematical steps for testing if the points satisfy the special conditions.