CS480/CS680 Problem Set 2

Due in class Tuesday, November 21 at the beginning of lecture.

Please prepare the answers to these questions, neatly written or typed, on separate paper.

1. (10 points) Prove the invariance property of NURBS under perspective viewing transforms. In other words, prove that applying a transform $\mathbf{M}_{per}$ to the control points $\mathbf{p}_k$ yields the same result as applying $\mathbf{M}_{per}$ directly to the NURBS curve function $\mathbf{P}(u)$.

2. (5 points) Are NURBS also invariant to affine transformations? Justify your answer.

3. We are given the following boundary conditions for a cubic spline section:

$$\begin{aligned} \mathbf{P}(0) &= \mathbf{p}_k \\ \mathbf{P}(1) &= \mathbf{p}_{k+1} \\ \mathbf{P}''(0) &= \mathbf{p}_{k+1} + \mathbf{p}_{k-1} - 2\mathbf{p}_k \\ \mathbf{P}''(1) &= \mathbf{p}_k + \mathbf{p}_{k+1} - 2\mathbf{p}_{k+2} \end{aligned}$$

   (a) (5 points) Do adjacent segments satisfy G1 continuity? Give a mathematical justification.

   (b) (5 points) Given the boundary conditions as specified above, derive $\mathbf{M}_{geom}$ for this curve.

   (c) (10 points) Given the boundary conditions as specified above, derive the 4×4 matrix $\mathbf{M}_{spline}$.

   (d) (5 points) Given your solution for $\mathbf{M}_{spline}$ above, write out the blending functions for this curve.

   (e) (5 points) Give the equation $\mathbf{P}'(u)$ for the tangent to this curve in terms of $\mathbf{M}_{spline}$ and $\mathbf{M}_{geom}$.

   (f) (10 points) Do adjacent segments satisfy C2 continuity? Give a mathematical justification.

   (g) (15 points) Say that we use this cubic spline formulation to define a 2D spline segment with control points $\mathbf{p}_{k-1}$, $\mathbf{p}_k$, $\mathbf{p}_{k+1}$, $\mathbf{p}_{k+2}$. We want to generate successive values along the curve at a fixed step size δ. Give the specific forward difference equations and initial values for this.

4. (30 points) We are given a 3D revolved surface (torus). The cross-section for the revolved surface is defined as a closed, quadratic periodic B-spline curve in the $x - y$ plane. The 3D surface is produced by revolving the cross-section around the y-axis.

   (a) Give the equation for the closed, quadratic periodic B-spline curve in the $x - y$ plane $\mathbf{P}(u)$.

   (b) Given $\mathbf{P}(u)$, derive the parametric equation for a point on the revolved surface $\mathbf{S}(u, \theta)$.

   (c) Given $\mathbf{P}(u)$, derive the parametric equation for the unit normal to the revolved surface $\mathbf{n}(u, \theta)$.

5. (CS680 only, 20 points) For the revolved surface in Problem 4, give the steps in the algorithm and equation(s) that can be used to determine if a point is *inside*, *on*, or *outside* the revolved surface via the even-odd parity rule.