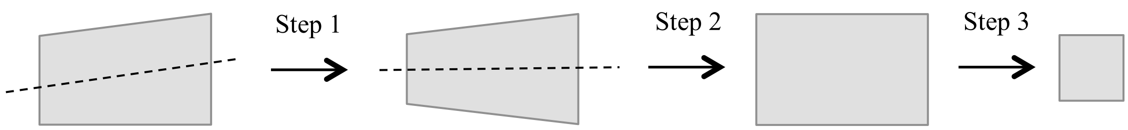
CS480 / CS680 MIDTERM – October 20, 2015

SUBMIT THIS EXAM WITH YOUR ANSWERS WRITTEN IN THE BLUE BOOK

Feel free to draw pictures to illustrate your answers.

1) (6 points) Briefly describe how the alpha channel works. Why is it used?
2) (6 points) In general, can bounding boxes be used to “trivially accept” intersection of the two convex polygons? If yes, how? If no, why?
3) (12 points) Depicted below is a pipeline of view volume transforms (cross section). For every one of the three steps: (a) name the type of transformation (b) explain why it is performed.



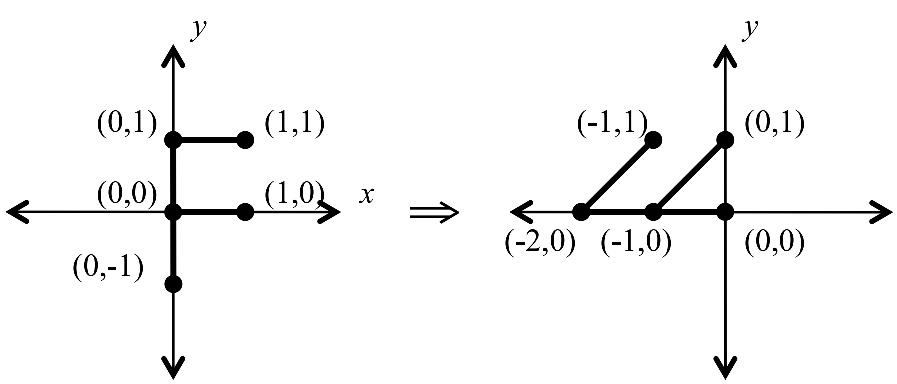
4) (12 points) Consider the unit quaternions

$$q_1 = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0,0\right)$$

$$q_2 = \left(\frac{\sqrt{3}-1}{2\sqrt{2}}, \frac{\sqrt{3}-1}{2\sqrt{2}}, 0,0\right)$$

   a. Does rotation by q_1 and q_2 commute? In other words, given a point **p**, we define **P**=(0,**p**). Does $q_1 q_2 \mathbf{P} q_2^{-1} q_1^{-1} = q_2 q_1 \mathbf{P} q_1^{-1} q_2^{-1}$? Give mathematical reasoning.
   b. Now consider the rotation represented using the unit quaternion $q_3 = (-1,0,0,0)$. What is the angle of rotation?

5) (12 points) A 2D affine transform **A** maps the “F” shape on the left to the “F” on the right.



Derive the 3x3 matrix that represents **A** over homogeneous coordinates. You can give the specific matrices, along with the order of multiplication. There is no need to multiply out.

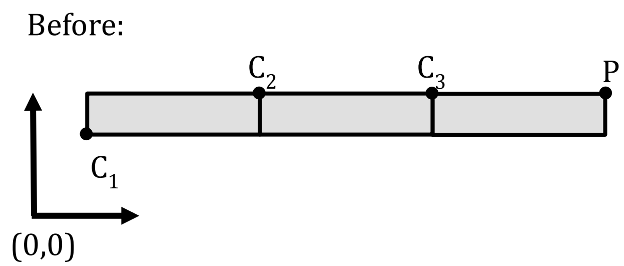
6) (12 points) We are given vertices for a line segment $\mathbf{v}_1 = (x_1, y_1, z_1)$ and $\mathbf{v}_2 = (x_2, y_2, z_2)$. The vertices have colors $\mathbf{c}_1 = (r_1, g_1, b_1)$ and $\mathbf{c}_2 = (r_2, g_2, b_2)$.

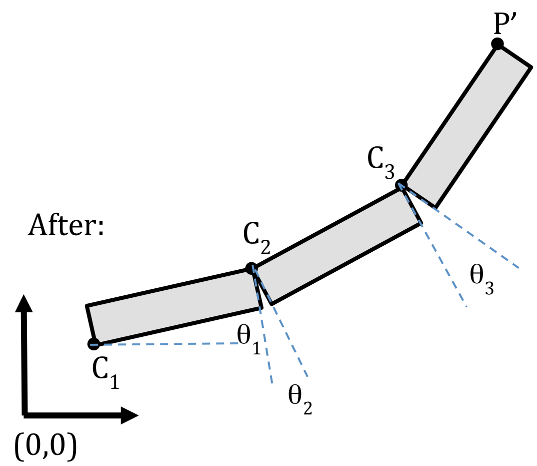
a. Give the parametric equation for the directed line segment from $\mathbf{v}_1$ to $\mathbf{v}_2$.
b. Give an expression for the linearly interpolated color at u=0.75.

7) (15 points) We have the 2D linkage. There are three links, connected by hinges that rotate around points $\mathbf{C}_1$, $\mathbf{C}_2$, and $\mathbf{C}_3$, by angle θ_1, θ_2, and θ_3. We define the homogeneous transforms

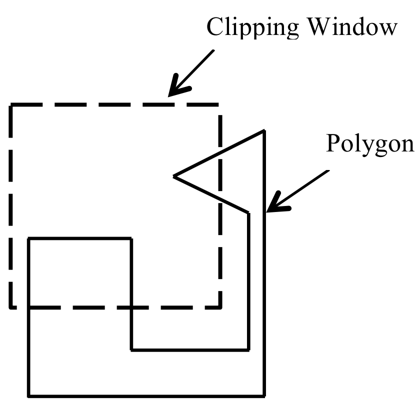
$$\mathbf{T}(\mathbf{C}_i) = \begin{bmatrix} 1 & 0 & x_c \\ 0 & 1 & y_c \\ 0 & 0 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{R}(\theta_i) = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 \\ \sin\theta_i & \cos\theta_i & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Give an expression for the position of the point P' in terms of C_1, C_2, C_3, θ_1, θ_2, and θ_3. Assume C_1 is fixed (but not at the origin) and the linkage moves as shown below.





8) (10 points) Assume we run Sutherland-Hodgman polygon clipping algorithm to clip the polygon below to the shown clipping window:



a. What is the shortcoming (bug) of the Sutherland-Hodgman algorithm for this case?
b. What algorithm in the textbook can addresses this shortcoming?

9) (15 points) Assume $\mathbf{p} = (x, y, z, 1)$ and $\mathbf{p}' = (x', y', z', 1)$ where

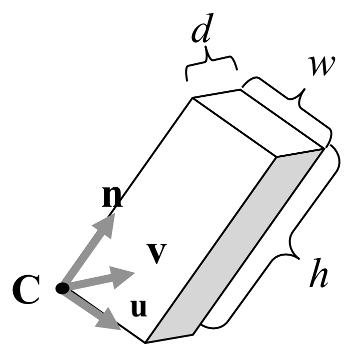
$$x' = 4$$

$$y' = \frac{4y + 6z + 8}{2y + x - 1}$$

$$z' = \frac{4x + 6z + 8}{4y + 2x - 2}$$

a. What is the *plane of projection* (also called *view plane*) in this case?
b. Give a homogeneous transform **M** such that when we homogenize $\mathbf{Mp}$ we get $\mathbf{p}'$.
c. **M** an oblique projection matrix. Give a unit vector for the direction of projection.

10) (25 points, **CS680 only**) A 3D oriented bounding box is determined by a corner point **C** (taken as the box origin), unit vectors **u**, **v**, **n** defining the box local coordinate system, and the box dimensions *w, d, h* along the **u**, **v**, **n** directions respectively.



Describe an algorithm that takes as input two such bounding boxes and determines if they intersect or not. Give pseudo-code for the algorithm, as well as specific transformation matrices employed. If you invoke an algorithm from the textbook as a subroutine, it is sufficient to just give the name of the algorithm (unnecessary to give the subroutine's pseudo-code).