CS480/CS680 Linear Algebra Self-Assessment

Due: September 8 at 11:00

For related review, see: Appendix A1-A5 in Hearn & Baker textbook

1. (a) Given points $\mathbf{p}_1 = (1, 6, 5)$ and $\mathbf{p}_2 = (-2, 2, 5)$, solve for $\mathbf{v}_1$ the vector from $\mathbf{p}_1$ to $\mathbf{p}_2$.
   (b) Given a third point $\mathbf{p}_3 = (0, 6, 5)$, solve for $\mathbf{v}_2$ the vector from $\mathbf{p}_1$ to $\mathbf{p}_3$.

2. (a) Find the value for the magnitude of $\mathbf{v}_1$.
   (b) Find the value for the magnitude of $\mathbf{v}_2$.

3. (a) Solve for the unit vector in the direction of $\mathbf{v}_1$.
   (b) Solve for the unit vector in the direction of $\mathbf{v}_2$.

4. (a) Solve for the vector (cross) product $\mathbf{v}_1 \times \mathbf{v}_2$.
   (b) Solve for $\mathbf{v}_2 \times \mathbf{v}_1$.

5. Solve for the scalar (dot) product $\mathbf{v}_2 \cdot \mathbf{v}_1$.

6. If two vectors $\mathbf{u}, \mathbf{v} \in \Re^n$ are orthogonal, what is the value of their scalar (dot) product?

7. Which of the following are unit vectors?

   $(\frac{1}{2}, -\frac{1}{2}, 0)$ $\qquad$ $(0, -1, 0)$ $\qquad$ $\frac{1}{25}(-3, 0, 4)$

8. We are given two non-zero vectors $\mathbf{u}, \mathbf{v} \in \Re^3$. Assume the angle between $\mathbf{u}$ and $\mathbf{v}$ satisfies $0 < \theta < \frac{\pi}{2}$. Use dot products and/or cross products of $\mathbf{u}$ and $\mathbf{v}$ to give expressions for:

   (a) $\cos\theta$
   (b) $\sin\theta$
   (c) A vector perpendicular to both $\mathbf{u}$ and $\mathbf{v}$

9. Given three square matrices $\mathbf{Q}, \mathbf{R}, \mathbf{S} \in \Re^{n \times n}$, which statements are true **in general**?

   (a) $(\mathbf{QRS})^{-1} = \mathbf{S}^{-1}\mathbf{R}^{-1}\mathbf{Q}^{-1}$
   (b) $\mathbf{QR} = \mathbf{RQ}$
   (c) $(\mathbf{QRS})^T = \mathbf{Q}^T\mathbf{R}^T\mathbf{S}^T$
   (d) $\mathbf{Q}(\mathbf{R} + \mathbf{S}) = \mathbf{QS} + \mathbf{QR}$

10. Given a square matrix $\mathbf{A} \in \Re^{n \times n}$ whose columns form an orthonormal basis

    (a) What is the dot product of any pair of columns in $\mathbf{A}$?
    (b) What is the inverse of $\mathbf{A}$?