Binocular Stereo, Part 2
Multiview Stereo
Epipolar Geometry
Active Stereo with Structured Light
Photometric Stereo

Lecture by Margrit Betke, CS 585, April 28, 2020
Camera Transformation Problems

1. **Interior Orientation = Camera Calibration = Intrinsic Calibration:**
   - Simple version: Find focal length $f$ and principal point $p$ (= point where optical axis intersects image plane)
   - Better: Correct for lens distortion, check if angle between $x$ & $y$ axes is $90^0$

2. **Exterior Orientation = Hand-Eye Calibration in Robotics:**
   - Find Center of Projection (CoP) of camera in world coordinate system

3. **Absolute Orientation = Alignment of 2 Cameras**
   - Find relationship between cameras. 3D coordinates of points are known

4. **Relative Orientation = Alignment of 2 Cameras**
   - Find relationship between cameras. 3D coordinates not known, only rays known
Relative Orientation – Binocular Stereo

Goal: Recovery of position and orientation of one imaging system relative to another from correspondences between rays

Given: 2D coordinates of image points of same world point

(x_left, y_left) \quad (x_left, y_left, Z_left) \quad (x_right, y_right, Z_right)

(\mathbf{X_left}, \mathbf{Y_left}, \mathbf{Z_left}) \quad (\mathbf{X_right}, \mathbf{Y_right}, \mathbf{Z_right})
Special Case

Baseline $b =$ Distance between Center of Projections (CoPs)

$Z = \frac{bf}{\delta}$
Relative Orientation = Binocular Stereo

Use perspective projection equations:

\[ \frac{x_{\text{left}}}{f_{\text{left}}} = \frac{X_{\text{left}}}{Z_{\text{left}}} \]
\[ \frac{y_{\text{left}}}{f_{\text{left}}} = \frac{Y_{\text{left}}}{Z_{\text{left}}} \]
\[ \frac{x_{\text{right}}}{f_{\text{right}}} = \frac{X_{\text{right}}}{Z_{\text{right}}} \]
\[ \frac{y_{\text{right}}}{f_{\text{right}}} = \frac{Y_{\text{right}}}{Z_{\text{right}}} \]

Transformation equation: \[ R \ r_{\text{left}} + r_0 = r_{\text{right}} \]

R = rotation matrix, \( r_0 \) = translation
Relative Orientation = Binocular Stereo

Transformation equation: \( R \mathbf{r}_{\text{left}} + \mathbf{r}_0 = \mathbf{r}_{\text{right}} \)

Unknown: Rotation matrix \( R \), translation \( \mathbf{r}_0 \), Z coordinates of \( \mathbf{r}_{\text{left}}, \mathbf{r}_{\text{right}} \)
Relative Orientation = Binocular Stereo

\[ R \cdot \mathbf{r}_{\text{left}} + \mathbf{r}_0 = \mathbf{r}_{\text{right}} \]

is equivalent to:

\[ r_{11} \cdot X_{\text{left}} + r_{12} \cdot Y_{\text{left}} + r_{13} \cdot Z_{\text{left}} + r_{14} = X_{\text{right}} \]

\[ r_{21} \cdot X_{\text{left}} + r_{22} \cdot Y_{\text{left}} + r_{23} \cdot Z_{\text{left}} + r_{24} = Y_{\text{right}} \]

\[ r_{31} \cdot X_{\text{left}} + r_{32} \cdot Y_{\text{left}} + r_{33} \cdot Z_{\text{left}} + r_{34} = Z_{\text{right}} \]

Insert Perspective Projection Equations:

\[ x_{\text{left}}/f = X_{\text{left}}/Z_{\text{left}} \quad y_{\text{left}}/f = Y_{\text{left}}/Z_{\text{left}} \]

\[ x_{\text{right}}/f = X_{\text{right}}/Z_{\text{right}} \quad y_{\text{right}}/f = Y_{\text{right}}/Z_{\text{right}} \]

\[ r_{11} \cdot X_{\text{left}} \cdot Z_{\text{left}}/f + r_{12} \cdot Y_{\text{left}} \cdot Z_{\text{left}}/f + r_{13} \cdot Z_{\text{left}} + r_{14} = x_{\text{right}} \cdot Z_{\text{right}}/f \]

\[ r_{21} \cdot X_{\text{left}} \cdot Z_{\text{left}}/f + r_{22} \cdot Y_{\text{left}} \cdot Z_{\text{left}}/f + r_{23} \cdot Z_{\text{left}} + r_{24} = y_{\text{right}} \cdot Z_{\text{right}}/f \]

\[ r_{31} \cdot X_{\text{left}} \cdot Z_{\text{left}}/f + r_{32} \cdot Y_{\text{left}} \cdot Z_{\text{left}}/f + r_{33} \cdot Z_{\text{left}} + r_{34} = Z_{\text{right}} \]

Multiply by \( f/\ Z_{\text{left}} \)
Relative Orientation = Binocular Stereo

\[ R \left( \begin{array}{c} r_{left} \\ r_{0} \end{array} \right) = \left( \begin{array}{c} r_{right} \end{array} \right) \]

is equivalent to:

\[ r_{11} x_{left} + r_{12} y_{left} + r_{13} f + r_{14} f/Z_{left} = x_{right} Z_{right}/Z_{left} \]
\[ r_{21} x_{left} + r_{22} y_{left} + r_{23} f + r_{24} f/Z_{left} = y_{right} Z_{right}/Z_{left} \]
\[ r_{31} x_{left} + r_{32} y_{left} + r_{33} f + r_{34} f/Z_{left} = Z_{right}/Z_{left} \]

One measurement pair \((x_{left}, y_{left})\) and \((x_{right}, y_{right})\) => 3 equations

with 14 unknowns \(r_{11}, r_{12}, ..., r_{34}\), and \(Z_{right}, Z_{left}\)
Relative Orientation

One measurement pair \((x_{\text{left}}, y_{\text{left}})\) and \((x_{\text{right}}, y_{\text{right}})\) => 3 equations
with 12 unknowns \(r_{11}, r_{12}, \ldots, r_{34}\) and 2 unknowns \(Z_{\text{right}}, Z_{\text{left}}\)

Trick: To solve for 14 unknowns:

Use \(n\) measurements => 3\(n\) equations
Find additional equations

\[
\begin{align*}
    r_{11} x_{\text{left}} + r_{12} y_{\text{left}} + r_{13} f + r_{14} f/Z_{\text{left}} &= x_{\text{right}} Z_{\text{right}}/Z_{\text{left}} \\
    r_{21} x_{\text{left}} + r_{22} y_{\text{left}} + r_{23} f + r_{24} f/Z_{\text{left}} &= y_{\text{right}} Z_{\text{right}}/Z_{\text{left}} \\
    r_{31} x_{\text{left}} + r_{32} y_{\text{left}} + r_{33} f + r_{34} f/Z_{\text{left}} &= Z_{\text{right}}/Z_{\text{left}} \\
\end{align*}
\]
Relative Orientation

One measurement pair \((x_{\text{left}}, y_{\text{left}})\) and \((x_{\text{right}}, y_{\text{right}})\) => 3 equations

with 12 unknowns \(r_{11}, r_{12}, \ldots, r_{34}\), and 2 unknowns \(Z_{\text{right}}, Z_{\text{left}}\)

One extra equation:

Scale factor ambiguity \(r_0, Z_{\text{right}}, Z_{\text{left}}\)

\[ \iff kr_0, kZ_{\text{right}}, kZ_{\text{left}} \]

Force \(r_0\) to be unit vector

\[ \Rightarrow |r_0|=1 \]
Relative Orientation

One measurement pair \((x_{\text{left}}, y_{\text{left}})\) and \((x_{\text{right}}, y_{\text{right}})\) => 3 equations with 14 unknowns \(r_{11}, r_{12}, \ldots, r_{34}\), and \(Z_{\text{right}}, Z_{\text{left}}\)

# unknowns: 12 for \(R, r_0\)
2n for \(Z_{\text{right}}, Z_{\text{left}}\) for each of \(n\) pairs of measurements

12 + 2n unknowns
Relative Orientation

\[
\begin{align*}
    r_{11} x_{\text{left}} + r_{12} y_{\text{left}} + r_{13} f + r_{14} f/Z_{\text{left}} &= x_{\text{right}} Z_{\text{right}}/Z_{\text{left}} \\
    r_{21} x_{\text{left}} + r_{22} y_{\text{left}} + r_{23} f + r_{24} f/Z_{\text{left}} &= y_{\text{right}} Z_{\text{right}}/Z_{\text{left}} \\
    r_{31} x_{\text{left}} + r_{32} y_{\text{left}} + r_{33} f + r_{34} f/Z_{\text{left}} &= Z_{\text{right}}/Z_{\text{left}}
\end{align*}
\]

One measurement pair \((x_{\text{left}}, y_{\text{left}})\) and \((x_{\text{right}}, y_{\text{right}})\) \(\Rightarrow\) 3 equations

with 14 unknowns \(r_{11}, r_{12}, ..., r_{34}\), and \(Z_{\text{right}}, Z_{\text{left}}\)

Number of equations:

- 6 for orthonormal \(R\) (columns sum to 1, dot products 0)
- 1 for unit length translation: \(|r_0|=1\)
- 3n for 3 equations per measurement pair

\(7 + 3n\) equations
Relative Orientation

One measurement pair \((x_{\text{left}}, y_{\text{left}})\) and \((x_{\text{right}}, y_{\text{right}})\) => 3 equations with 14 unknowns \(r_{11}, r_{12}, \ldots, r_{34}\), and \(Z_{\text{right}}, Z_{\text{left}}\)

\[
\begin{align*}
    r_{11} x_{\text{left}} + r_{12} y_{\text{left}} + r_{13} f + r_{14} f/Z_{\text{left}} &= x_{\text{right}} Z_{\text{right}}/Z_{\text{left}} \\
    r_{21} x_{\text{left}} + r_{22} y_{\text{left}} + r_{23} f + r_{24} f/Z_{\text{left}} &= y_{\text{right}} Z_{\text{right}}/Z_{\text{left}} \\
    r_{31} x_{\text{left}} + r_{32} y_{\text{left}} + r_{33} f + r_{34} f/Z_{\text{left}} &= Z_{\text{right}}/Z_{\text{left}}
\end{align*}
\]

# unknowns: 12 for R, \(r_0\)
2n for \(Z_{\text{right}}, Z_{\text{left}}\) for each of n pairs of measurements

Number of equations: 6 for orthonormal R (columns sum to 1, dot products 0)
1 for unit length translation \(r_0 F\)
3n for 3 equations per measurement pair

Need at least n ? measurement pairs: 12 + 2 * n = 7 + 3*n
Relative Orientation

\[ r_{11} x_{\text{left}} + r_{12} y_{\text{left}} + r_{13} f + r_{14} f / Z_{\text{left}} = x_{\text{right}} Z_{\text{right}} / Z_{\text{left}} \]
\[ r_{21} x_{\text{left}} + r_{22} y_{\text{left}} + r_{23} f + r_{24} f / Z_{\text{left}} = y_{\text{right}} Z_{\text{right}} / Z_{\text{left}} \]
\[ r_{31} x_{\text{left}} + r_{32} y_{\text{left}} + r_{33} f + r_{34} f / Z_{\text{left}} = Z_{\text{right}} / Z_{\text{left}} \]

One measurement pair \((x_{\text{left}}, y_{\text{left}})\) and \((x_{\text{right}}, y_{\text{right}})\) => 3 equations

with 14 unknowns \(r_{11}, r_{12}, ..., r_{34}\) and \(Z_{\text{right}}, Z_{\text{left}}\)

# unknowns: 12 for \(R, r_0\)
2n for \(Z_{\text{right}}, Z_{\text{left}}\) for each of \(n\) pairs of measurements

Number of equations: 6 for orthonormal \(R\) (columns sum to 1, dot products 0)
1 for unit length translation \(r_0\)
3n for 3 equations per measurement pair

Need at least 5 measurement pairs: 12 + 2 * 5 = 22 = 7 + 3*5
Algorithms to Solve the Problem of Binocular Stereo = Relative Orientation

• Longuet-Higgins’ 8-point Algorithm (1981):
  
  \[
  (x_{\text{left}}, y_{\text{left}}, 1) \trans F (x_{\text{right}}, y_{\text{right}}, 1) = 0
  \]

  $F$ is called the 3x3 \textit{“fundamental matrix”}

  Algorithm is sensitive to how accurate point pairs were located (= numerically unstable)

• Variations of the 8-point Algorithm
  
  e.g. Hartley’s Normalized 8-point algorithm (1997)

• Horn’s Iterative Method (1991)

• Deep Fundamental Matrix Estimation without Correspondences (e.g., Poursaeed et al. 2018)
Multi-Camera Stereo

View from above:
Z axis = direction of gravity

$P_1$ is projection of scene point $P_1$
$i_1$ is projection of scene point $P_1$ in view of camera $C_1$

$P_2$ is projection of scene point $P_2$
$i_2$ is projection of scene point $P_2$ in view of camera $C_1$
Multi-Camera Stereo

\[ \begin{align*}
\text{i}_1 & \text{ could be projection of scene point } G_1 \\
\text{i}_2 & \text{ could projection of scene point } G_2
\end{align*} \]
3rd Camera resolves the ambiguity:
$G_1$ and $G_2$ are "ghosts" (non-existing points)
$P_1$ and $P_2$ are the true scene points
Green line is ray from $P_1$ into camera $C_3$. It appears as an “epipolar line” in the image of camera $C_1$. 
The green line is the ray from $P_1$ into the 3\textsuperscript{rd} camera. The orange line is the ray from $P_1$ into the 2\textsuperscript{nd} camera.

They appear as “epipolar lines” in the image of camera $C_1$ and must intersect at the same image point $i_1$. 
The green and red epipolar lines in the camera $C_2$ intersect at image point $i_3$. The orange and red epipolar lines in the camera $C_3$ intersect at image point $i_3$. 
Epipolar Geometry

1. All possible scene points $M$ ($M'$, $M''$, ... ) that produce image $m_{\text{left}}$ are on a half line through $m_{\text{left}}$ and $\text{CoP}_{\text{left}}$

2. All possible images $m_r$ of $M$ are images of this half line called “epipolar line.”

3. The image of $\text{CoP}_{\text{left}}$ in the right image plane is called “epipole” ($e_{\text{right_epipole}}$)
Epipolar Geometry

left image

right image

Image Credit: OpenCV.org
Epipolar Geometry: Special Case Parallel Optical Axes

left image

right image

Image Credit: Scharstein, 2014
left image
right image

Image Credit: OpenCV.org
Epipolar Geometry

Epipolar lines are parallel = along image rows (epipoles are at infinity)
Algorithm: Find corresponding points in same image rows, e.g., via template matching
Result of Binocular Stereo Matching: Depth Map

\[ Z = \frac{bf}{\delta} \]

http://vision.middlebury.edu/stereo/data/scenes2014/
Rectification of Binocular Stereo Images: Undo Foreshortening

Why?

Epipolar lines are now parallel, enabling a simple search for corresponding points along image rows.

Image Source: Alyosha Efros
Rectification of Binocular Stereo Images: Undo Foreshortening

How?
Iterative Scheme

We want

$$I_{\text{left}}(x + \delta/2, y) = I_{\text{right}}(x - \delta/2, y)$$

Least Squares Method:

$$\min_{\delta} \sum_p [I_{\text{left}}(x + \delta/2, y) - I_{\text{right}}(x - \delta/2, y)]^2$$

$p = \text{patch size of patch}$ $p$: tradeoff
too small instability
too large smearing

Use current estimate of disparity $\delta$ to warp

Then solve LSM & update disparity
Binocular Stereo Solution Paths: 2 Alternatives

1. “Weak Calibration”
   • If needed: Use rectification to ensure epipolar lines are along image rows
   • Find corresponding points in both views and calculate disparity $\delta$
   • Compute depth: $Z = bf/\delta$

2. “Strong Calibration”
   • Calibrate each camera (= interior orientation): $f$, pp
   • Find geometric transformation of cameras (= relative orientation): $R$, $r_0$
   • Find 3D coordinates
Binocular Stereo Solution Paths: 2 Alternatives

1. “Weak Calibration”
   • If needed: Use rectification to ensure epipolar lines are along image rows
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2. “Strong Calibration”
   • Calibrate each camera (= interior orientation): $f$, pp
   • Find geometric transformation of cameras (= relative orientation): $R$, $r_0$
   • Find 3D coordinates

In our animal tracking research, “strong calibration” was the better solution
Binocular Stereo Solution Path: “Strong Calibration”

Images & Method: Theriault et al. 2014

Throw wand in the air several times (mark out bird flying space)

Identify wand position in all views
Take advantage of knowing the dimensions of the wand

Estimate $R$ and $r_0$
Binocular Stereo Solution Path: “Strong Calibration”
Binocular Stereo for 3D Bird Flight Analysis

Images & Method:
Theriault et al. 2014
Binocular Stereo for 3D Bat Flight Analysis

Images & Method:
Theriault et al. 2014
Binocular Stereo Solution Path: “Strong Calibration”

Indoor scenario is much easier:

Instead of wand, use “checker board” as calibration device

Take many images at different positions & orientations

Image Source: Jean-Yves Bouguet
Binocular Stereo Solution Path: “Strong Calibration”

Indoor scenario is much easier:

Instead of wand, use “checker board” as calibration device

Take many images at different positions & orientations

Use
http://www.vision.caltech.edu/bouguetj/calib_doc/index.html
Or OpenCV

Image Source: Jean-Yves Bouguet
Active stereo with structured light

Image credit: Li Zhang
Active stereo with structured light

View without structured light

Image credit: Li Zhang
Active stereo with structured light

Project “structured” light patterns onto the object simplifies the correspondence problem

Image credit: Li Zhang
Active stereo with structured light

Image credit: Li Zhang
Active stereo with structured light

Image credit: Li Zhang et al.
What if we do not have 2 cameras?

Can we still do 3D reconstruction of a scene?

YES !!!

2 Algorithms
Problem Definitions

• Shape from Shading (SfS)
  Find 3D shape in scene from a single 2D image

• Photometric Stereo (binocular stereo)
  Find 3D shape in scene from a set of 2D images that are taken under different lighting conditions

“stereo” = “solid” in Greek, used to refer to solidity, three-dimensionality
Photometric Stereo

Example:
Find 3D shape in scene from these images of faces
Photometric Stereo

3D shape visualized with texture from 1\textsuperscript{st} image
Photometric Stereo & SfS

Light reflected at surface patch depends on

• surface orientation
• reflectance properties of surface
• distribution of light sources illuminating surface
Light source $s$, Viewer Direction $v$, Image $E$
Photometric stereo & SfS

Light reflected at surface patch depends on
- surface orientation
- reflectance properties of surface
- distribution of light sources illuminating surface

Reconstruction Method:
- Determine surface reflectance properties and direction of light source(s)
- Compute surface orientation
Light source $\mathbf{s}$, Viewer Direction $\mathbf{v}$, Surface Normal $\mathbf{n}$, Image $E$

Brightness

$$E = \text{fct}(\theta_i, \theta_e)$$
or

$$E = \text{fct}(\mathbf{n}, \mathbf{s}, \mathbf{v})$$
Ideal Lambertian Surface looks equally bright from all directions

Brightness

\[ E = \text{fct}(\theta_i, \theta_e) \]

or \[ fct(n, s, v) \]

\[ E = \cos \theta_i = n \cdot s \]
Example of Lambertian Surface: Matte Sphere

Synthetic image with deliberately few gray values
Brightness (gray levels) does not change linearly from bright to dark
Equation?  \( \cos(n, s) \)
Lambertian reflectance model popular in graphics -- but something is wrong!
Surface Orientation

surface

surface normal $\mathbf{n}$

tangent plane

optical axis

$z$

$y$

$x$
Surface Orientation

Optical axis z

Surface Gradient

\((z_x, z_y)^T\)

\[ a = (\delta x, 0, z_x \delta x)^T = \delta x (1, 0, z_x)^T \]

\[ b = (0, \delta y, z_y \delta y)^T = \delta y (0, 1, z_y)^T \]

\[ n \parallel (a \times b) \]

\[ n = (-z_x, -z_y, 1)^T = (-p, -q, 1)^T \]
Reflectance Map of Matte Surface

Ideal Lambertian surface looks equally bright from all directions

\[ \cos \theta = n \cdot s \]

Example:
light source near viewer

Reflectance Map of Matte Surface
R(p,q) of Lambertian Surface
R(p,q) of Lambertian Surface

Nalwa '93
Reflectance Map

Two different projections can create maps of the surface gradients on “Gaussian” (or unit) sphere:

Stereographic plane:
- Whole sphere is projected
- Includes occluding boundary of sphere

Reflectance map:
- Upper hemisphere of sphere is projected
- Isobrightness lines extend to infinity
Photometric Stereo

Goal: Given images $E_1$ and $E_2$ under 2 lighting conditions $(p_1,q_1)$ and $(p_2,q_2)$, find surface orientation $n = (-p,-q,1)^T$, i.e., find $p$ & $q$.

2 nonlinear equations:
$E_1 = R_1(p,q)$
$E_2 = R_2(p,q)$

If $(p_1,q_1) = (p_2,q_2)$
   infinite number of solutions
else 0, 1, or 2 solution(s)

Better, use $N$ images &
least-squares method
Mars

Viking
Lander I
1977

Image Credit: Horn
Image Credit: Horn