Boston University
CAS CS 585:
Image and Video Computing

Lecture on Convolution, Correlation, Object Recognizability, CNNs, Image Net

by Margrit Betke

March 5, 2024

Learning Objectives for this Lecture

Computer Science

- Understand differences and similarities between pre-2012 “traditional computer vision” and post-2012 neural-network-based computer vision & see examples
- Understand why convolution is powerful
- Understand the connection between convolution and correlation
- Understand how tools from estimation theory can be used to measure recognizability of objects in images
- Understand template matching with image pyramids
- Understand CNNs as a learning hierarchy of features
- Learn about early CNN used in computer vision: LeCun’s work on recognizing handwritten numbers
- Understand CNN concepts, e.g., convolution layers, fully connected (dense) layers, non-linearity (ReLU), pooling (downsampling)
- Learn about breakthrough dataset ImageNet

Today's Computer Vision: Mostly (but not all) Neural Networks

Computer Science

- Deep convolutional neural networks
- Transformers
- Diffusion models

\+ traditional computer vision algorithms, representations, geometry, and tricks

Deep learning does not work well for:

Multi-view geometry, i.e., 3D object pose and 3D scene representation

1D Discrete Convolution

Computer Science

1D Convolution:
Time signal *f* and shifted time signal *g* are multiplied and added:

$$(f * g)[n] \stackrel{\mathrm{def}}{=} \sum_{m=-\infty}^{\infty} f[m]\, g[n-m] = \sum_{m=-\infty}^{\infty} f[n-m]\, g[m].$$

2D generalization:
f = input image, g = template image
(or CNN function)

2D Convolution Example

Computer Science

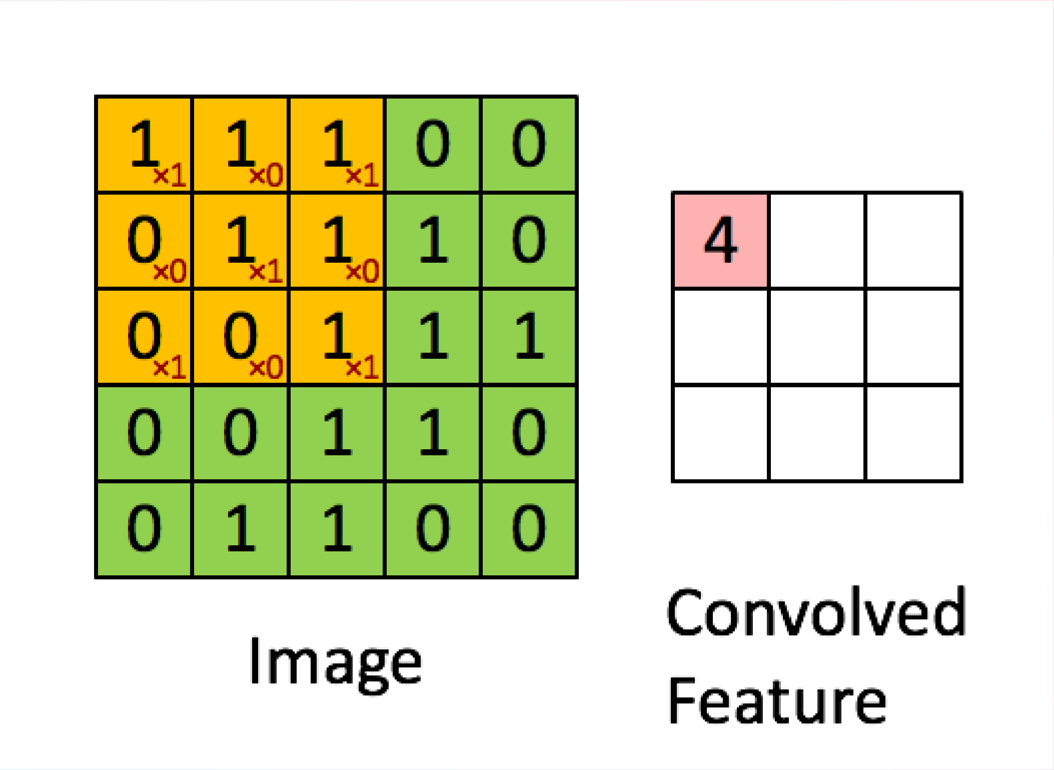
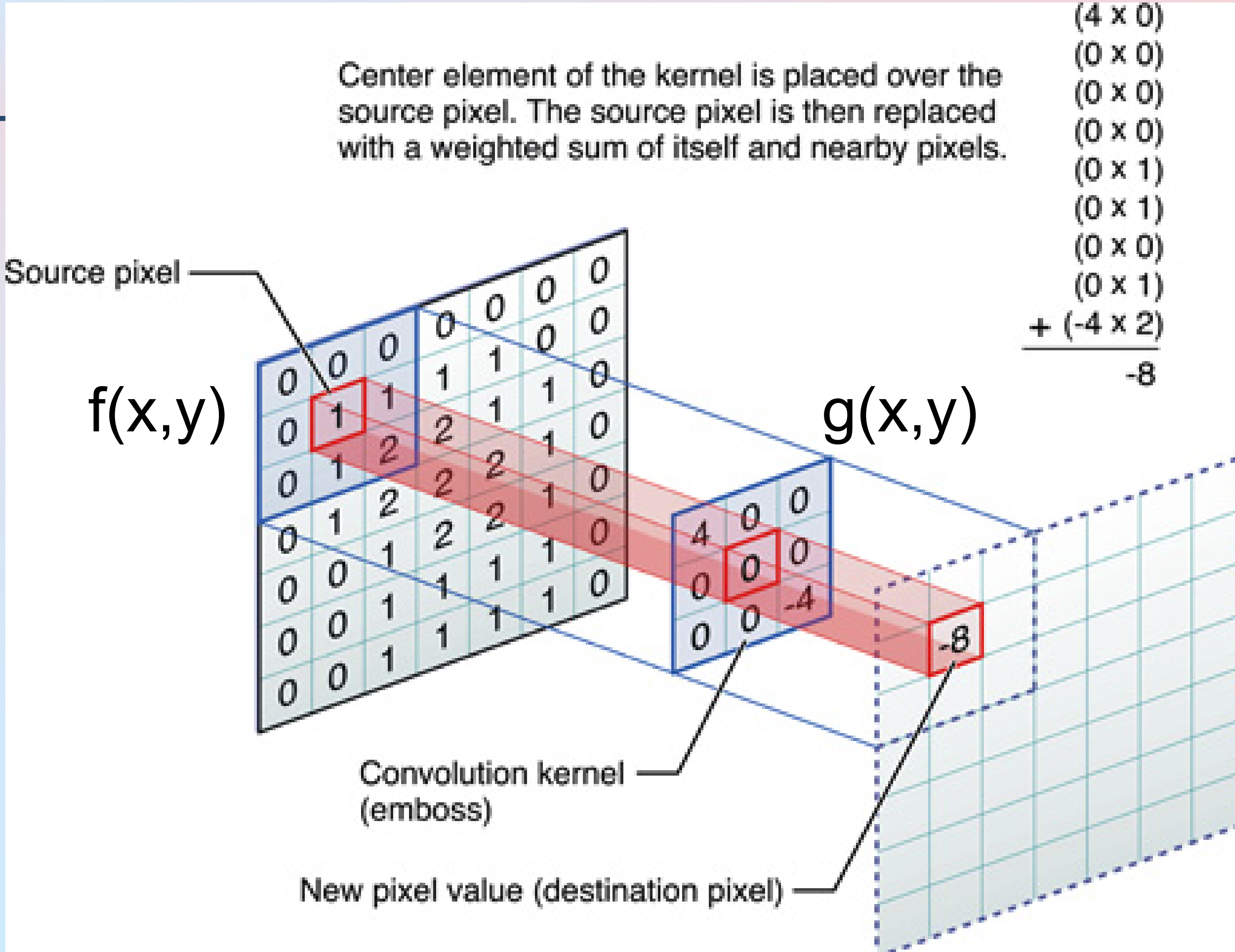


Image Credit: Nvidia



f*g

Computer Science

Image Credit: Madhushree Basavarajaiah

Computer Science

Why is Convolution Powerful?

Signal Processing:

Computer Science

Convolution is used to define a "matched filter" for locating "targets" in time signals

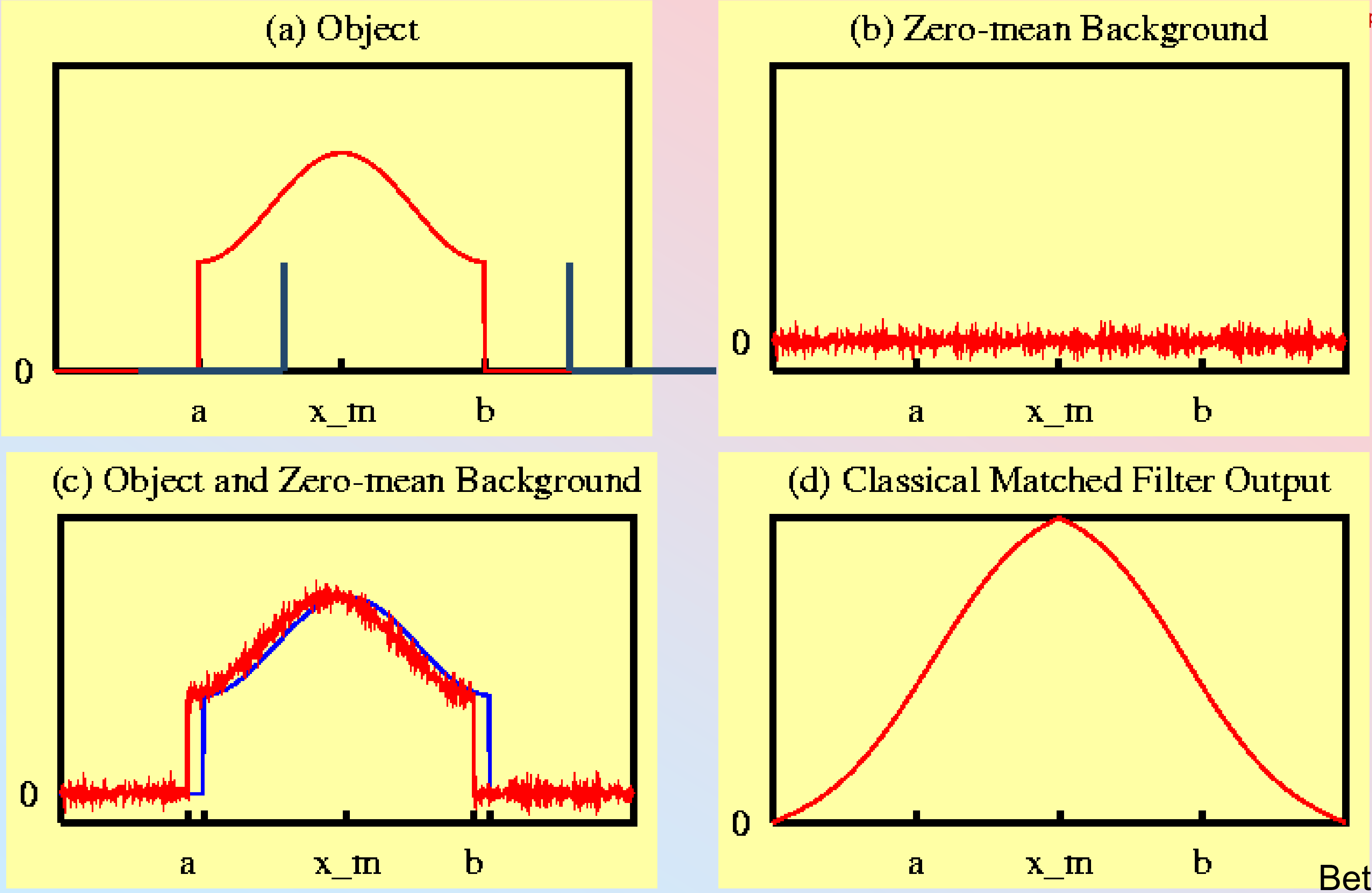
Template matching is optimal algorithm if noise is Gaussian.

Optimality of Template Matching

Computer Science

Betke, Makris, IJCV 2001

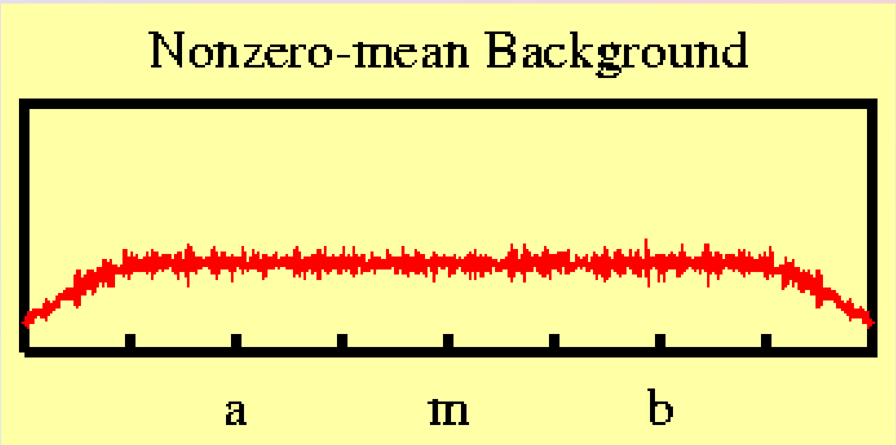
1D Position Estimation: Σ object*background

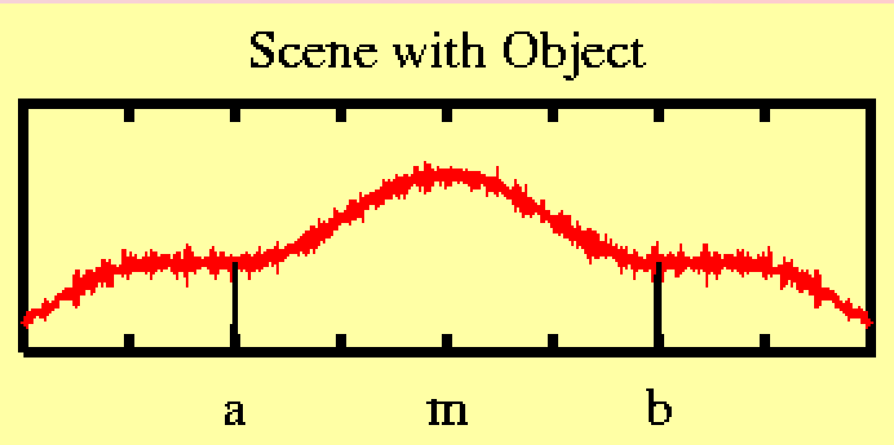


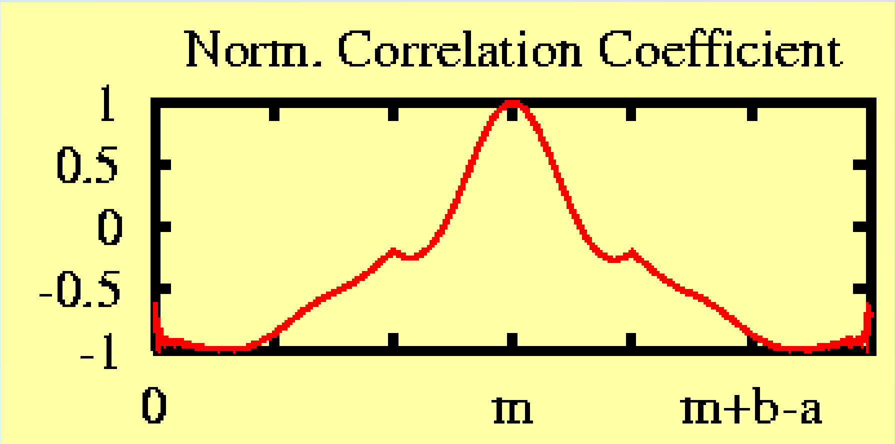
Betke, Makris, IJCV 2001

Boston University
Computer Science

Another 1D convolution example:



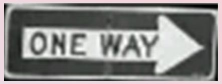


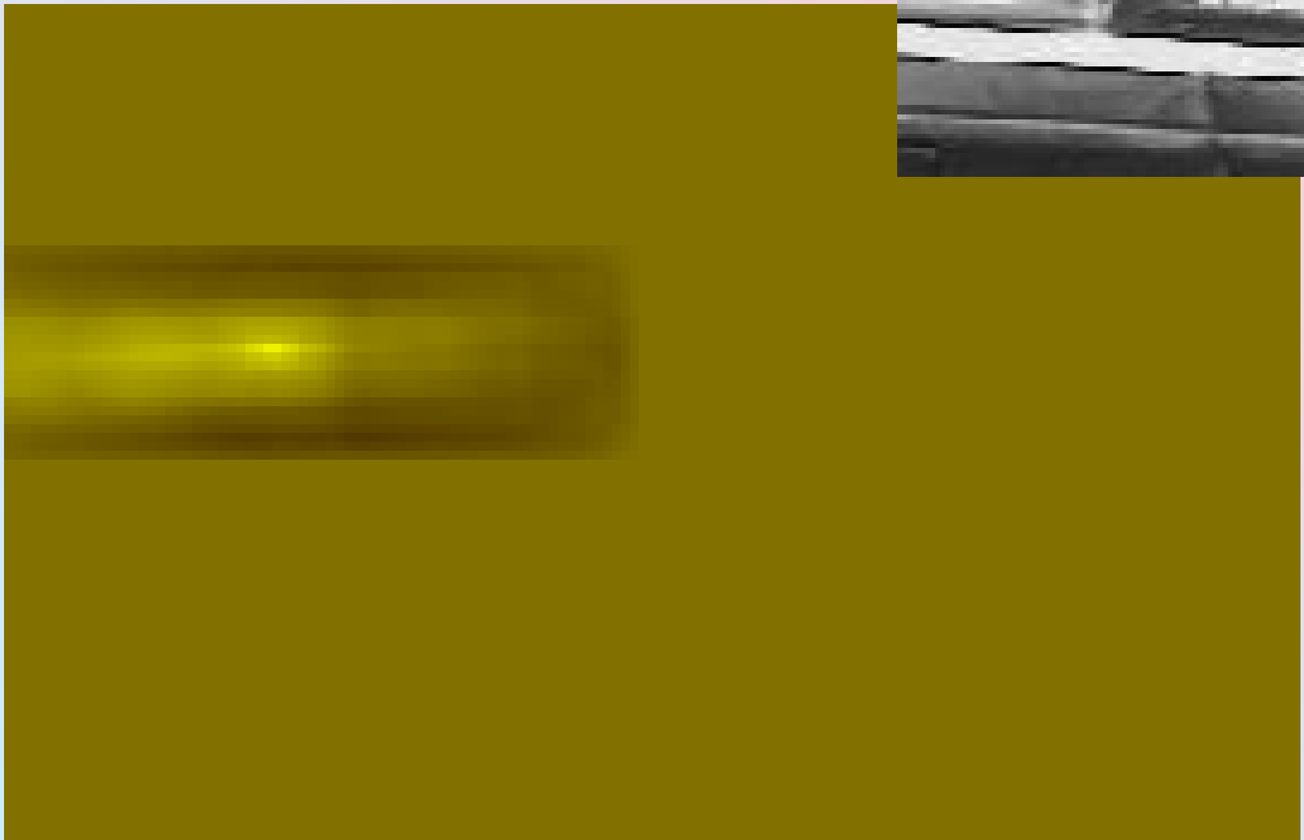


= convolution/std-devs

Betke, Makris, IJCV 2001

2D Position Estimation

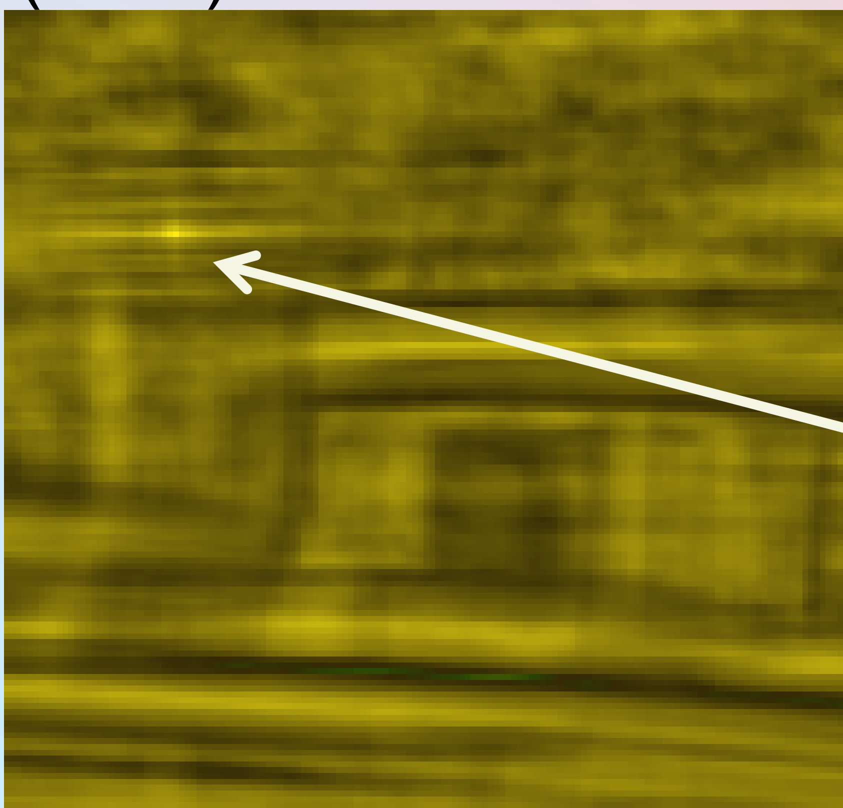
Convolution of one-way sign with itself 



Betke, Makris, IJCV 2001

2 D Position Estimation

Convolution of one-way sign with scene (NCC)

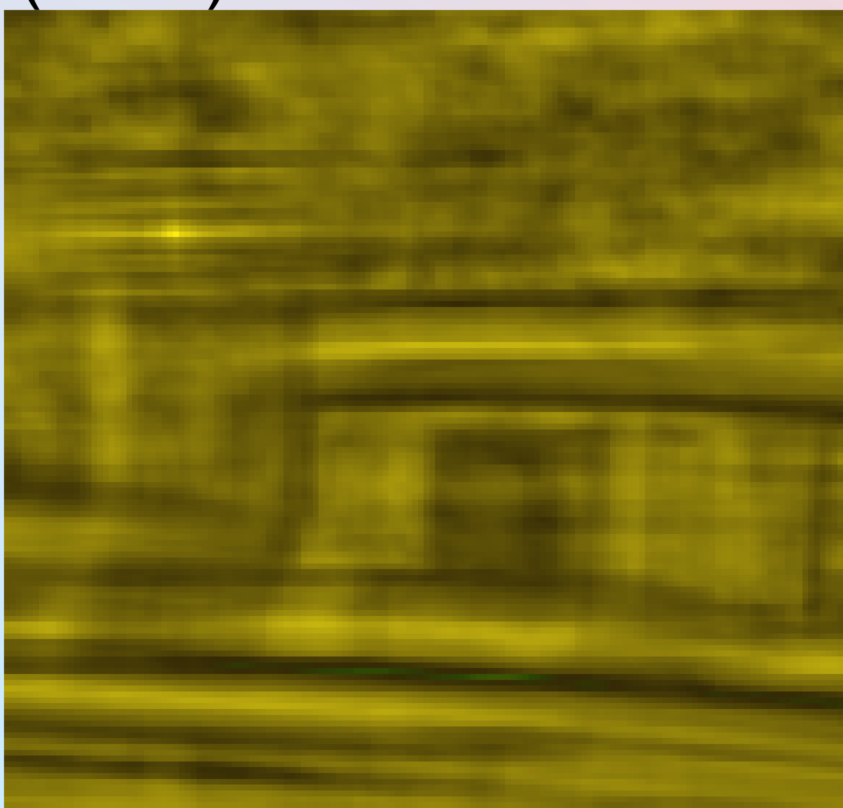


Peak in performance surface (= negative loss fct) at correct location

Betke, Makris, IJCV 2001

2 D Position Estimation

Convolution of one-way sign with scene (NCC)



This performance surface is computed for correct size of one-way sign

Different surfaces for different sizes of object

Sample Performance Surfaces

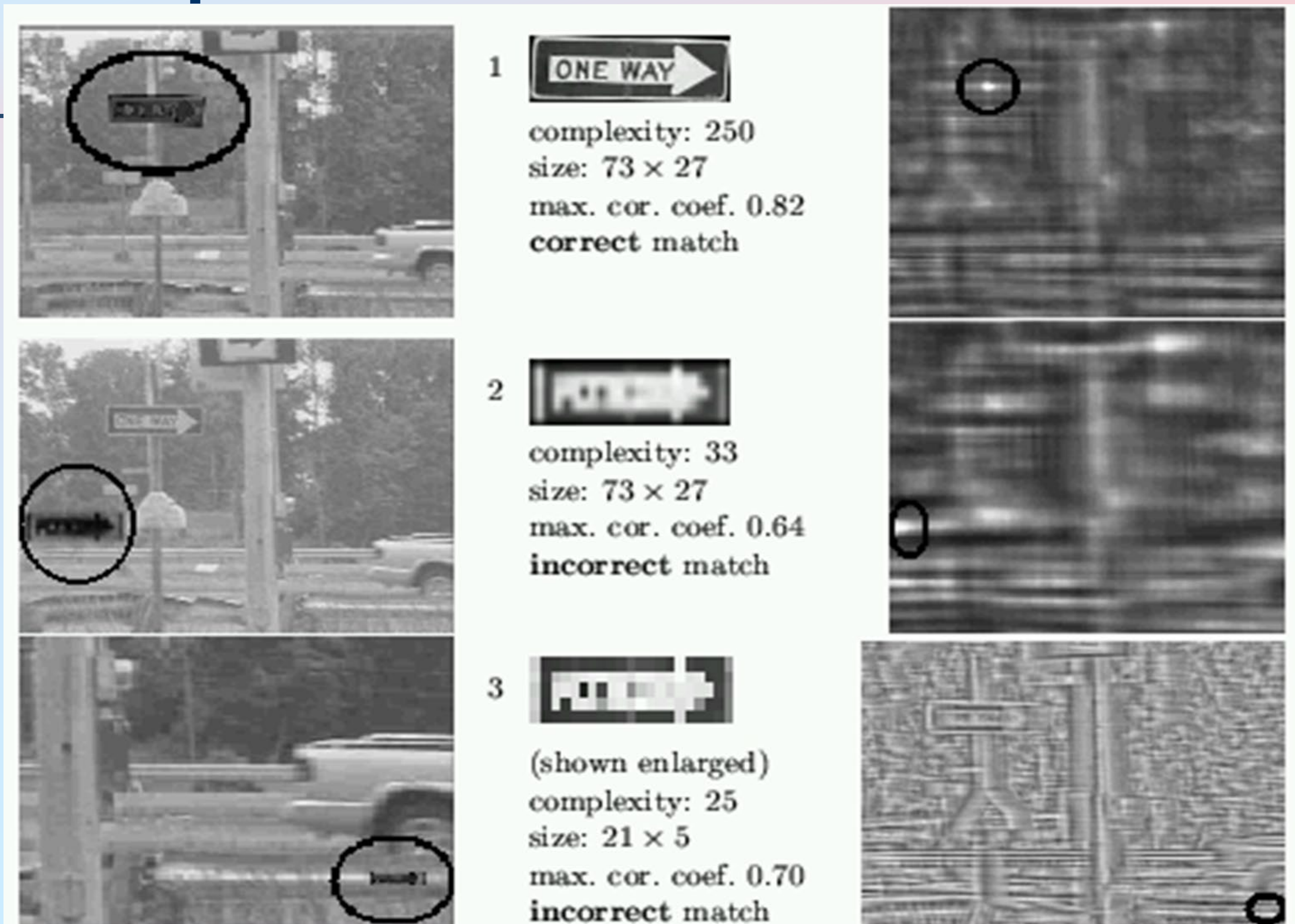
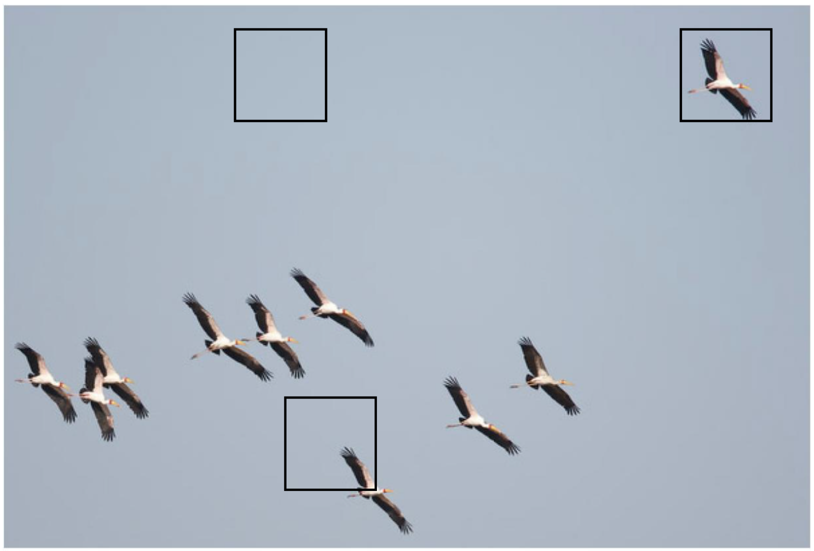


Image Credit: Efros/Freeman

Convolving template with subimage



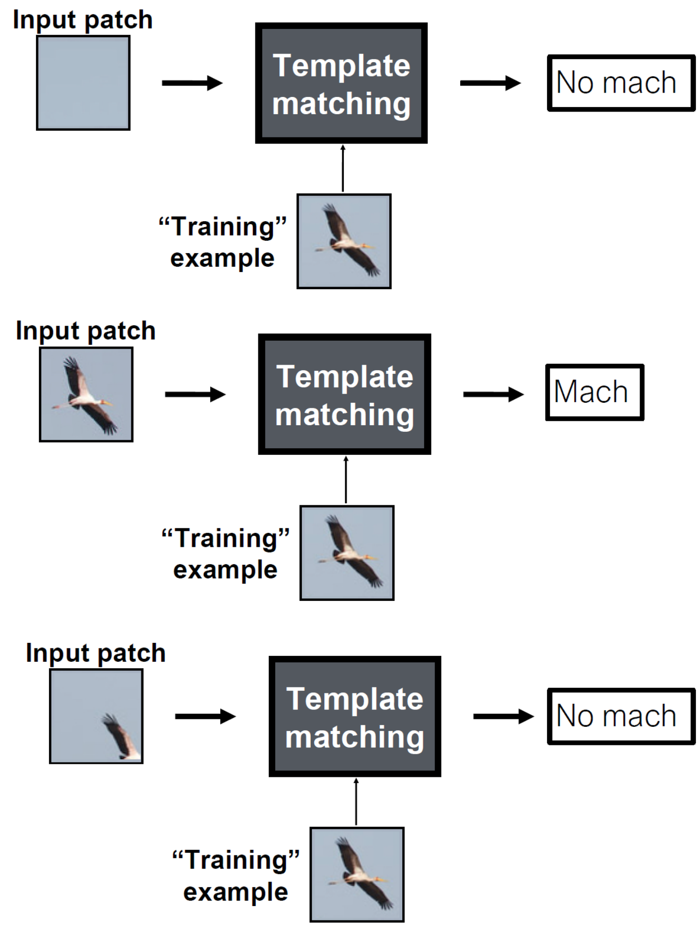
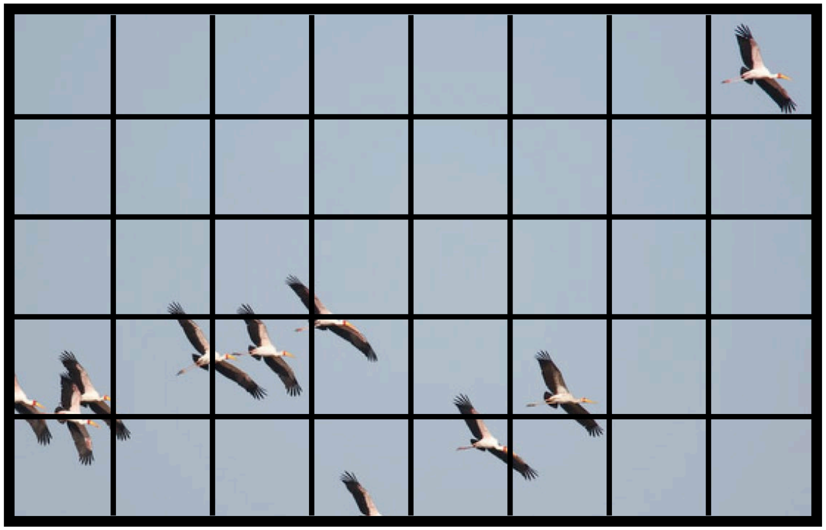


Image Credit: Efros



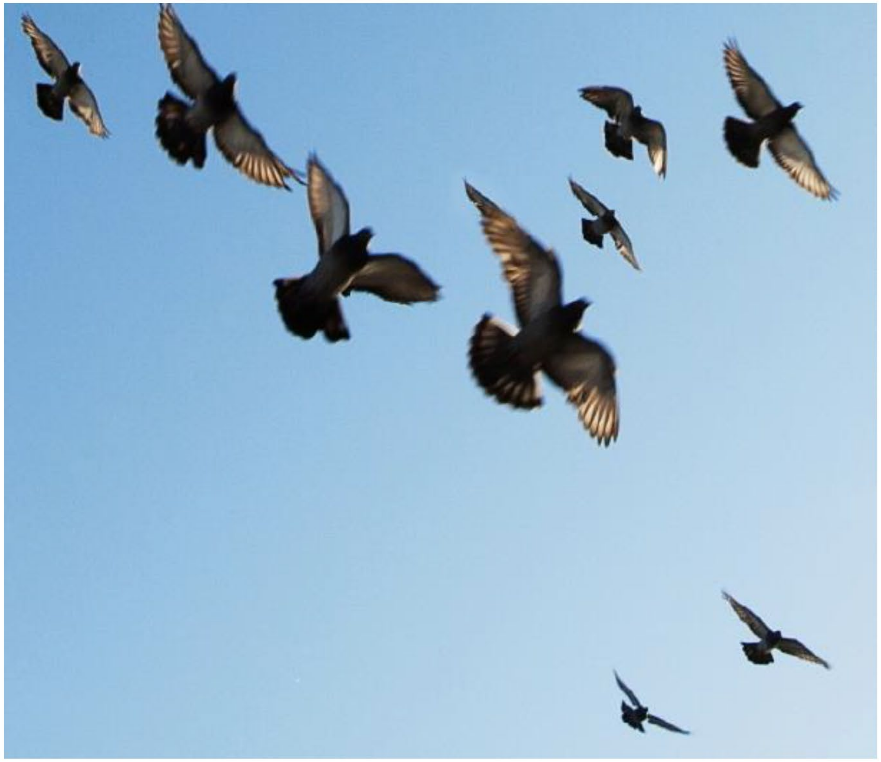
Sky	Sky	Sky	Sky	Sky	Sky	Sky	Bird
Sky	Sky	Sky	Sky	Sky	Sky	Sky	Sky
Sky	Sky	Sky	Sky	Sky	Sky	Sky	Sky
Bird	Bird	Bird	Sky	Bird	Sky	Sky	Sky
Sky	Sky	Sky	Bird	Sky	Sky	Sky	Sky

Image Credit: Freeman

What if object in image appears in a range of sizes?

Image Credit: Efros

Multi-Scale Pyramids

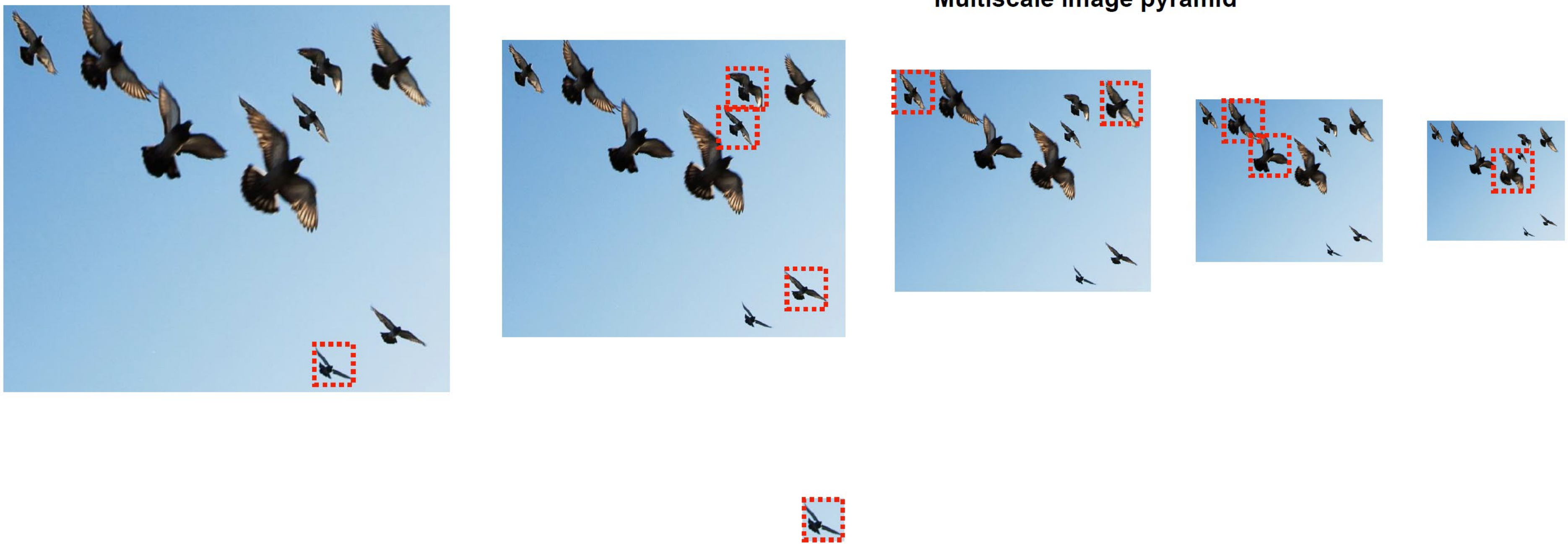


Template

Image Credit: Efros

Multi-Scale Pyramids

Multiscale image pyramid



Template

A multiscale image pyramid provides an alternative image representation to achieve translation and scale invariance

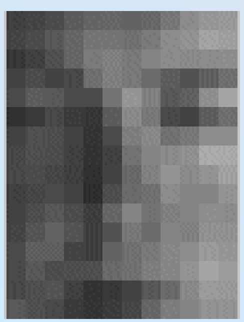
Image Credit: Efros

Multi-Resolution Matching

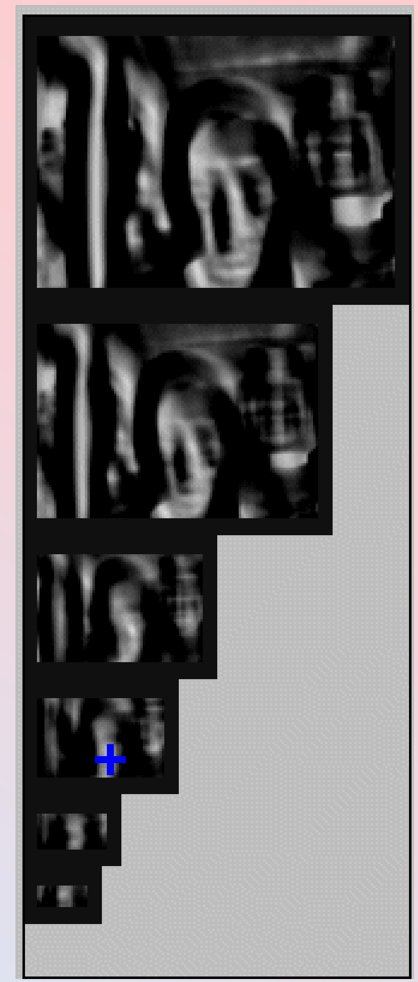
Computer Science

Normalized correlation coefficient over multi-resolution search space:

$$r = \frac{1}{n} \frac{\sum_i (s_i - mean(s))\,(m_i - mean(m))}{(\sigma_s\,\sigma_m)}$$



←Template matched over all resolutions →



Finding the Face and its Movement by Locating the Best Match of a Face Template

(a) Input

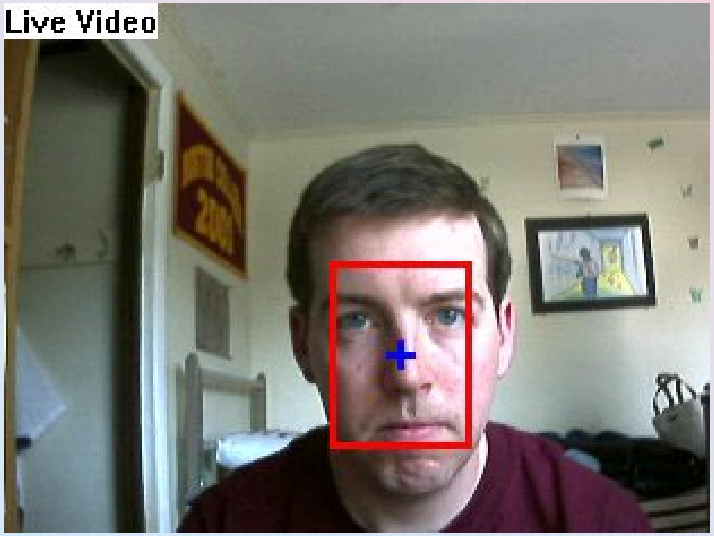
You can apply template matching to a small version of your input image and use that search result to start searching for a match in the 2nd smallest images. Repeat until the original size is processed.

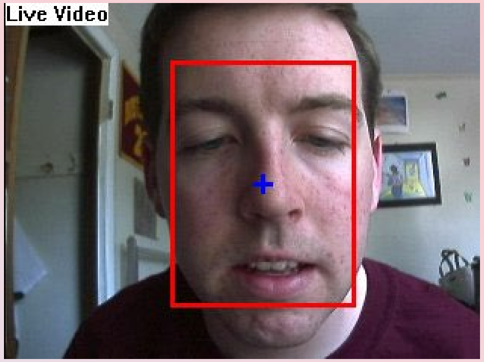
(d) Correlation

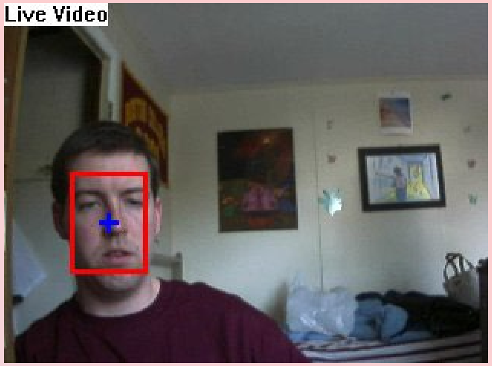
Face Detection

Computer Science

Data Variability





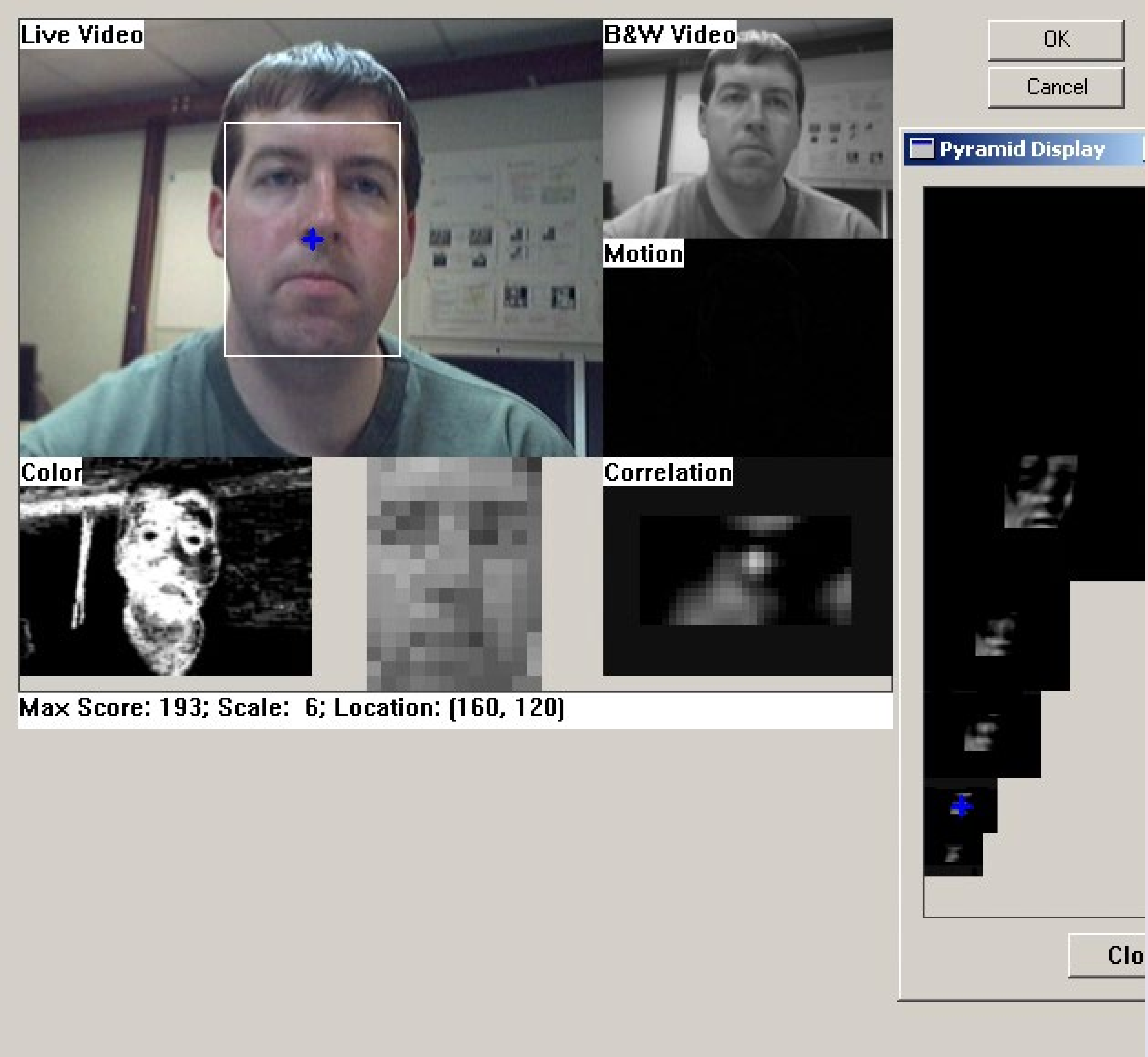


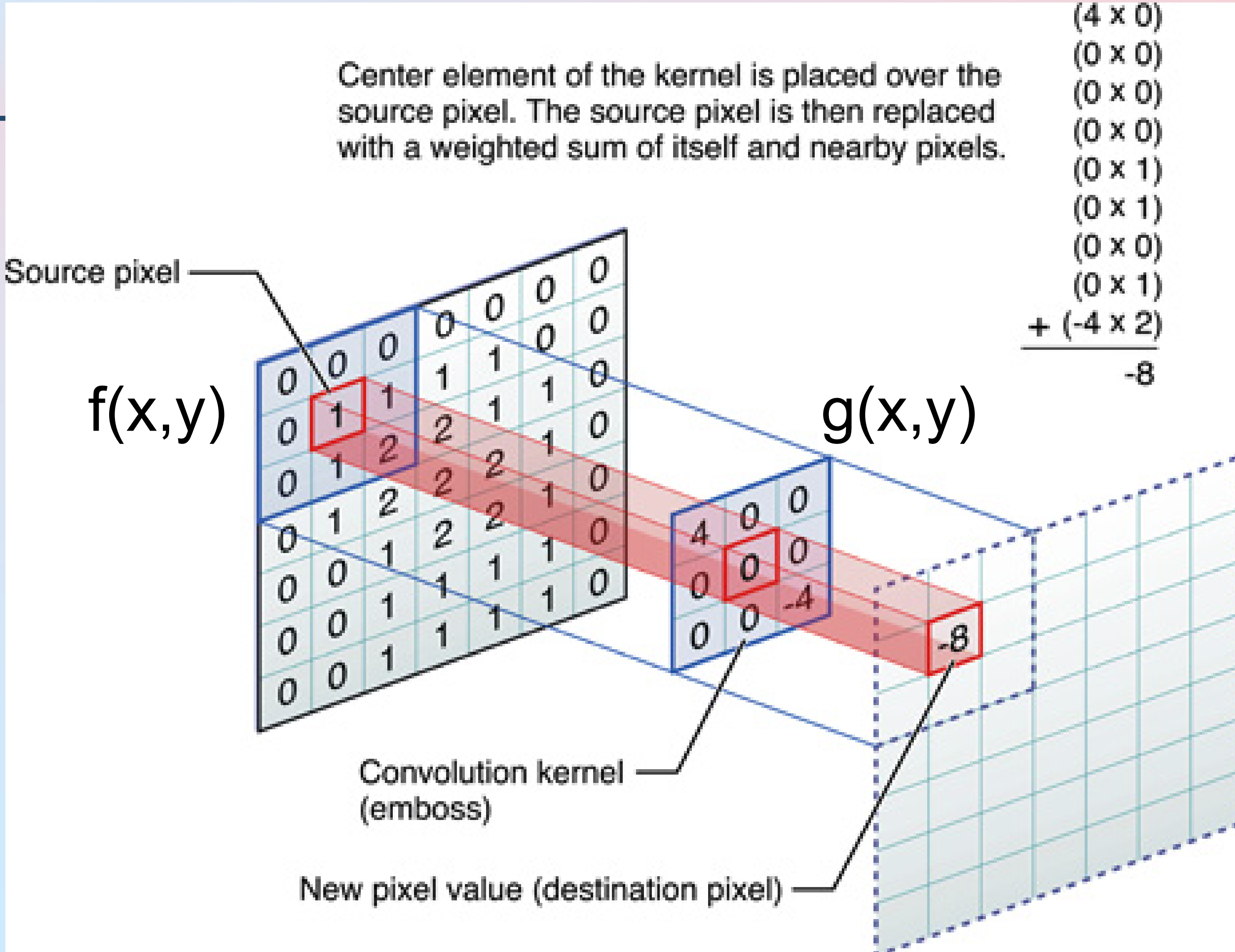
Large Face

Small Face

Shadows

Cluttered background


Live Video
B&W Video
OK
Cancel
Pyramid Display
Motion
Color
Correlation
Max Score: 193; Scale: 6; Location: (160, 120)
Clo



f*g

Image Credit: Madhushree Basavarajaiah

Object Recognition = Parameter Estimation

Computer Science

Affine parameterization **x**' = A**x** + b => estimate **a**

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} x_0 \\ y_0 \end{pmatrix}$$

2D translation

$$\mathbf{A} = \begin{pmatrix} s_x & 0 \\ 0 & s_y \end{pmatrix} \begin{pmatrix} \cos\theta_0 & \sin\theta_0 \\ -\sin(\theta_0+\alpha) & \cos(\theta_0+\alpha) \end{pmatrix}$$

scale, sheer in x & y, rotation

Betke, Makris, IJCV 2001

Object Recognition = Parameter Estimation

Computer Science

Affine parameterization **x**' = A**x** + b => estimate **a**

Likelihood function

$$P(\mathbf{I}\,|\,\mathbf{a}) = \frac{1}{(2\pi\sigma^2)^{MN/2}} \times \exp\left(-\frac{1}{2\sigma^2}\sum_{k=1}^{MN}(I_k - m_k(\mathbf{a}))^2\right)$$

General Camer-Rao lower bound:

$$\mathrm{E}[(\hat{\mathbf{a}} - \mathbf{a})(\hat{\mathbf{a}} - \mathbf{a})^T] \geq \mathbf{J}^{-1}$$

Betke, Makris, IJCV 2001

Fisher Information Matrix J

$$
\begin{aligned}
J_{ij} &= -\mathrm{E}\left[\frac{\partial^2}{\partial a_i \partial a_j} \ln P(\mathbf{I} \mid \mathbf{a})\right] \\
&= \frac{1}{\sigma^2} \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} \left(\frac{\partial m(x, y; \mathbf{a})}{\partial a_i} \frac{\partial m(x, y; \mathbf{a})}{\partial a_j} \right)
\end{aligned}
$$

a_4 = s

change in scale

a_1 = x

horizontal shift

a_2 = y

vertical shift

a_3 = θ

in-plane rotation

Object Coherence

Computer Science

CRLB: $$\mathrm{E}[(\hat{a}_i - a_i)^2] \geq [\mathbf{J}^{-1}]_{ii} = \frac{\sigma^2}{E}\ell_i^2$$

Energy for object q: $$E = \sum_{(x,y)\in O} |q(x, y; \mathbf{a})|^2$$

Coherence scale and volume:

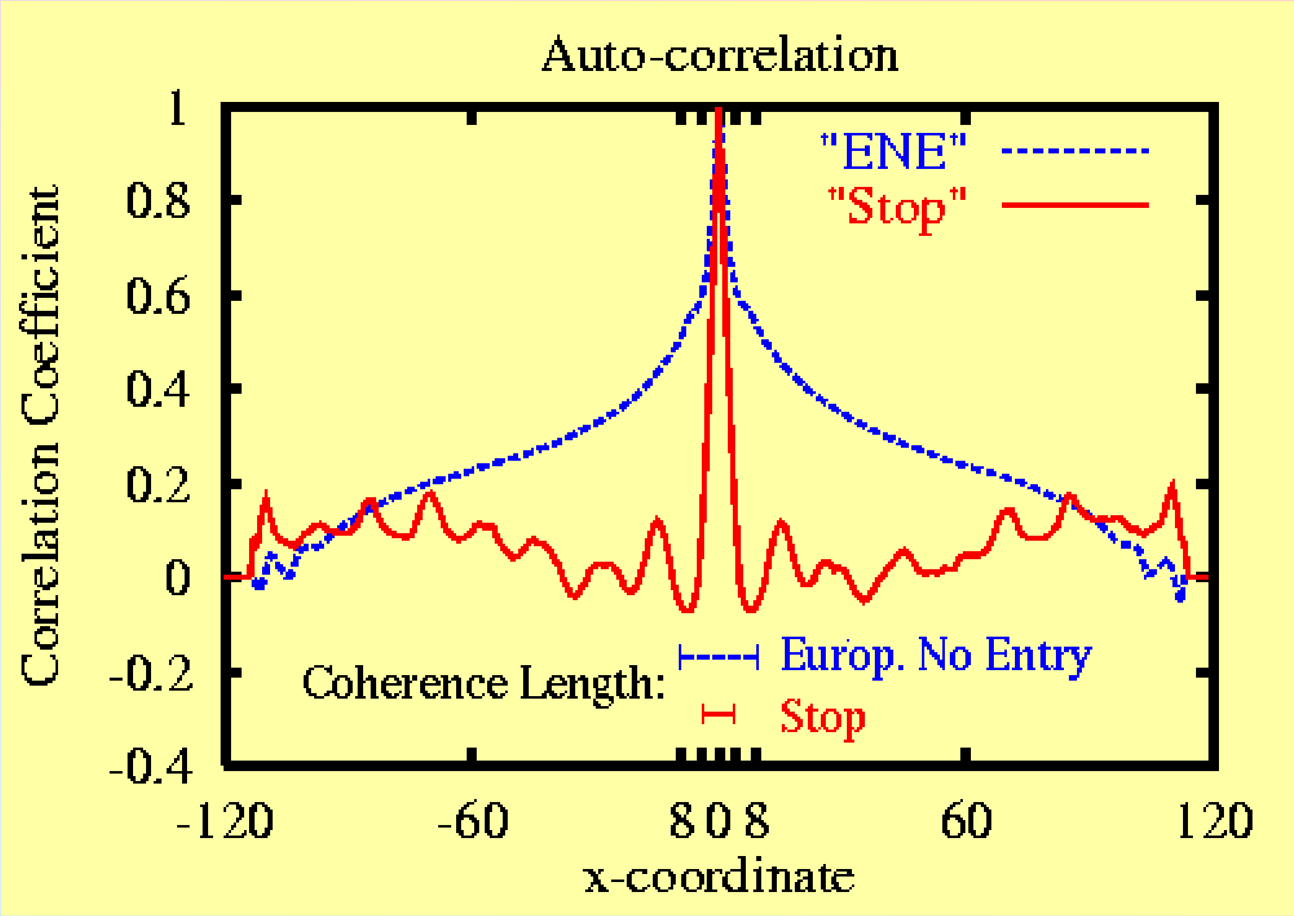
$$\ell_i = \left([\mathbf{J}^{-1}]_{ii}\,\frac{E}{\sigma^2}\right)^{\frac{1}{2}}$$

$$V = \left(\frac{E}{\sigma^2}\right)^{\frac{n_a}{2}} |\mathbf{J}|^{-\frac{1}{2}}$$

Affine:
$n_a = 6$

Coherence Length Scale ℓ_x

Computer Science



Since coherence length of Stop sign < No-Entry Sign, resolving location (x-coordinate) of Stop sign is easier

Coherence Area

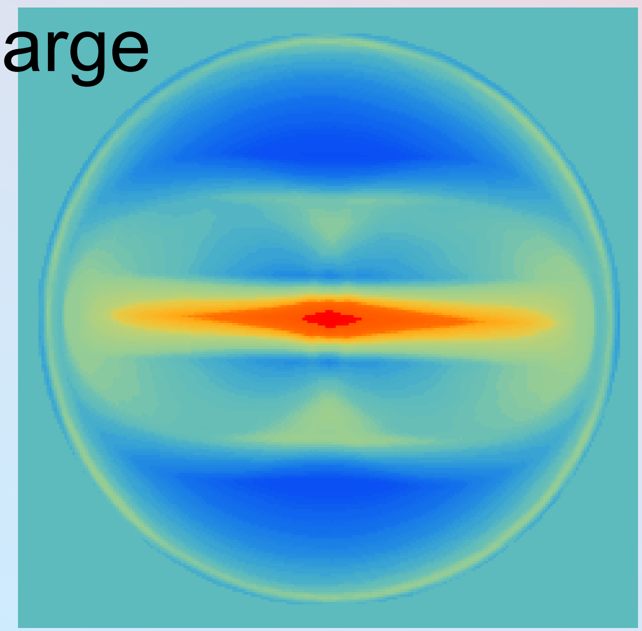
Computer Science

Betke, Makris, IJCV 2001

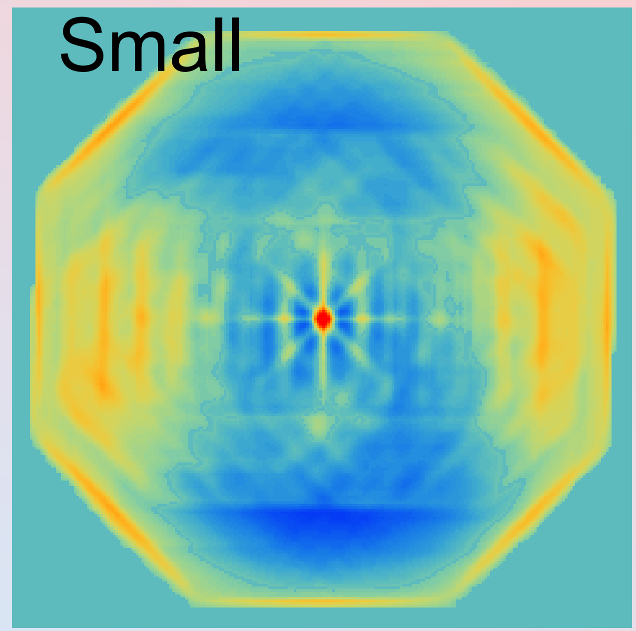
$$V = \left(\frac{E}{\sigma^2}\right)^{\frac{n_a}{2}} |\mathbf{J}|^{-\frac{1}{2}}$$

$n_a = 2$

Large



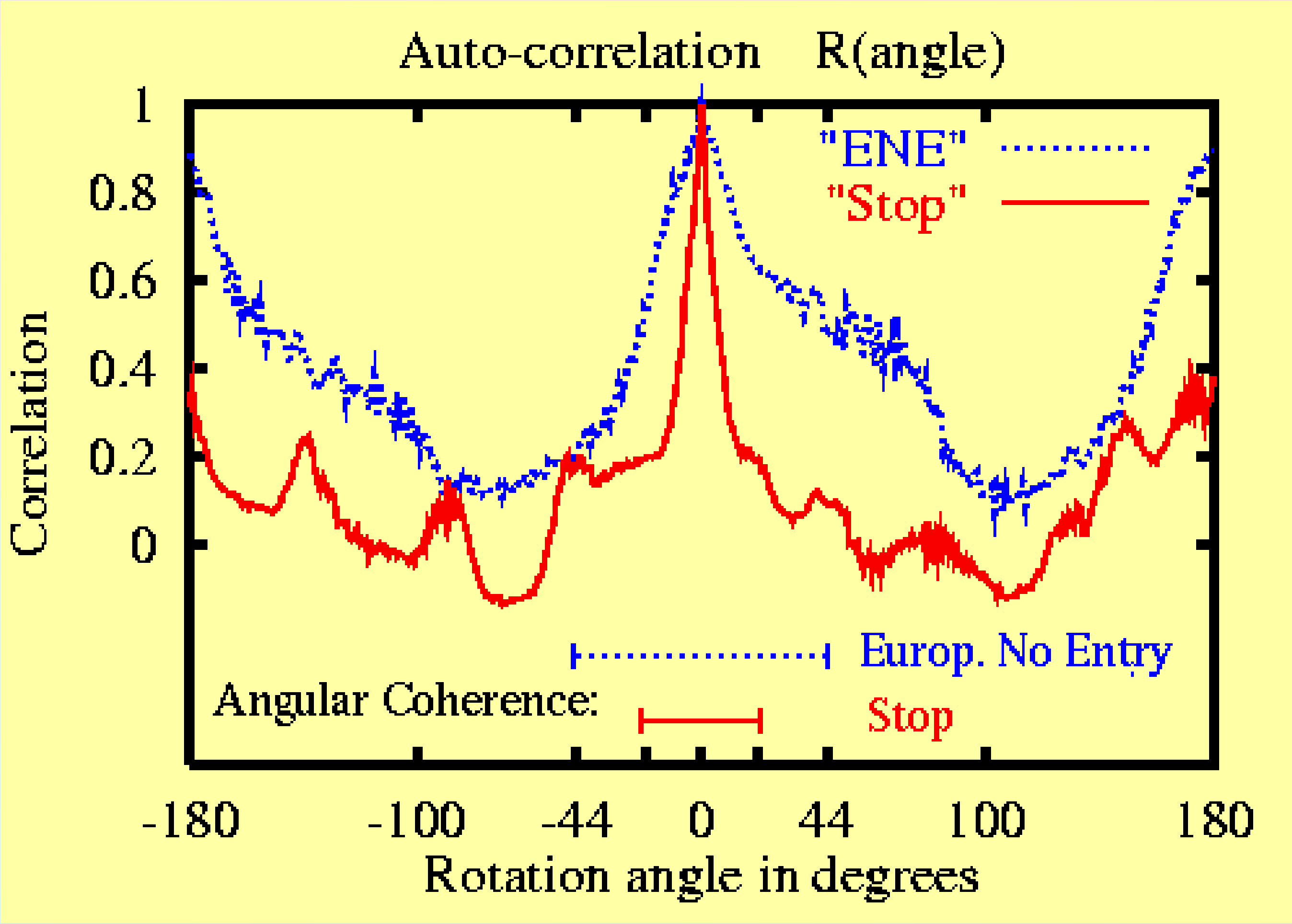
Small



Resolving (x,y) location is easier for Stop sign

Angular Coherence Scale

Computer Science



Peaks at ~45, 90, … degrees

Betke, Makris, IJCV 2001

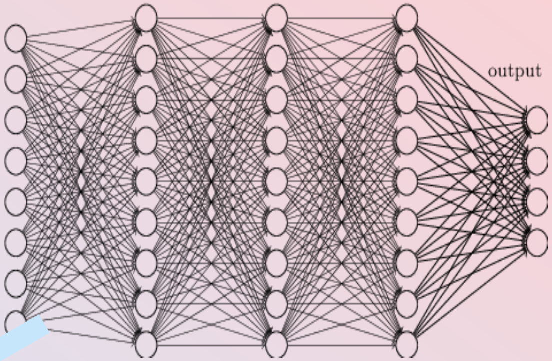
Conclusions on Coherence

Computer Science

- Using the Fisher Information matrix, we can compute the coherence scales of objects
- Coherence scales define the recognizability of object parameters
- Intuitively, coherence areas = “cells” = “interconnected parts” =“degrees of freedom”
- Coherence scales can be visualized with autocorrelations, i.e., “object convolution with itself”
- Neural nets compute many convolutions and memorize coherence scales of objects

Back to Neural Nets & their Success in Solving Computer Vision Problems

Large labeled datasets



Deep neural networks

GPU technology

Slide credit: Dinesh Jayaraman

Convolutional Neural Networks (CNN, ConvNet, DCN)

- CNN = a multi-layer neural network with
  - **Local** connectivity:
    - Neurons in a layer are only connected to a small region of the layer before it
  - **Share** weight parameters across spatial positions:
    - Learning shift-invariant filter

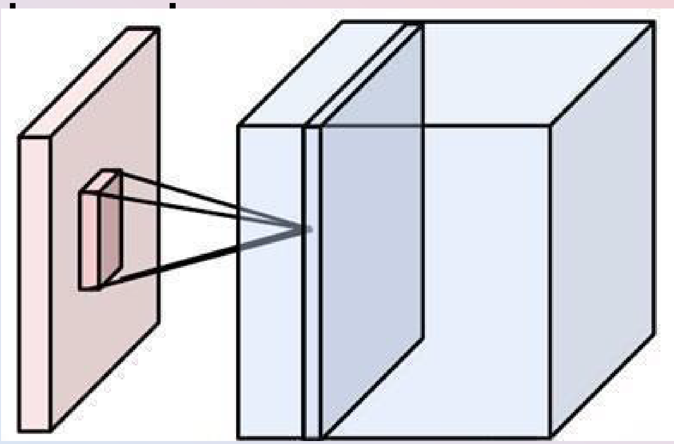
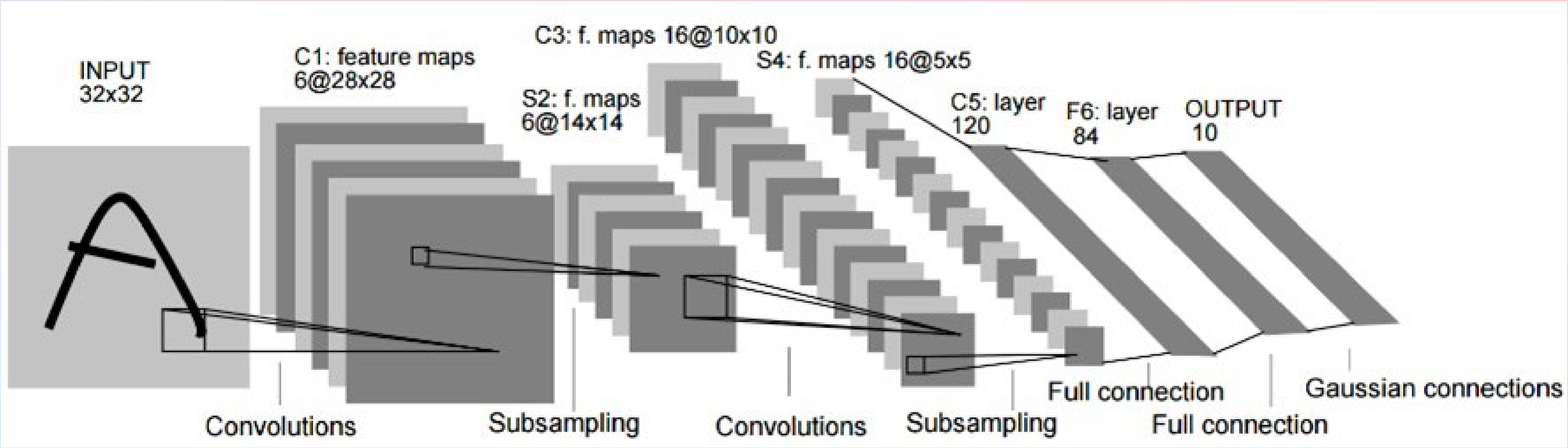


Image credit: A. Karpathy

LeNet [LeCun et al.]



1990: Zipcode recognition

http://yann.lecun.com/exdb/lenet/multiples.html

LeNet-1 from 1993

Gradient-based learning applied to document recognition [LeCun, Bottou, Bengio, Haffner 1998]

LeCun Interview, Oct. 5, 2023

Computer Science

- https://www.rsipvision.com/ICCV2023-Thursday/

Yann LeCun

- VP and Chief AI Scientist, Facebook
- Silver Professor of Computer Science, Data Science, Neural Science, and Electrical and Computer Engineering, New York University
- ACM Turing Award Laureate
- Member, National Academy of Engineering

LeCun's 2023 Focus: Predict Content of Masked-out Images/Video Frames

Boston University Computer Science

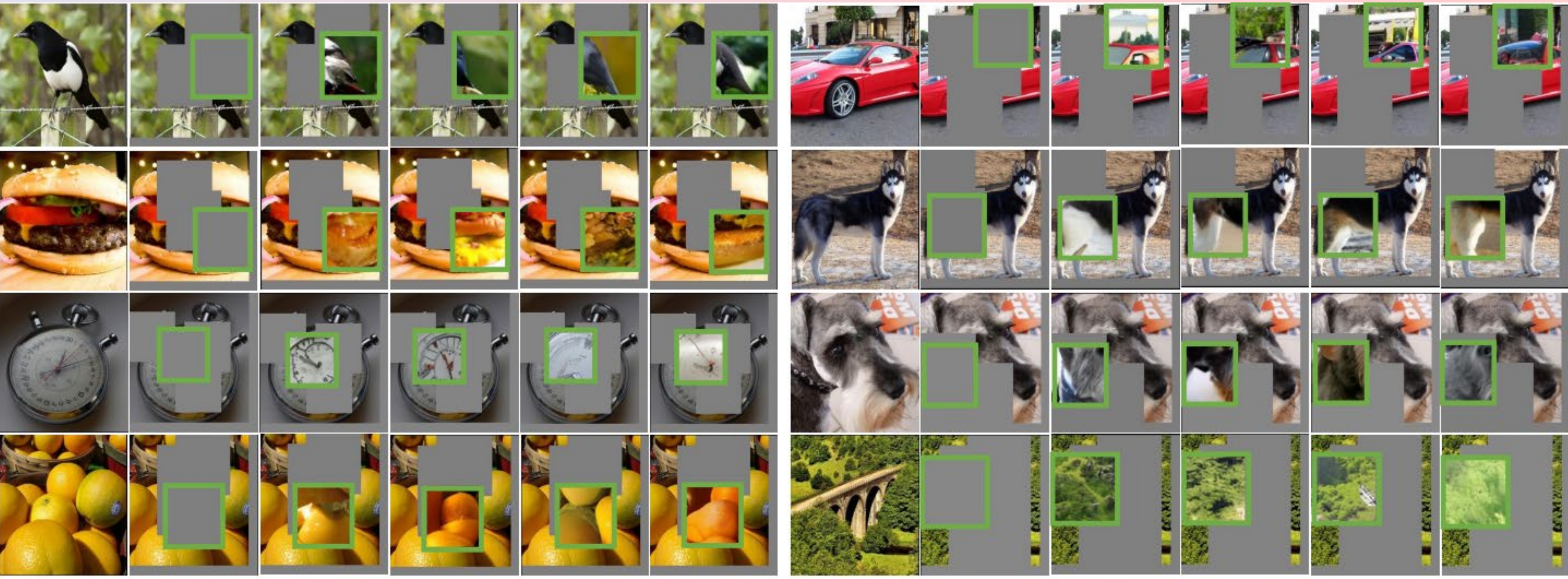
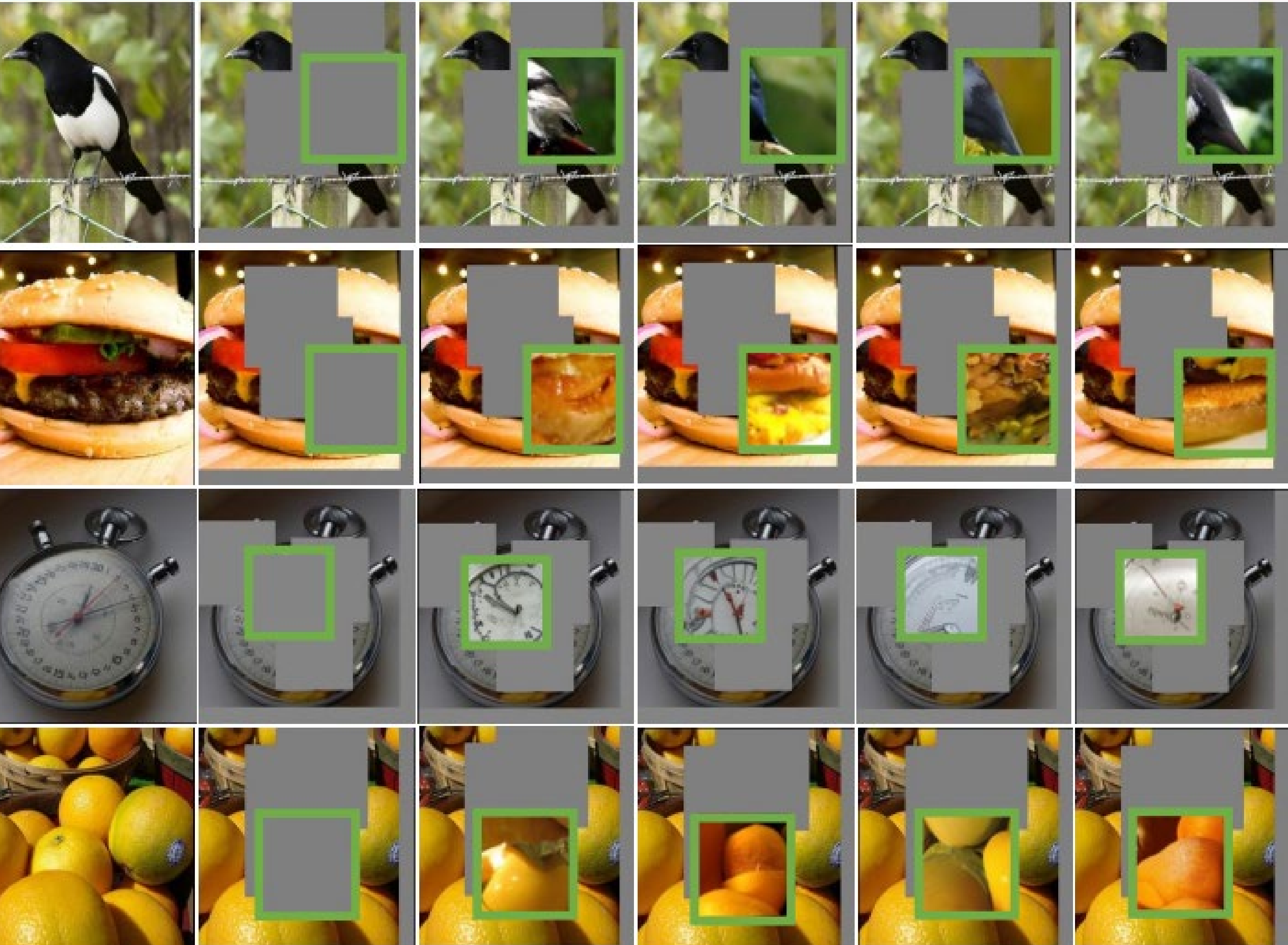
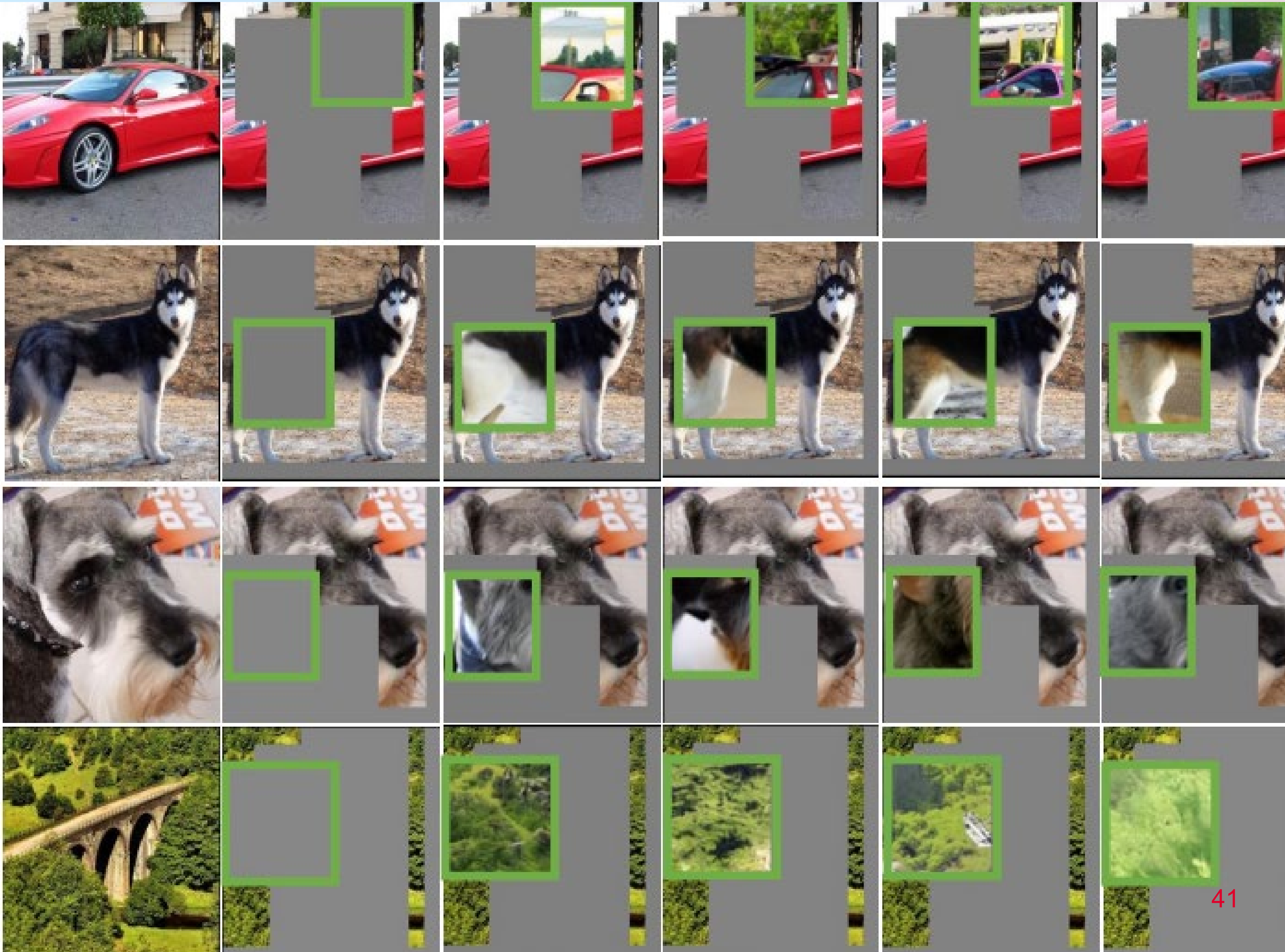


Image Credit: 2301.08243.pdf (arxiv.org)





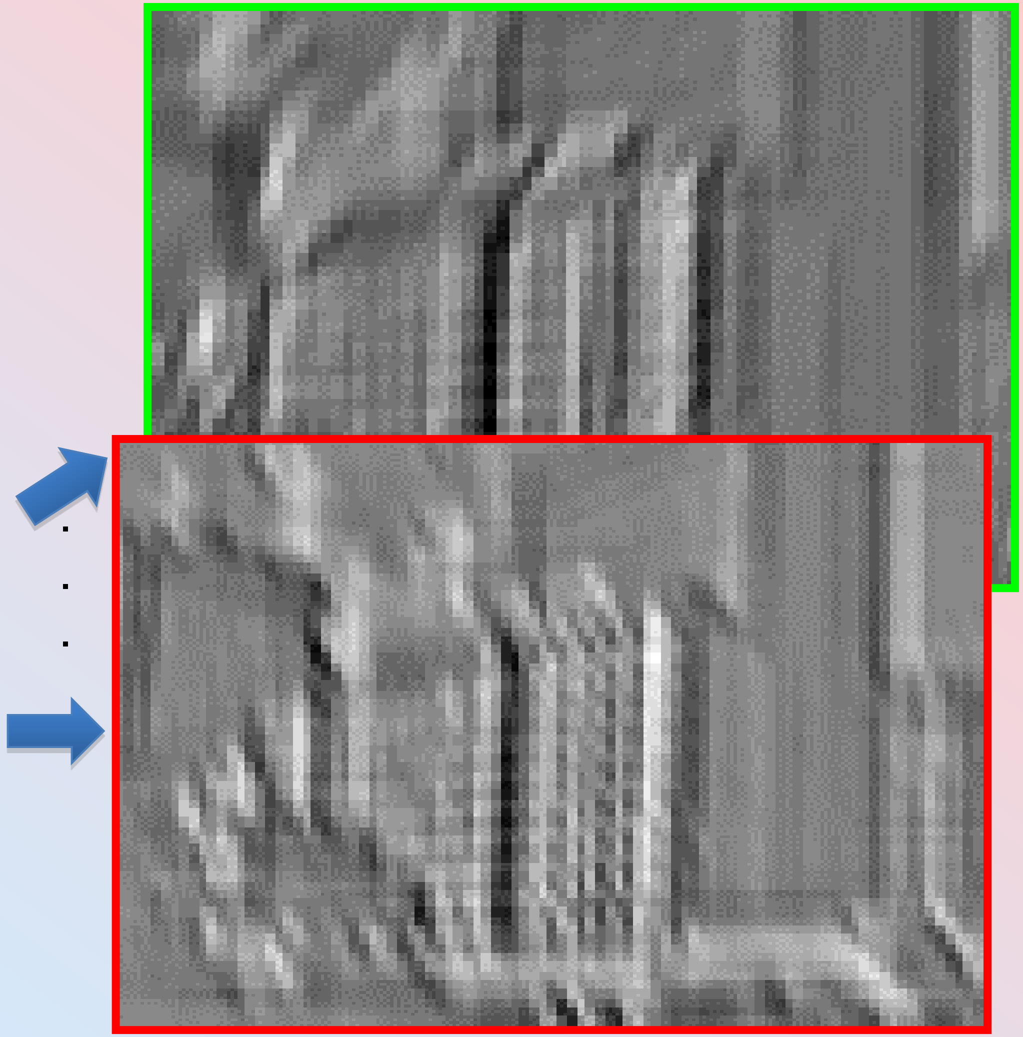

GT

Masked Siamese Networks
Assran et al., ECCV 2022

Another example of 2D Convolution

- Weighted moving sum

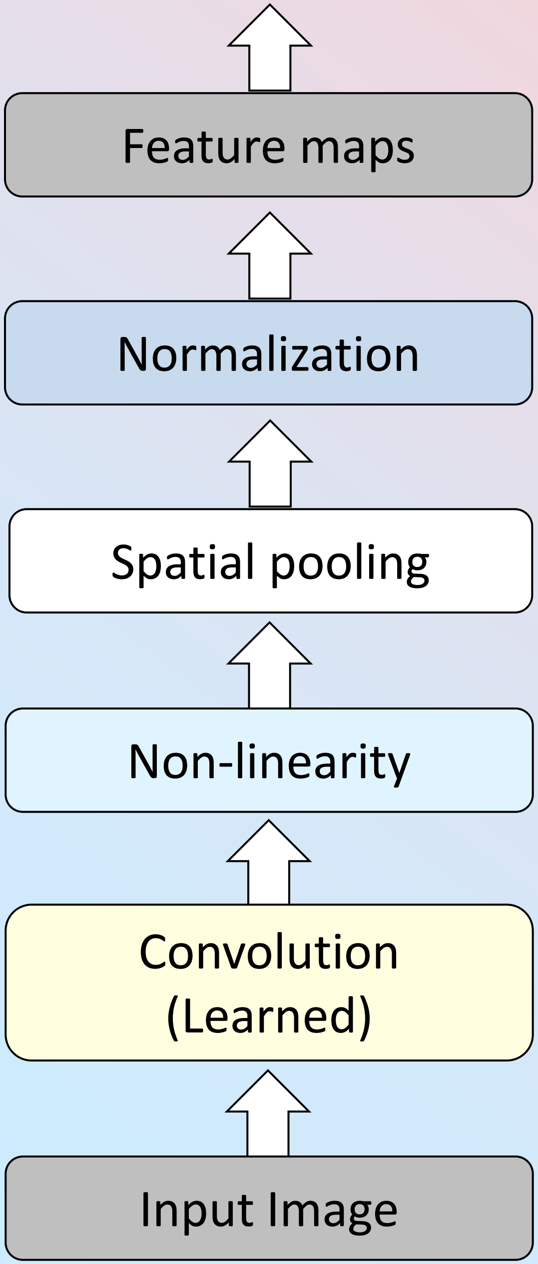
Input



Feature Activation Map

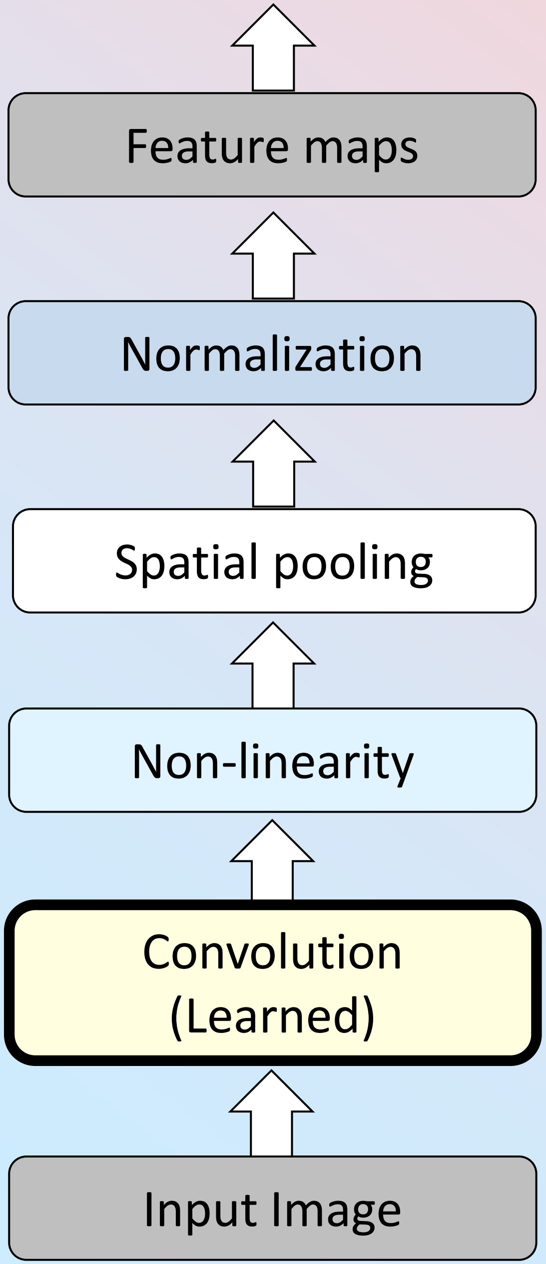
slide credit: S. Lazebnik

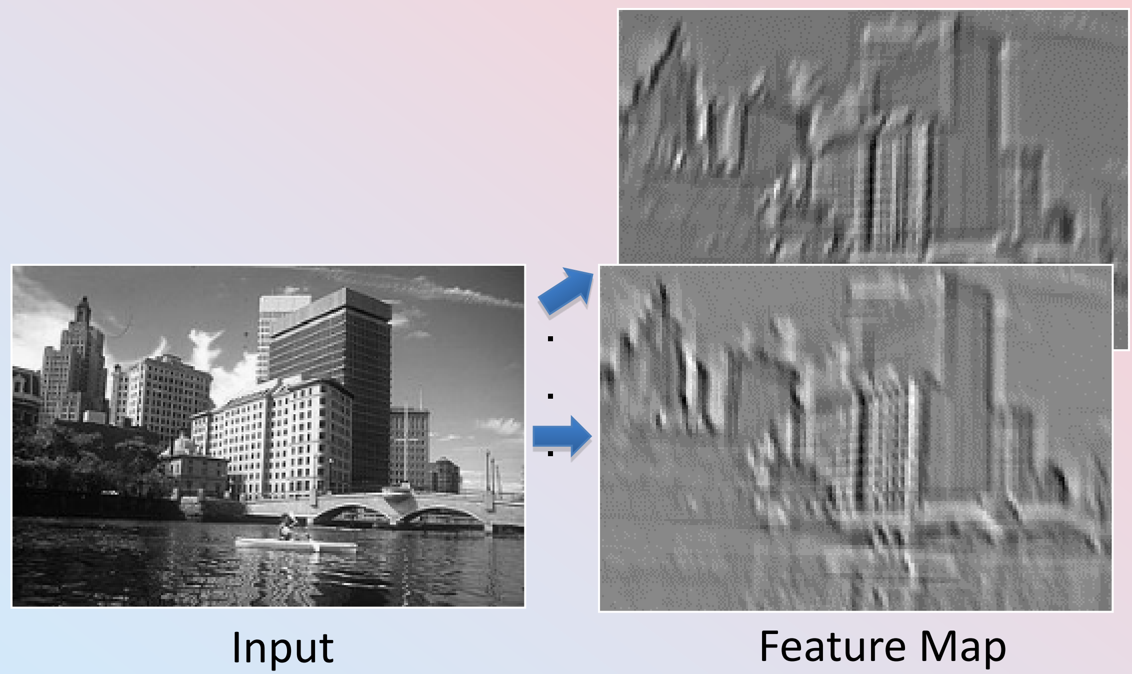
Convolutional Neural Networks



slide credit: S. Lazebnik

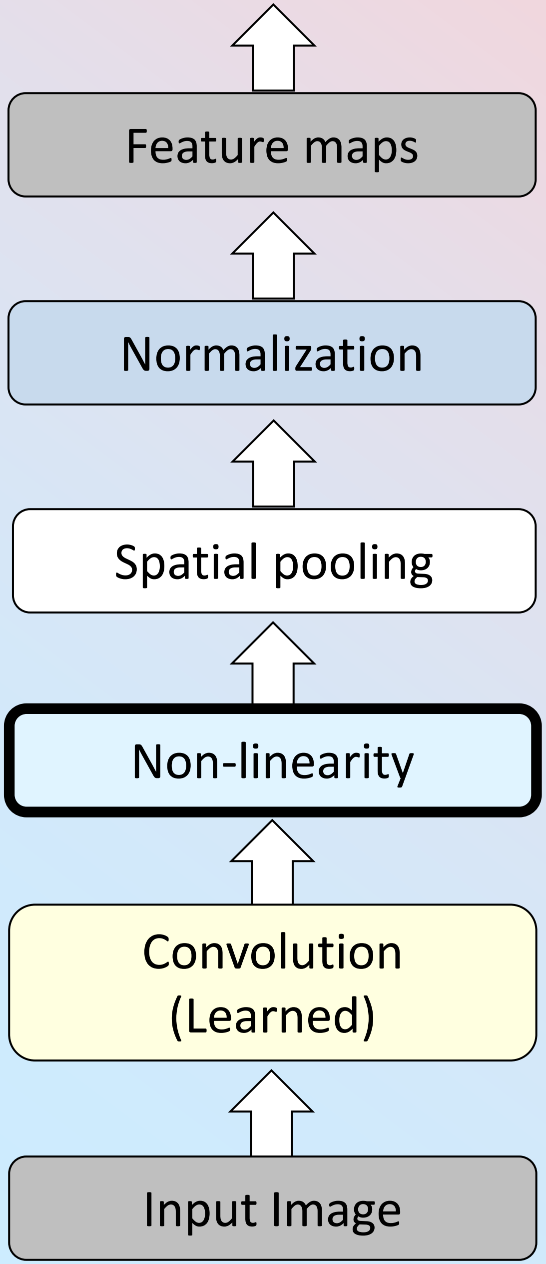
Convolutional Neural Networks



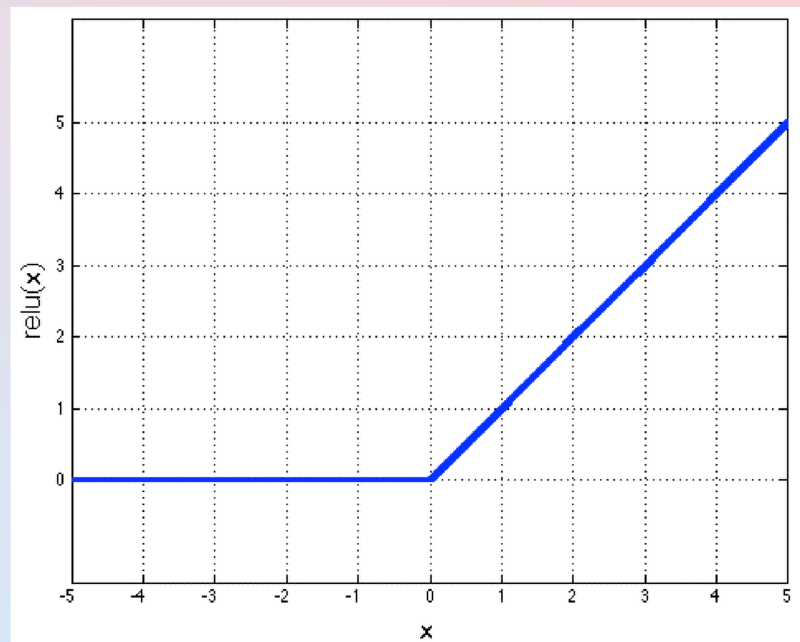


slide credit: S. Lazebnik

Convolutional Neural Networks

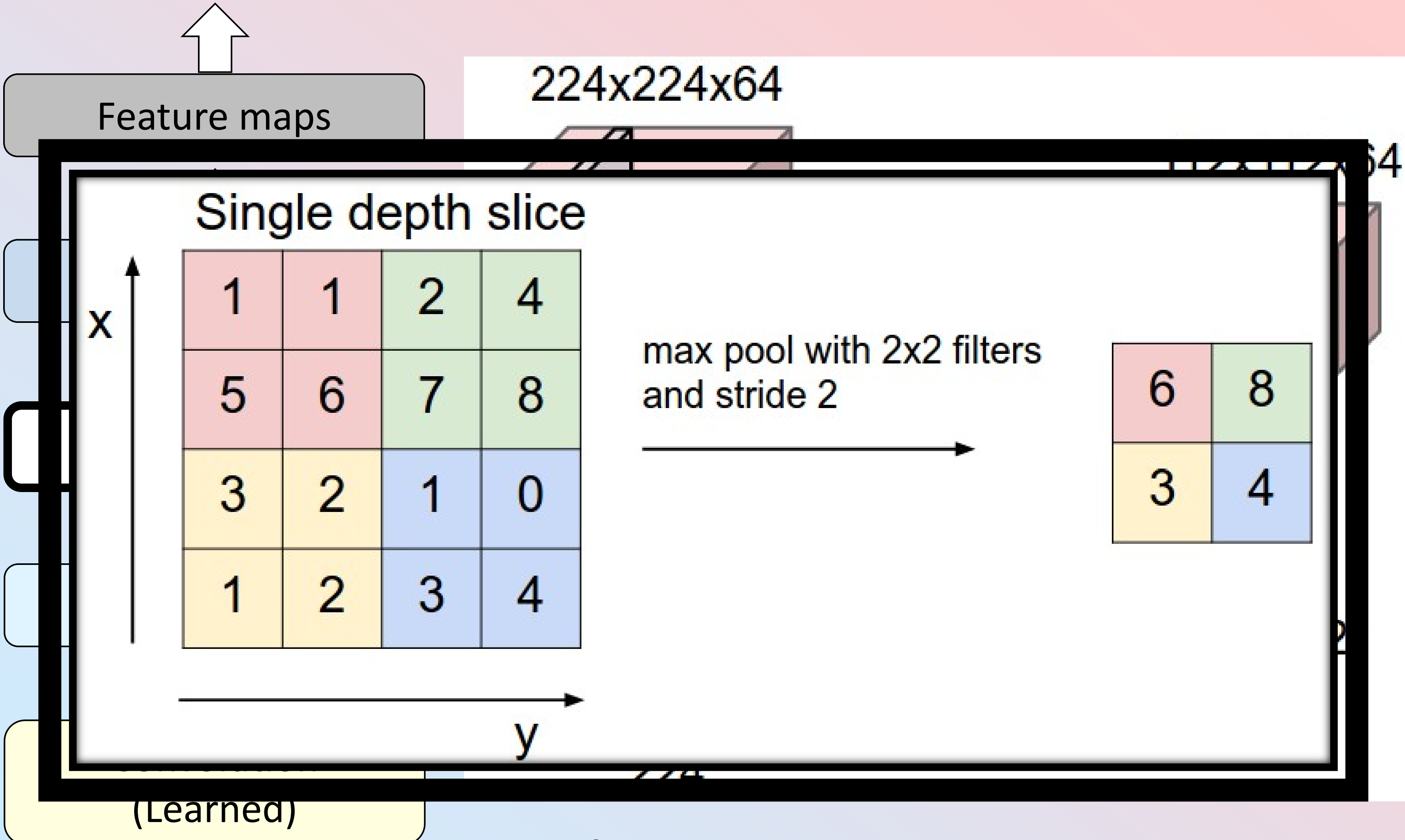


Rectified Linear Unit (ReLU)

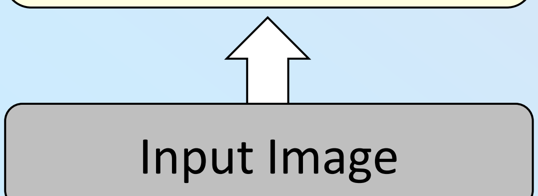


slide credit: S. Lazebnik

Convolutional Neural Networks

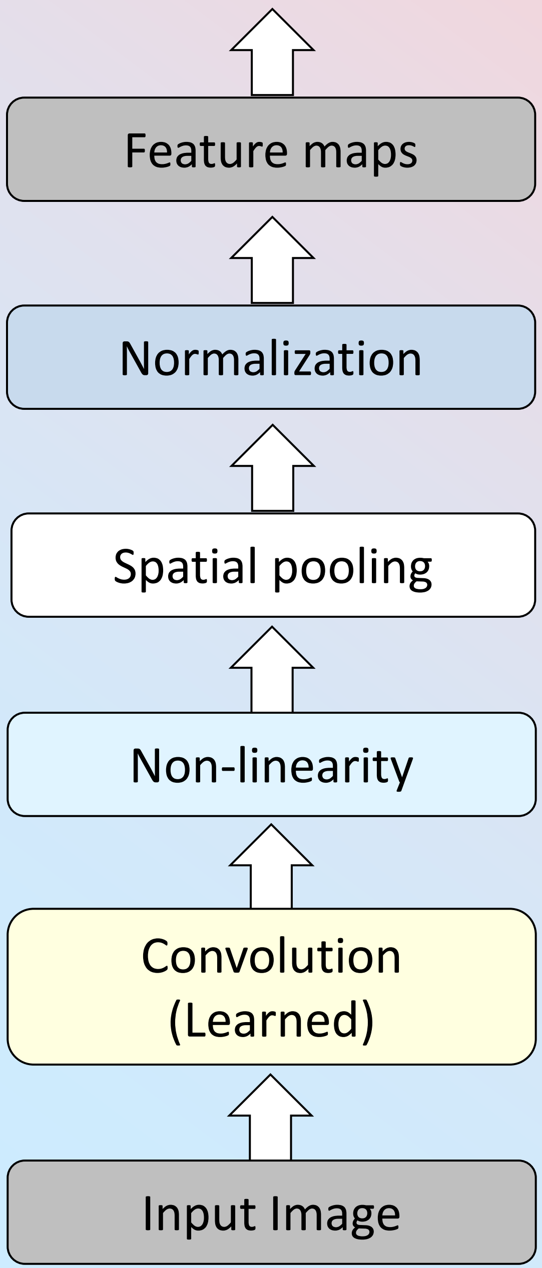


Provide *translation invariance*



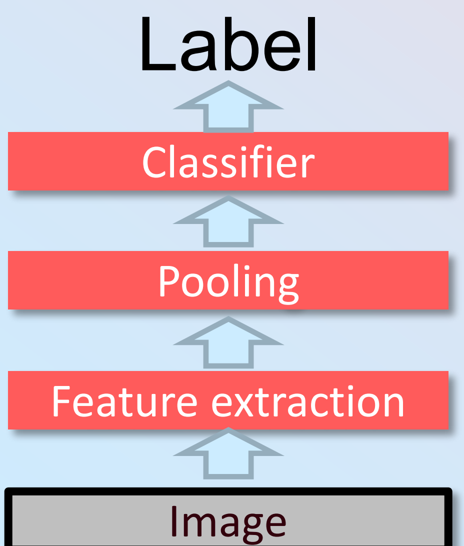
slide credit: S. Lazebnik

Convolutional Neural Networks

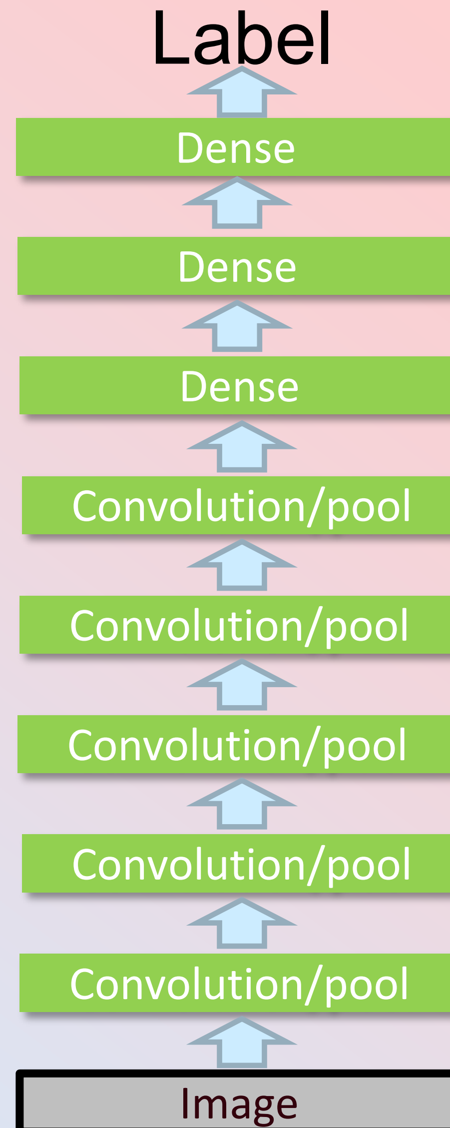


slide credit: S. Lazebnik

Traditional versus NN-based Computer Vision: Engineered versus Learned Features



Convolutional filters are trained in a supervised manner by back-propagating classification error



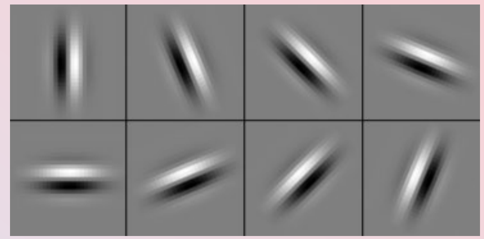
Jia-Bin Huang and Derek Hoiem, UIUC

SIFT Descriptor

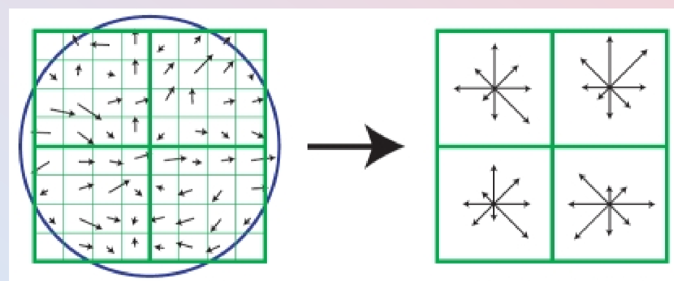
Lowe [IJCV 2004]

Image Pixels

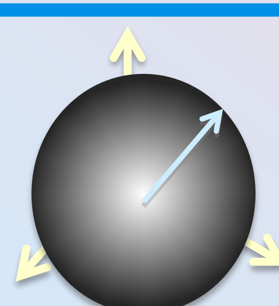
Apply oriented filters



Spatial pool (Sum)



Normalize to unit length

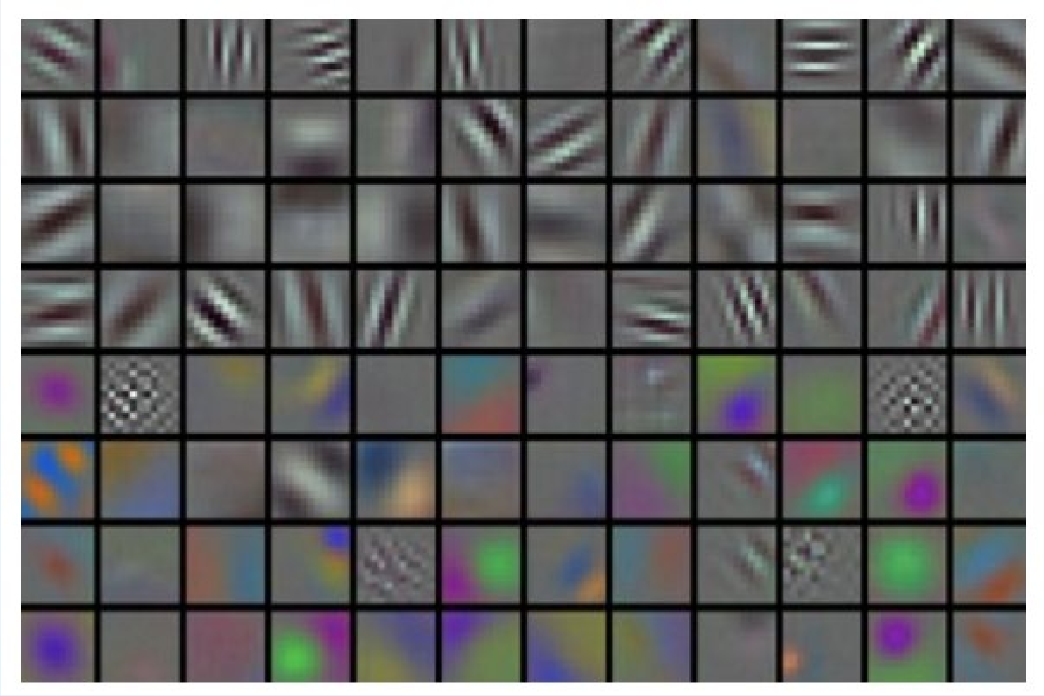


Feature Vector

Slide credit: R. Fergus

Visualizing what was learned

- What do the learned filters look like?



Typical first layer filters

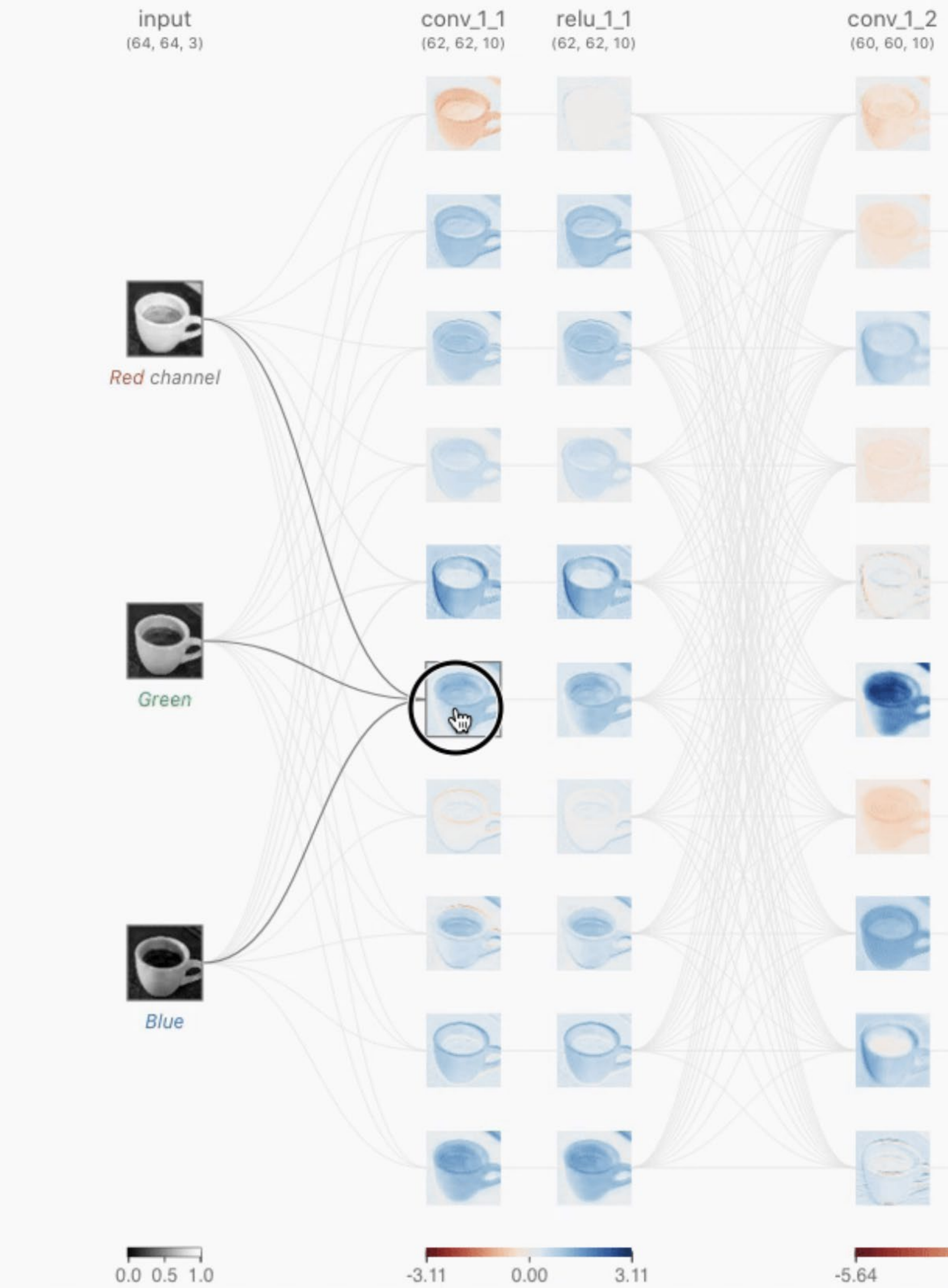
Image Credit: Kristen Grauman

The CNN Explainer

Thanks to CS640 classmate Mao Mao, we have a link to the *CNN Explainer*:

https://poloclub.github.io/cnn-explainer/

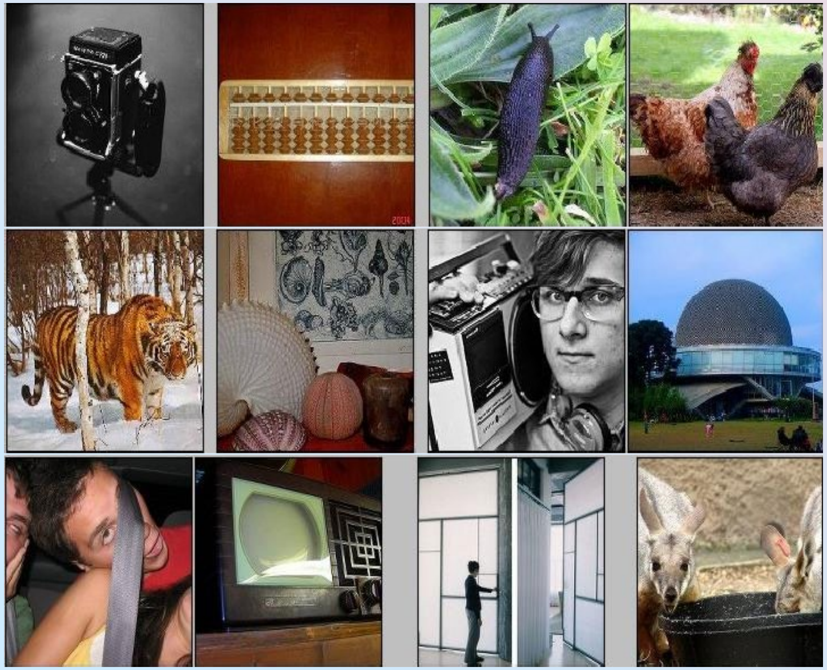
by Jay Wang, Robert Turko, Omar Shaikh, Haekyu Park, Nilaksh Das, Fred Hohman, Minsuk Kahng, and Polo Chau, a result of a research collaboration between Georgia Tech and Oregon State University



ImageNet –
The Data Set that Mattered and Still Matters!

IMAGENET

[Deng et al. CVPR 2009]

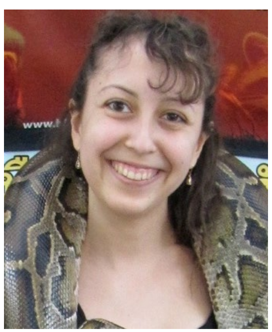
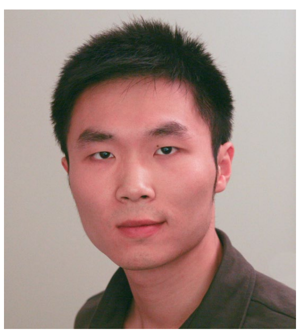
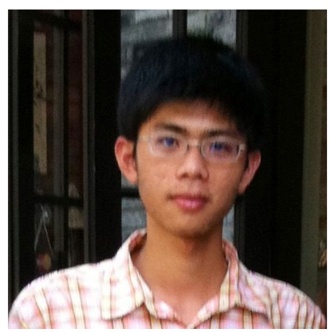
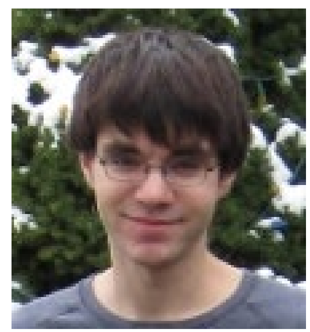


- 14 million labeled images
- 20 thousand object classes
- Images collected from the Internet
- Human labels obtained by crowdsourcing with Amazon Turk
- Still very important in 2024 because it is widely used for pretraining of “backbone neural nets” of current models

Analysis of Large Scale Visual Recognition

Adapted for BU CS 440/640 by M. Betke

Fei-Fei Li and Olga Russakovsky

Olga Russakovsky, Jia Deng, Zhiheng Huang, Alex Berg, Li Fei-Fei
Detecting avocados to zucchinis: what have we done, and where are we going?
ICCV 2013 **http://image-net.org/challenges/LSVRC/2012/analysis**

Backpack

Flute

Strawberry

Traffic light

Backpack

Matchstick

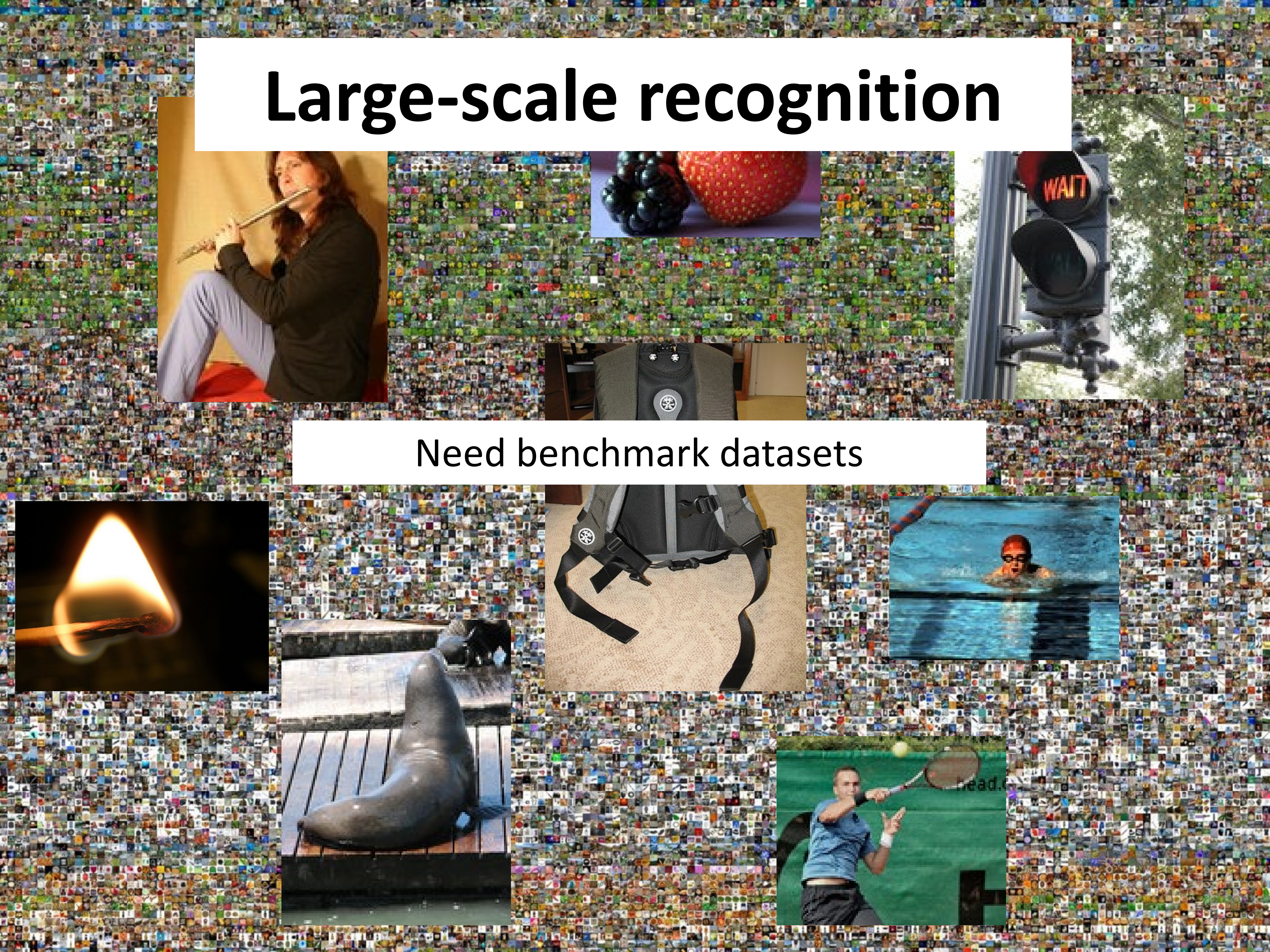
Bathing cap

Sea lion

Racket

Large-scale recognition

Large-scale recognition

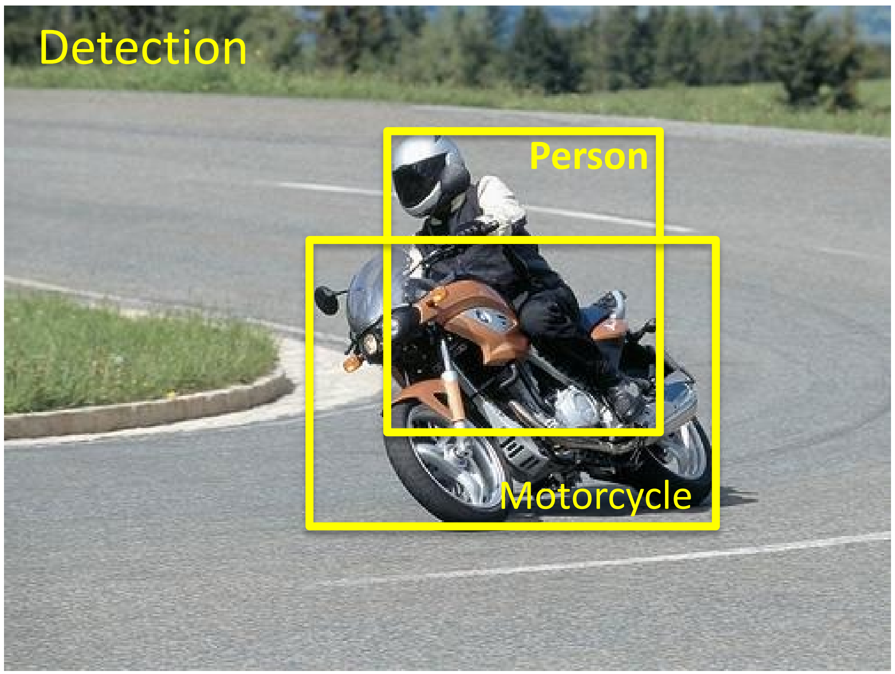


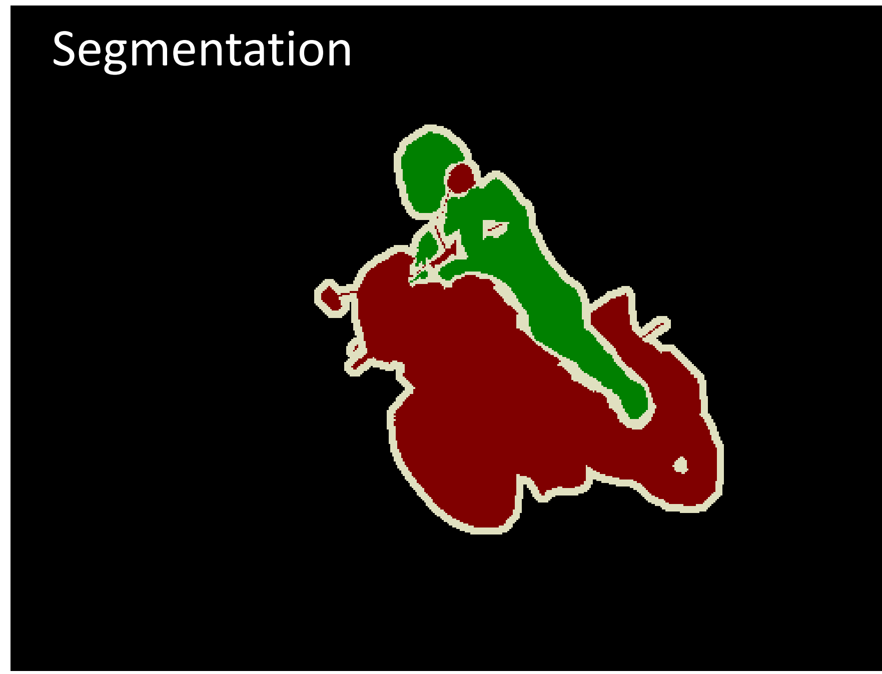
Need benchmark datasets

PASCAL VOC 2005-2012

20 object classes **22,591 images**

Classification: person, motorcycle





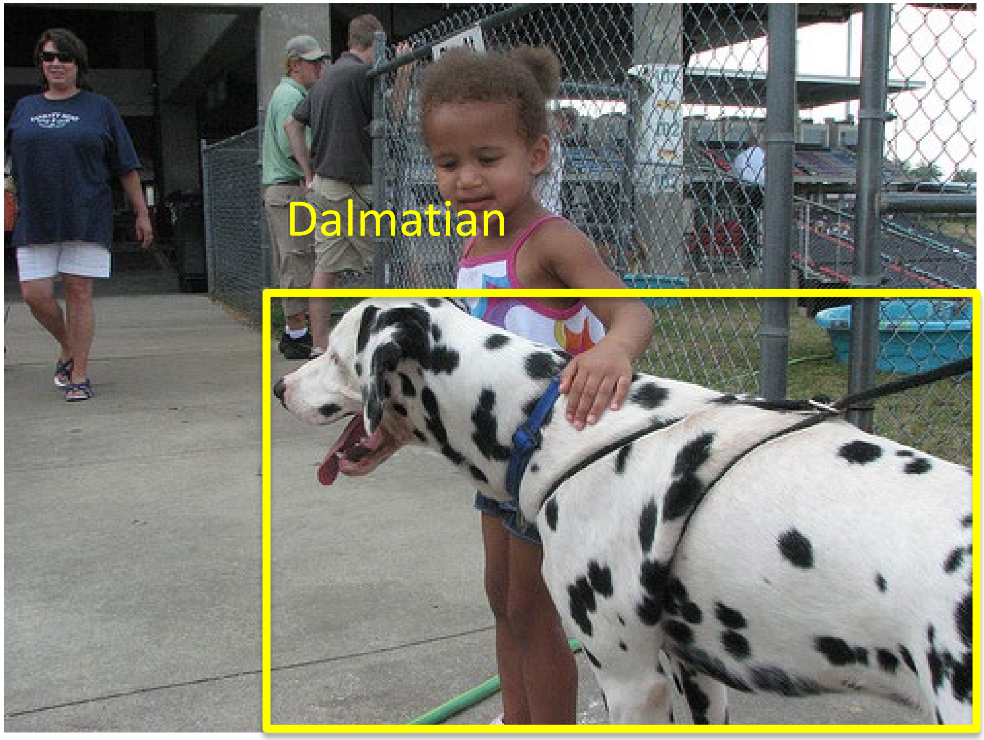
Action: riding bicycle

Everingham, Van Gool, Williams, Winn and Zisserman.
The PASCAL Visual Object Classes (VOC) Challenge. IJCV 2010.

IMAGENET Large Scale Visual Recognition Challenge (ILSVRC) 2010-2012

~~20 object classes 22,591 images~~

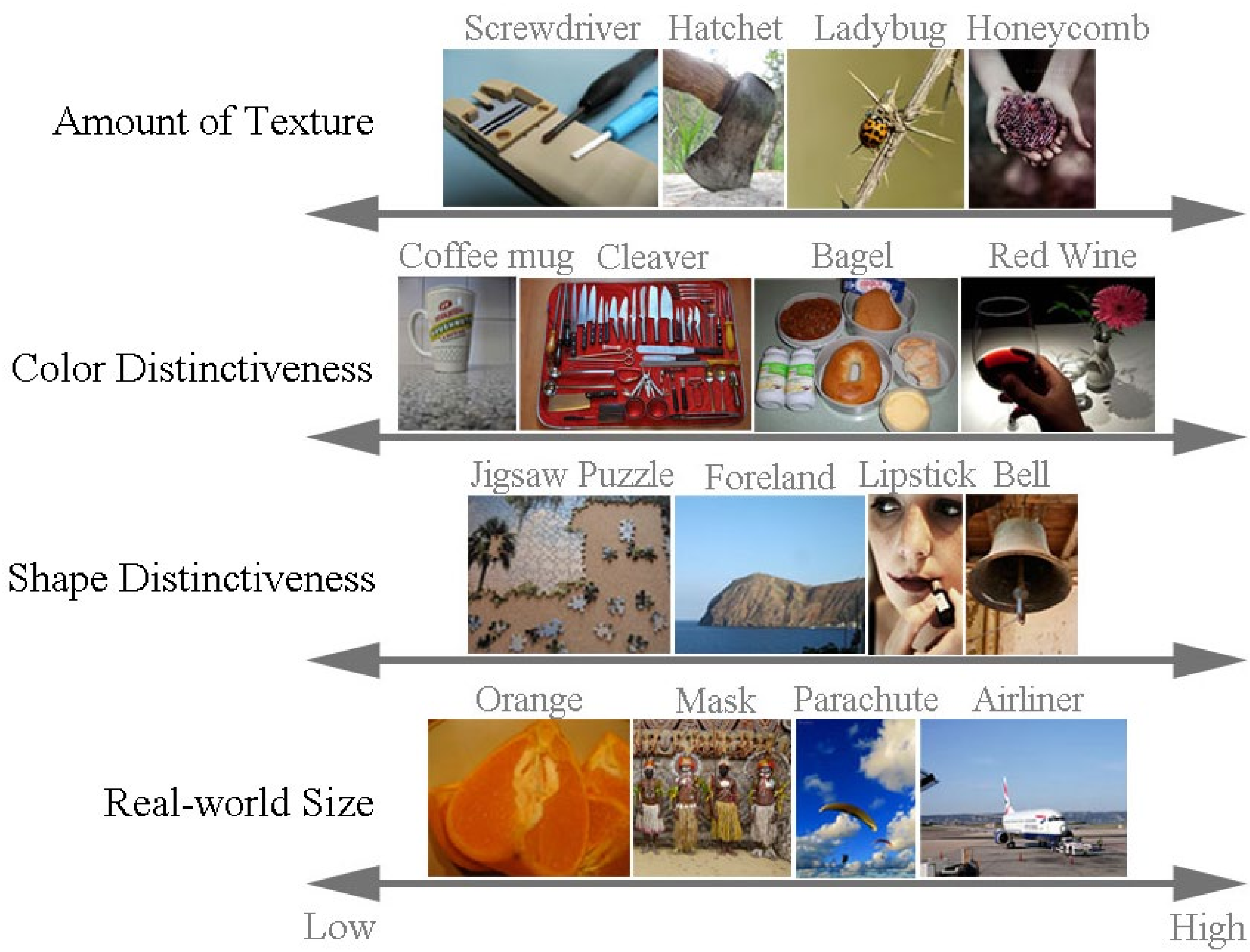
1000 object classes 1,431,167 images



http://image-net.org/challenges/LSVRC/{2010,2011,2012}

Variety of object classes in ILSVRC

Variety of object classes in ILSVRC



ILSVRC Task 1: Classification

Steel drum

ILSVRC Task 1: Classification

Allowed system output: 5 predictions per image

Goal: Get 1 of the 5 predictions correct

Steel drum

Output:
Scale
T-shirt
Steel drum
Drumstick
Mud turtle

Output:
Scale
T-shirt
Giant panda
Drumstick
Mud turtle

Indicator Function:

1[System output correct on this image] = 1 = 0

ILSVRC Task 1: Classification

Steel drum

Output:
Scale
T-shirt
<u>Steel drum</u>
Drumstick
Mud turtle

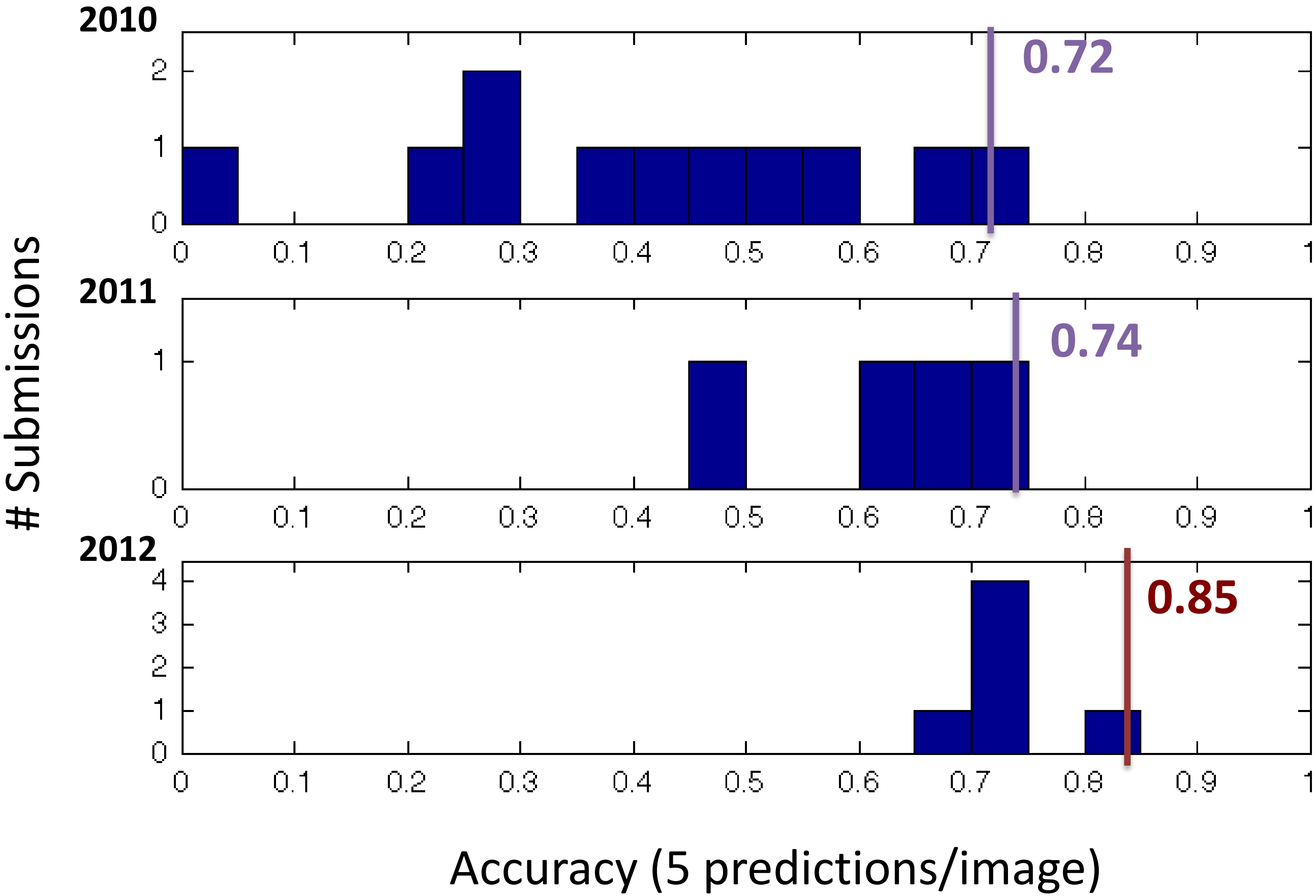
✓

Output:
Scale
T-shirt
Giant panda
Drumstick
Mud turtle

✗

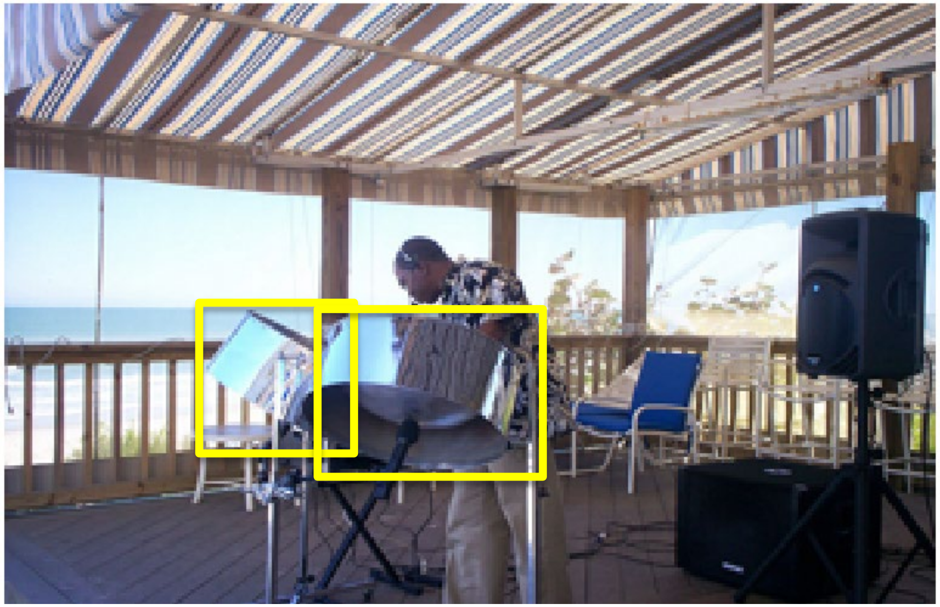
$$\text{Accuracy} = \frac{1}{100{,}000} \sum_{\substack{100{,}000 \\ \text{images}}} 1[\text{correct on image } i]$$

ILSVRC Task 1: Classification



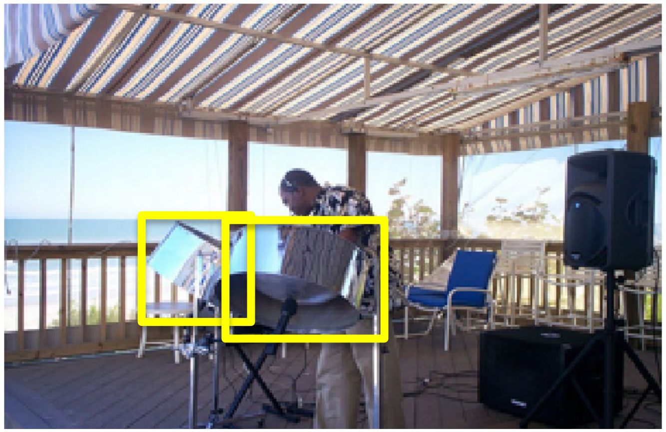
ILSVRC Task 2: Classification + Localization

Steel drum

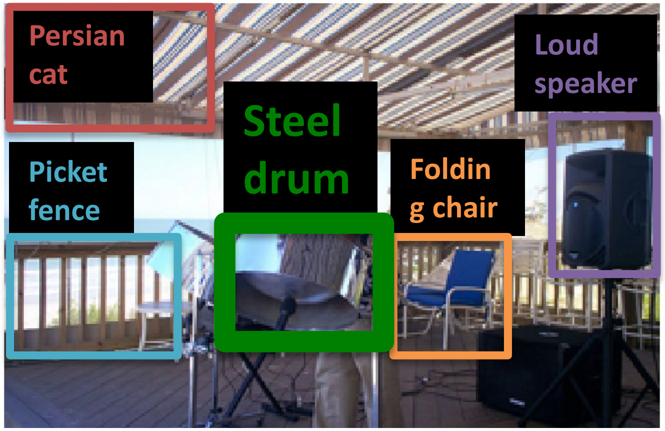


ILSVRC Task 2: Classification + Localization

Steel drum

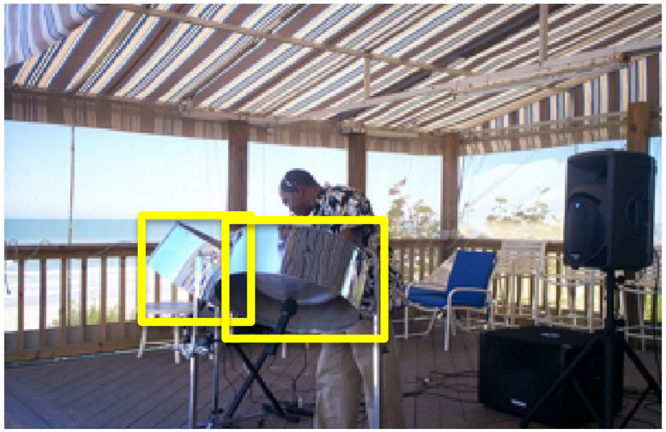


Output

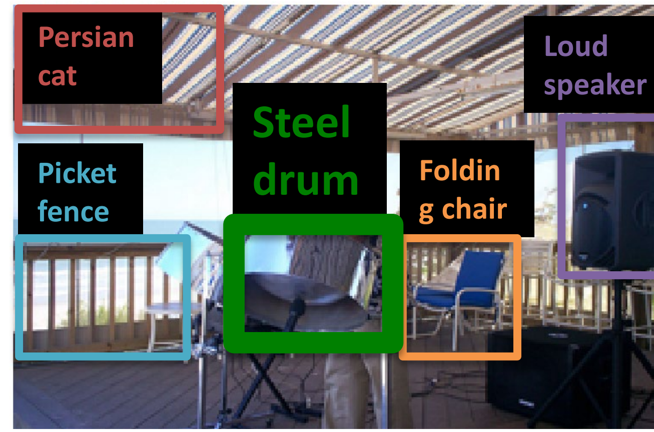


ILSVRC Task 2: Classification + Localization

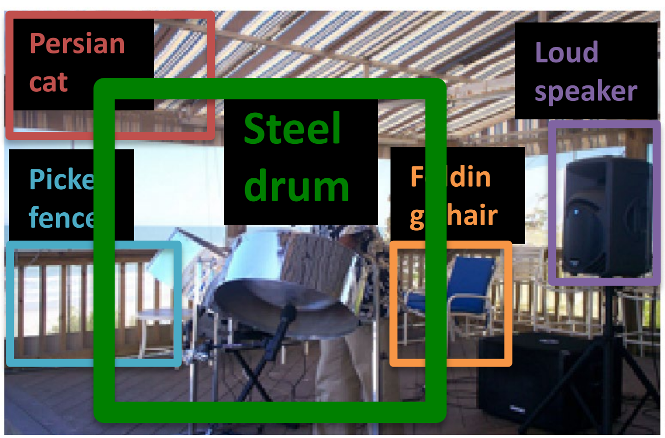
Steel drum



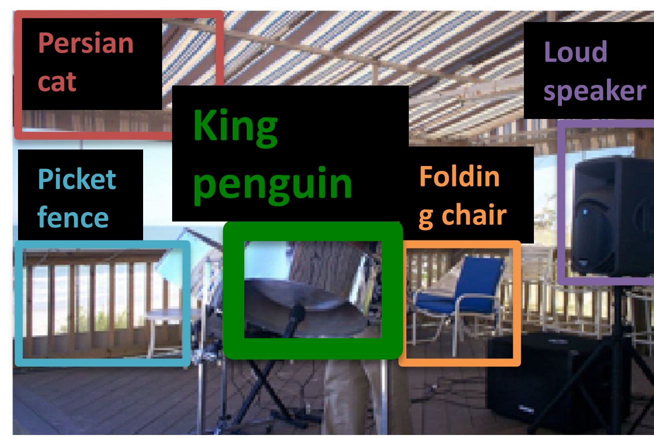
Output



Output (bad localization)

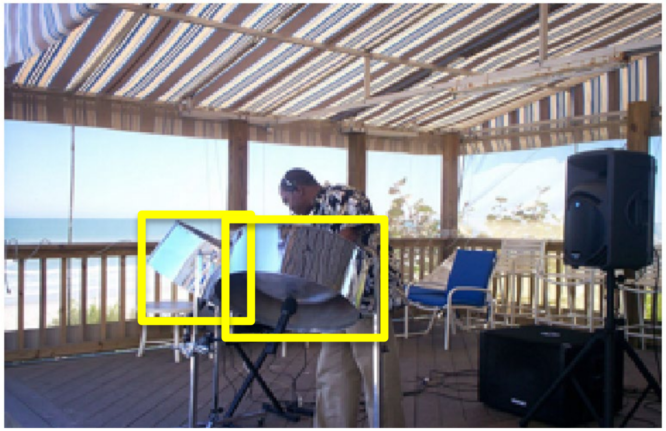


Output (bad classification)

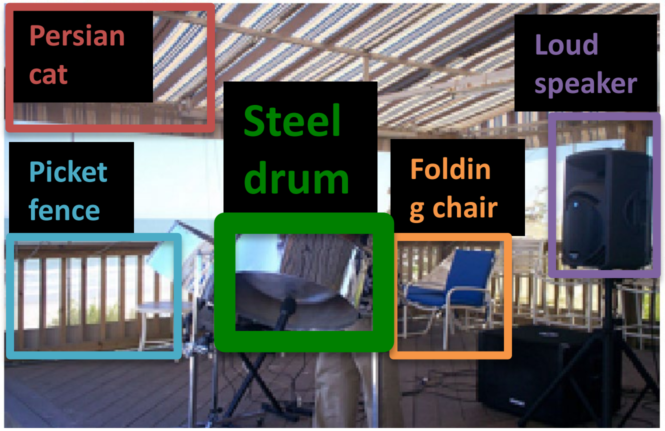


ILSVRC Task 2: Classification + Localization

Steel drum

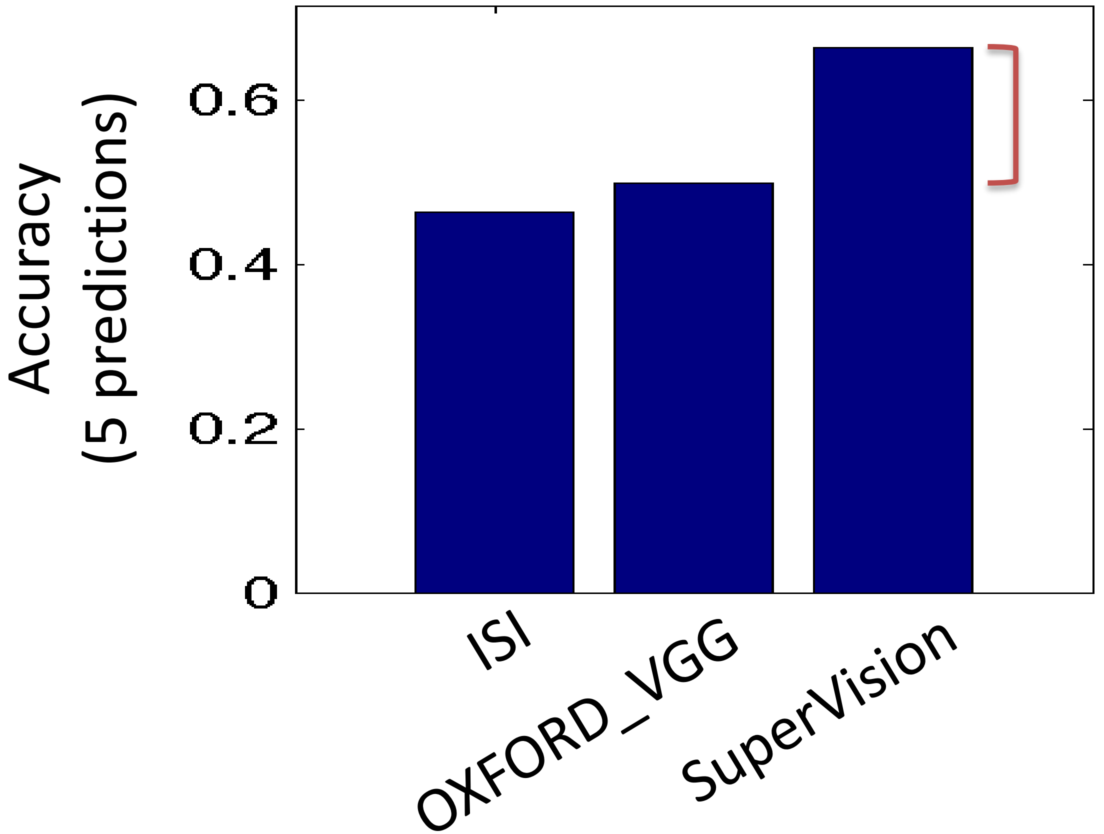


Output



$$\text{Accuracy} = \frac{1}{100{,}000} \sum_{\substack{100{,}000 \\ \text{images}}} 1[\text{correct on image i}]$$

ILSVRC Task 2: Classification + Localization



ISI=Uni. Tokyo Team

VGG=Uni. Oxford Team

SuperVision =
University of Toronto Team
Led by
Geoffrey Hinton,
Turing Award Winner

What happens under the hood?

Preliminaries:

- ILSVRC-500 (2012) dataset
- Leading algorithms

What happens under the hood on classification+localization?

- A closer look at small objects
- A closer look at textured objects

Olga Russakovsky, Jia Deng, Zhiheng Huang, Alex Berg, Li Fei-Fei
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ILSVRC (2012)

1000 object classes

ILSVRC-500 (2012)

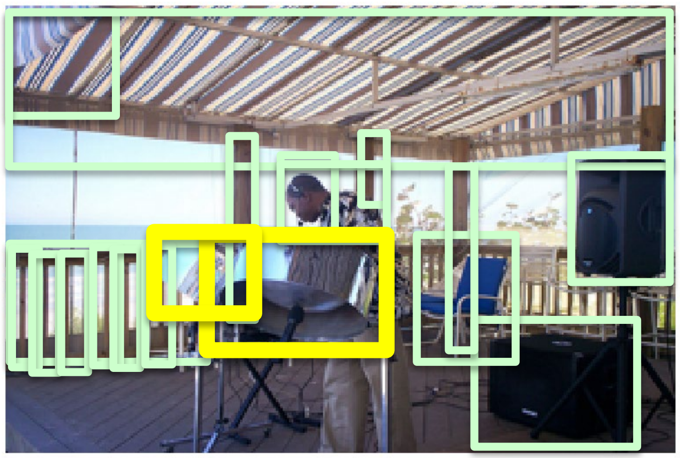
ILSVRC-500 (2012)

Object scale (fraction of image area occupied by target object)

ILSVRC-500 (2012)	500 object categories	25.3%
PASCAL VOC (2012)	20 object categories	25.2%

Level of clutter

Steel drum



- Generate candidate object regions using method of

  Selective Search for Object Detection vanDeSande et al. ICCV 2011
- Filter out regions inside object
- Count regions

ILSVRC-500 (2012)	500 object categories	128 ± 35
PASCAL VOC (2012)	20 object categories	130 ± 29

SuperVision = AlexNet

Alex Krizhevsky, Ilya Sutskever, Geoffrey Hinton (Krizhevsky NIPS12)

Image classification: Deep convolutional neural networks

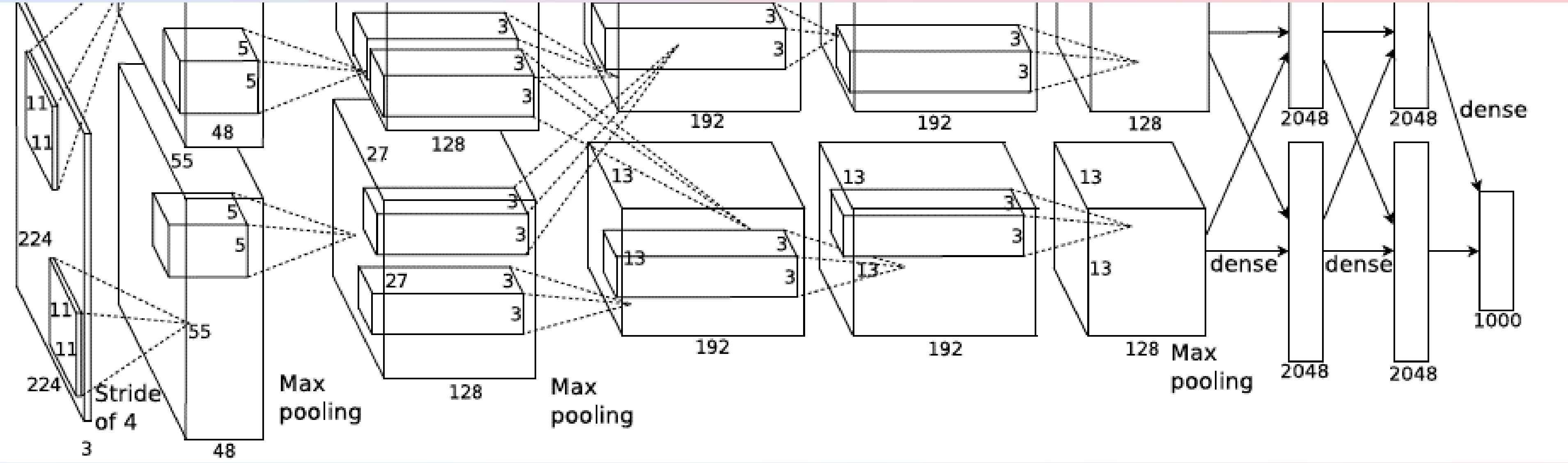
- 7 hidden "weight" layers, 650K neurons, 60M parameters, 630M connections
- Rectified Linear Units, max pooling, dropout trick
- Randomly extracted 224x224 patches for more data
- Trained with Stochastic Gradient Descent on two GPUs for a week, fully supervised (50x speed-up over CPU)

Localization: Regression on (x,y,w,h)

http://www.cs.toronto.edu/~fritz/absps/imagenet.pdf

AlexNet

- Similar to the model proposed by LeCun in 1998 but:
  - Larger model (7 hidden layers, 650,000 units, 60,000,000 params)
  - More data (10^6 vs. 10^3 images)



A. Krizhevsky, I. Sutskever, and G. Hinton,
ImageNet Classification with Deep Convolutional Neural Networks, NIPS 2012

Jia-Bin Huang and Derek Hoiem, UIUC

Details of the Oxford VGG

This is **not** the neural net VGG but uses traditional computer vision techniques!

Karen Simonyan, Yusuf Aytar, Andrea Vedaldi, Andrew Zisserman

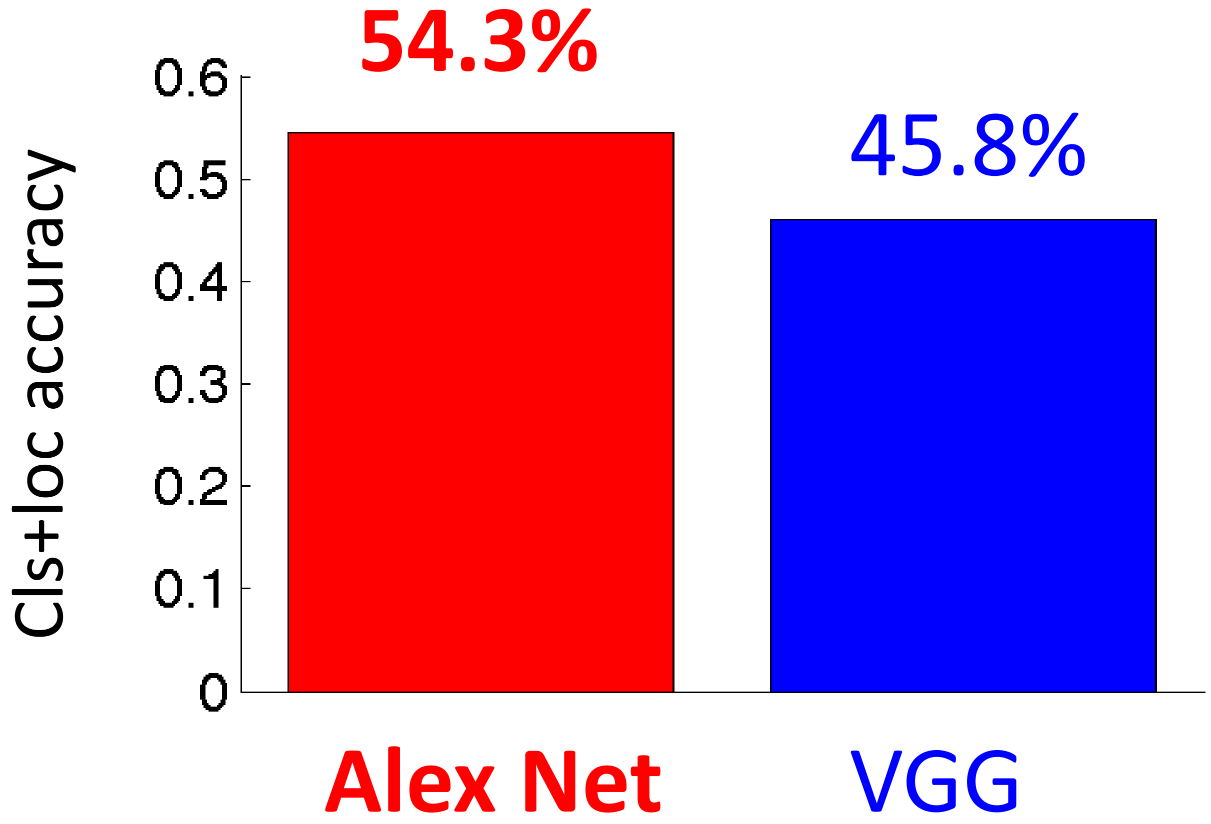
Image classification: Fisher vector + linear SVM (Sanchez CVPR11)

- Root-SIFT (Arandjelovic CVPR12), color statistics, augmentation with patch location (x,y) (Sanchez PRL12)
- Fisher vectors: 1024 Gaussians, 135K dimensions
- No SPM, product quantization to compress
- Semi-supervised learning to find additional bounding boxes
- 1000 one-vs-rest SVM trained with Pegasos SGD
  - 135M parameters!

Localization: Deformable part-based models (Felzenszwalb PAMI10), without parts (root-only)

http://image-net.org/challenges/LSVRC/2012/oxford_vgg.pdf

Results on ILSVRC-500



Preliminaries:

- ILSVRC-500 (2012) dataset – similar to PASCAL
- Leading algorithms: Alex Net and VGG

What happens under the hood on classification+localization?

- Alex Net always great at classification, but VGG does better than Alex Net localizing small objects
- A closer look at textured objects

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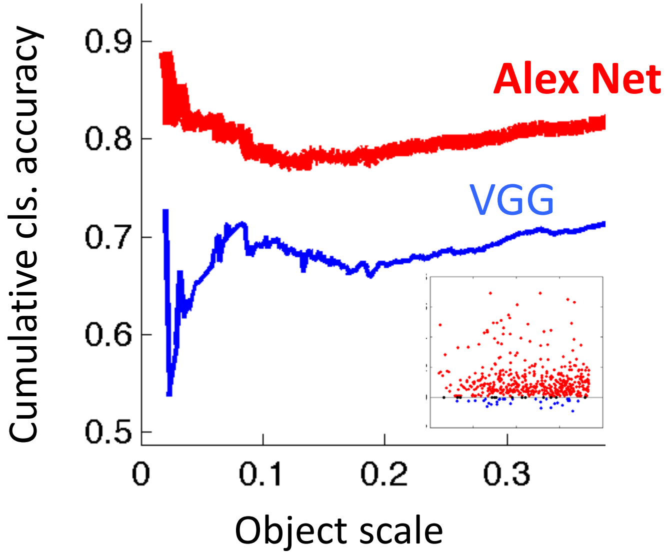
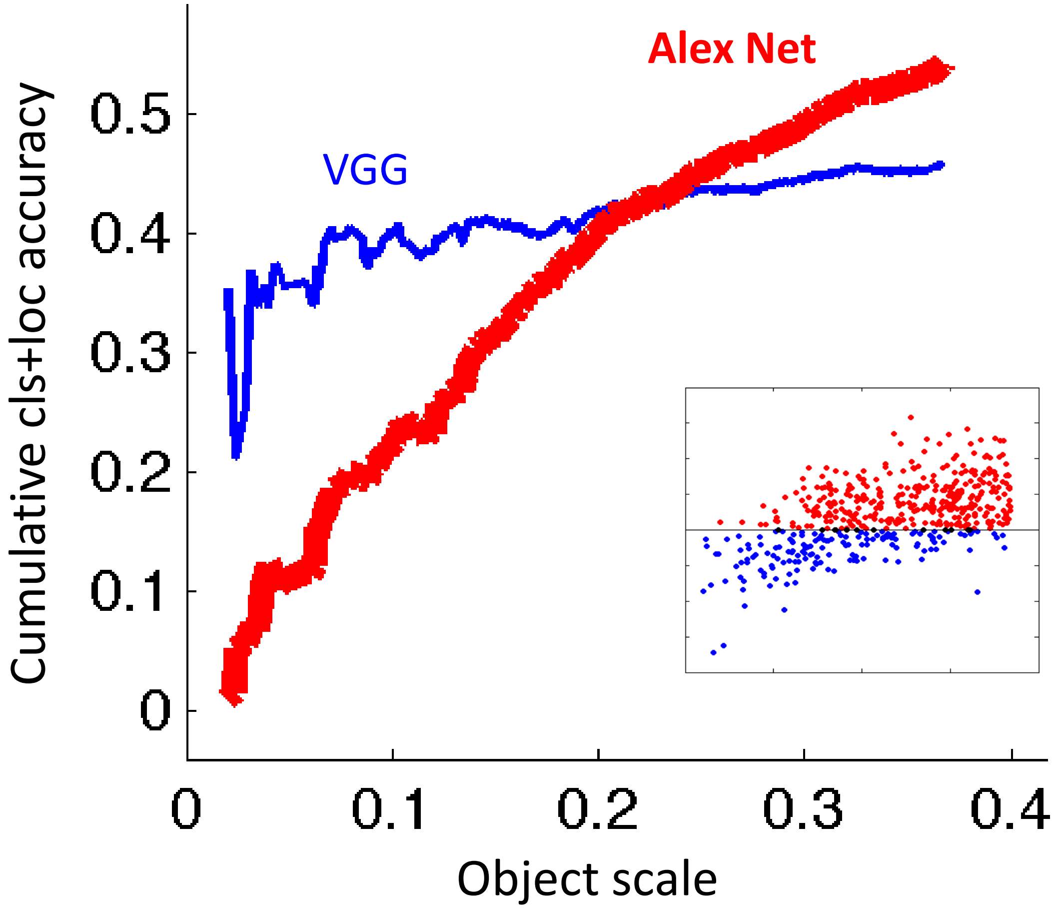
Cumulative accuracy across scales

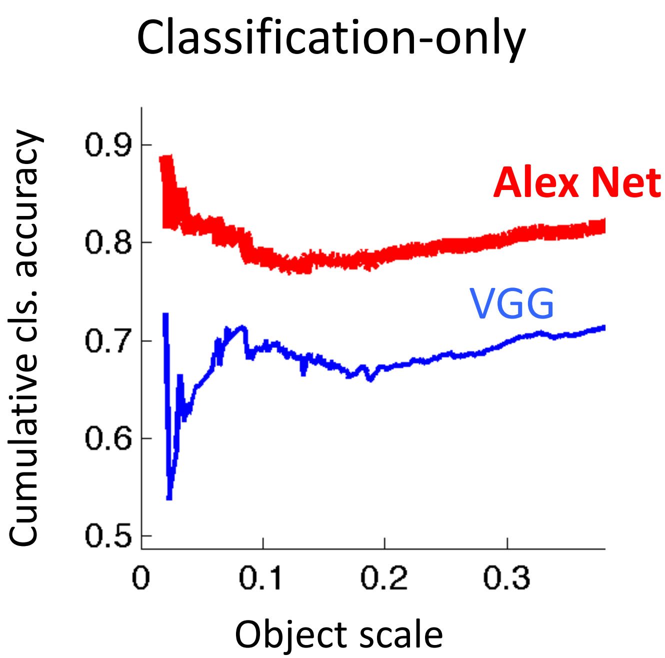
Classification-only

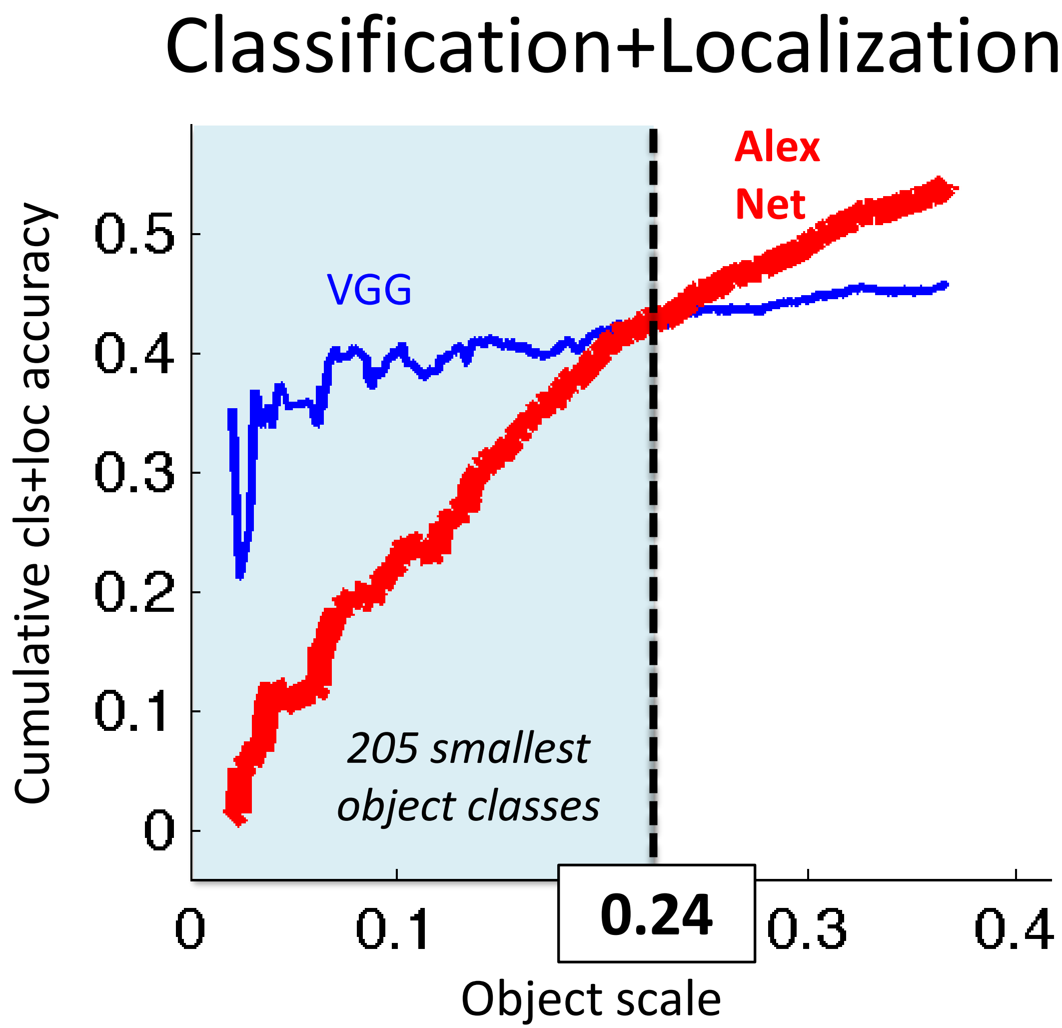


Classification+Localization



Cumulative accuracy across scales





Textured objects (ILSVRC-500)

Textured objects (ILSVRC-500)

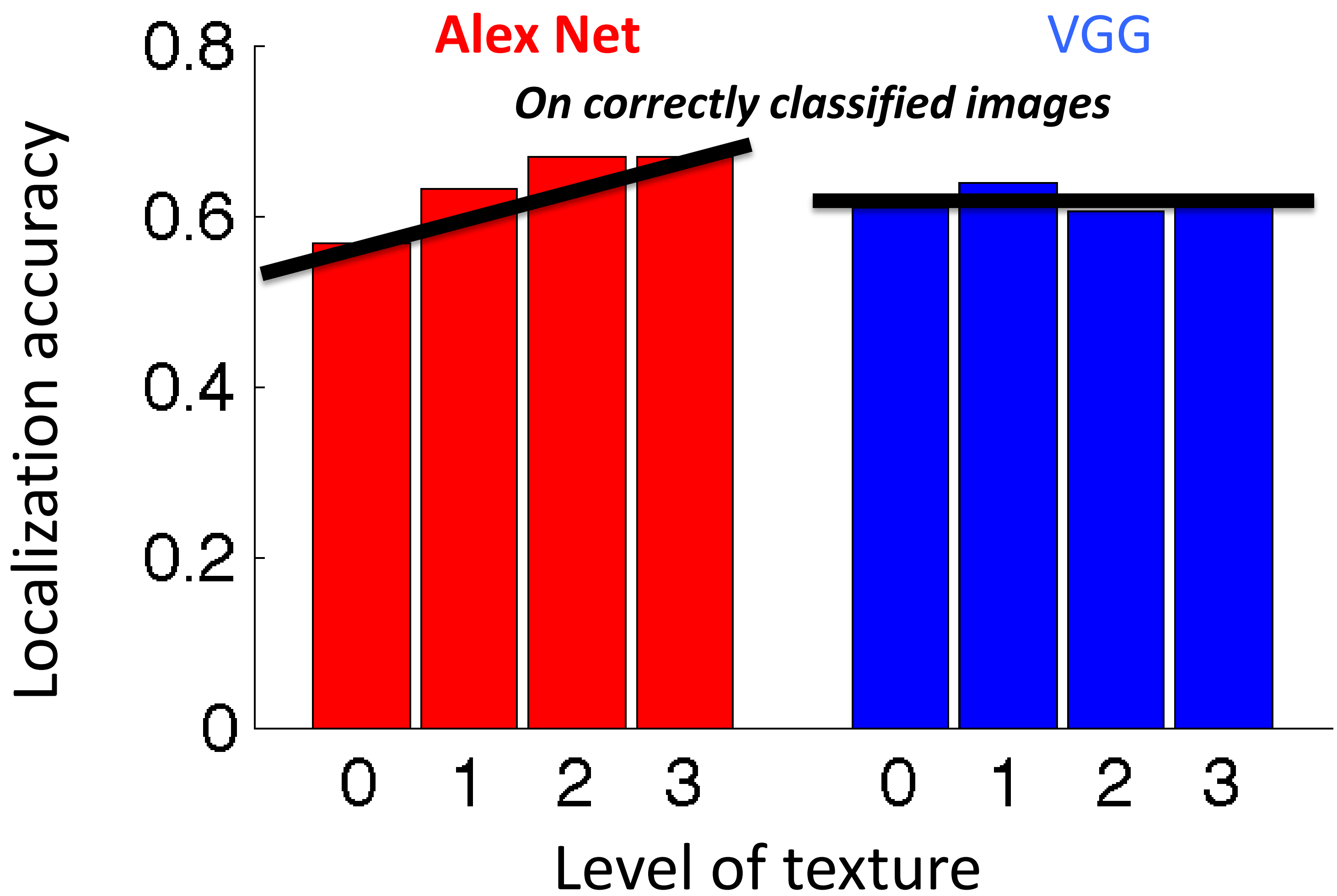
	No texture	Low texture	Medium texture	High texture
# classes	116	189	143	52
Object scale	20.8%	23.7%	23.5%	25.0%

Textured objects (416 classes)

	No texture	Low texture	Medium texture	High texture
# classes	116	~~189~~ 149	~~143~~ 115	~~52~~ 35
Object scale	20.8%	~~23.7%~~ 20.8%	~~23.5%~~ 20.8%	~~25.0%~~ 20.8%

Localizing textured objects

(416 classes, same average object scale at each level of texture)



Conclusions on analysis of classification+localization results

- Alex Net always great at classification, but VGG does better than Alex Net localizing small objects
- Textured objects: VGG broadly successful. Alex Net better at higher textures, worse at smaller.

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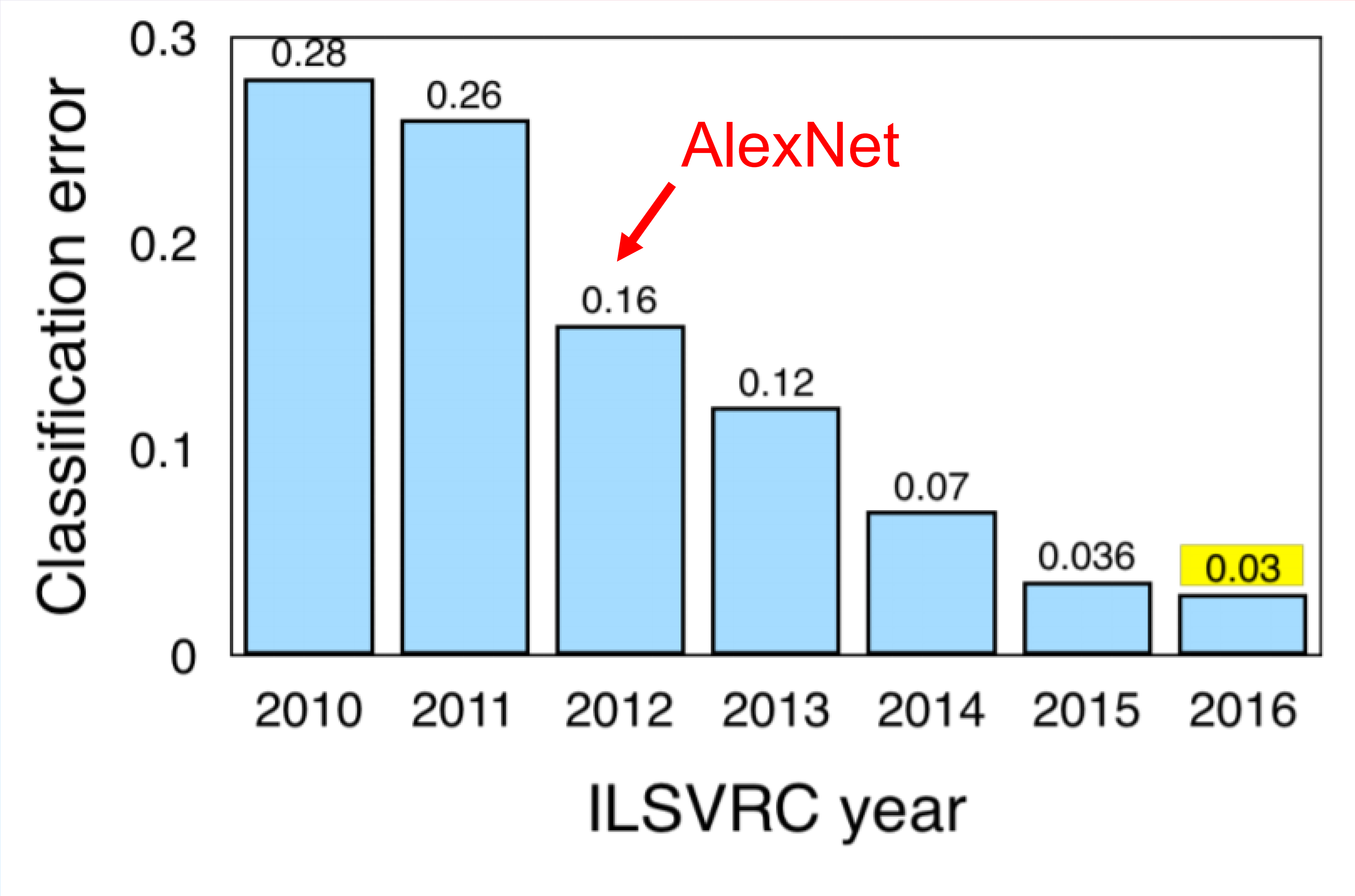
ImageNet Classification Challenge



http://image-net.org/challenges/talks/2016/ILSVRC2016_10_09_clsloc.pdf

Recap of NN-based Computer Vision

- Neural networks
  - View of neural networks as learning hierarchy of features
- Convolutional neural networks
  - Architecture of network accounts for image structure
  - “End-to-end” recognition from pixels
  - Together with large labeled datasets and lots of computation → major success on benchmark ImageNet, i.e., object classification and localization

Learning Objectives for this Lecture

Computer Science

- Understand differences and similarities between pre-2012 “traditional computer vision” and post-2012 neural-network-based computer vision & see examples
- Understand why convolution is powerful
- Understand the connection between convolution and correlation
- Understand how tools from estimation theory can be used to measure recognizability of objects in images
- Understand template matching with image pyramids
- Understand CNNs as a learning hierarchy of features
- Learn about early CNN used in computer vision: LeCun’s work on recognizing handwritten numbers
- Understand CNN concepts, e.g., convolution layers, fully connected (dense) layers, non-linearity (ReLU), pooling (downsampling)
- Learn about breakthrough dataset ImageNet