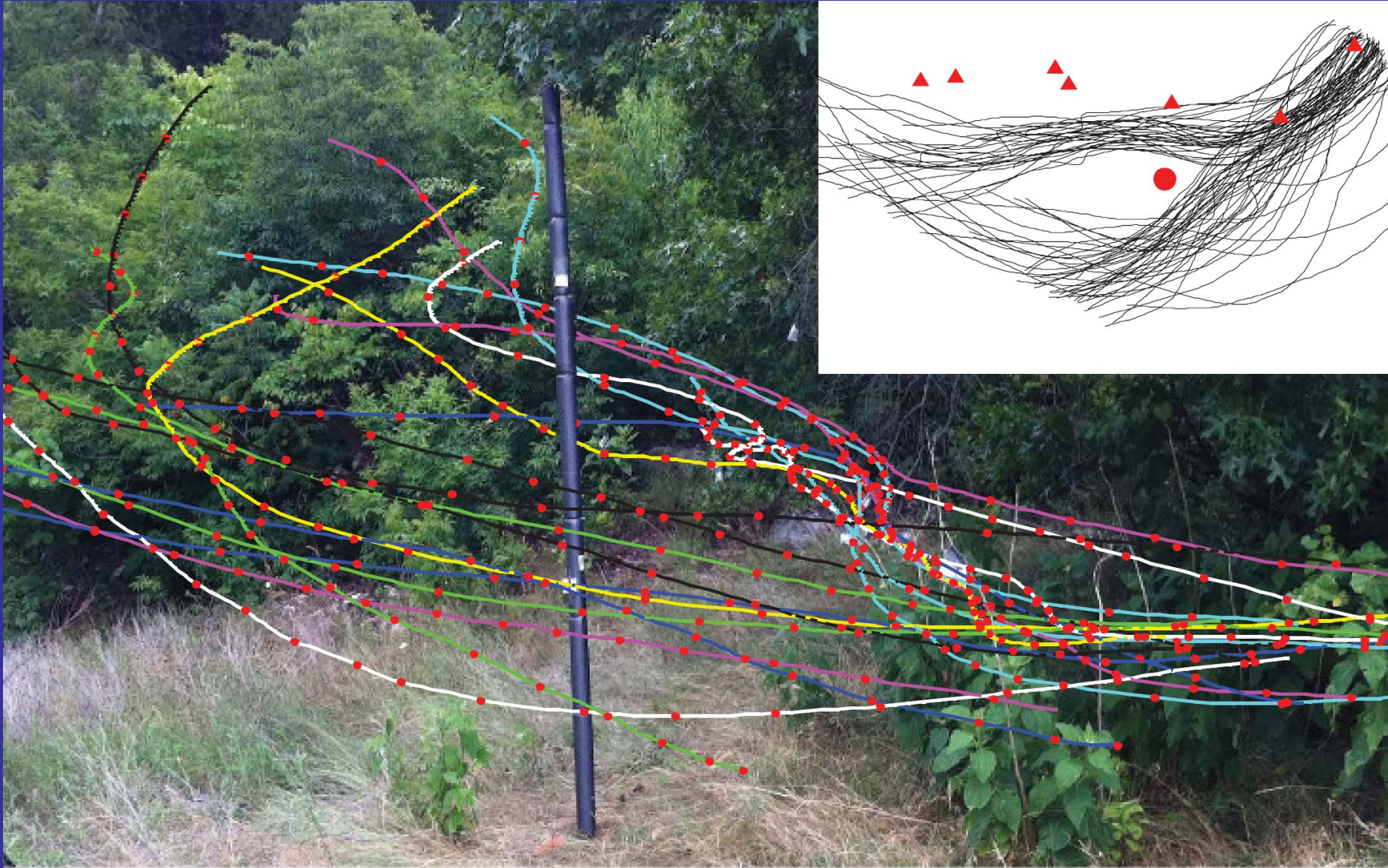


CS 585 Lecture on Multi-View Multi-Object Tracking

Margrit Betke
Computer Science Department
Boston University
April 23, 2024

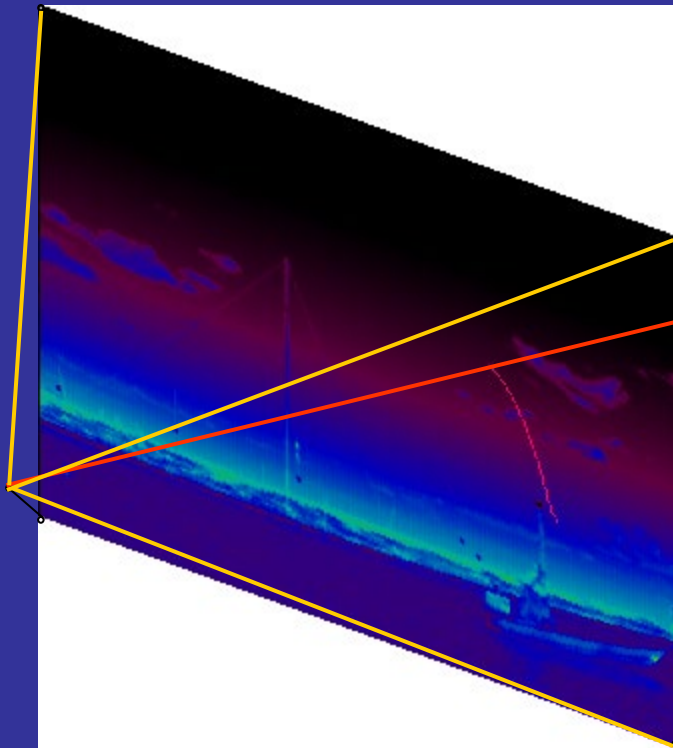
Opportunities for Studying Wildlife

How do bats fly with respect to their environment?

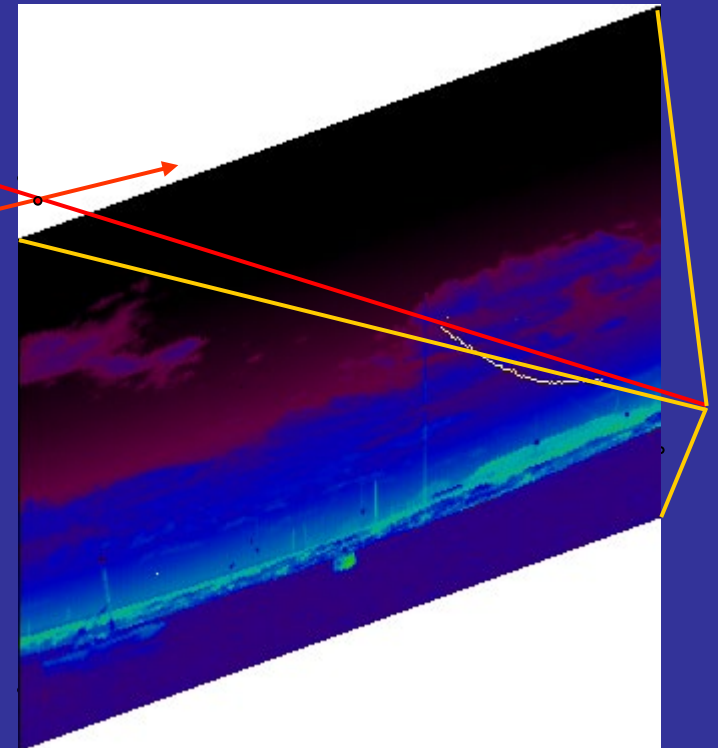


Stereoscopic 3D Reconstruction via Triangulation

3D position of single object



left camera



right camera

Tracking Multiple Objects via Two Data Association Methods

Previous Class:

- ✱ **Cluster-based Approach:** uses Hungarian method to match measurements and objects in cluster.
- ✱ **Greedy Approach:** “greedily” favors objects with long observation histories. Matching process is started by matching longest-observed object and its nearest measurement. Then second-longest observed object in cluster is matched with its nearest measurement, etc.

- ❖ We learned methods to track multiple objects
- ❖ We learned methods to estimate the 3D position of an object seen in multiple views

Multiview 2D measurements \rightarrow 3D estimate

Multiple moving objects in 2D \rightarrow 2D tracks

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Multiview 2D measurements -> 3D estimate

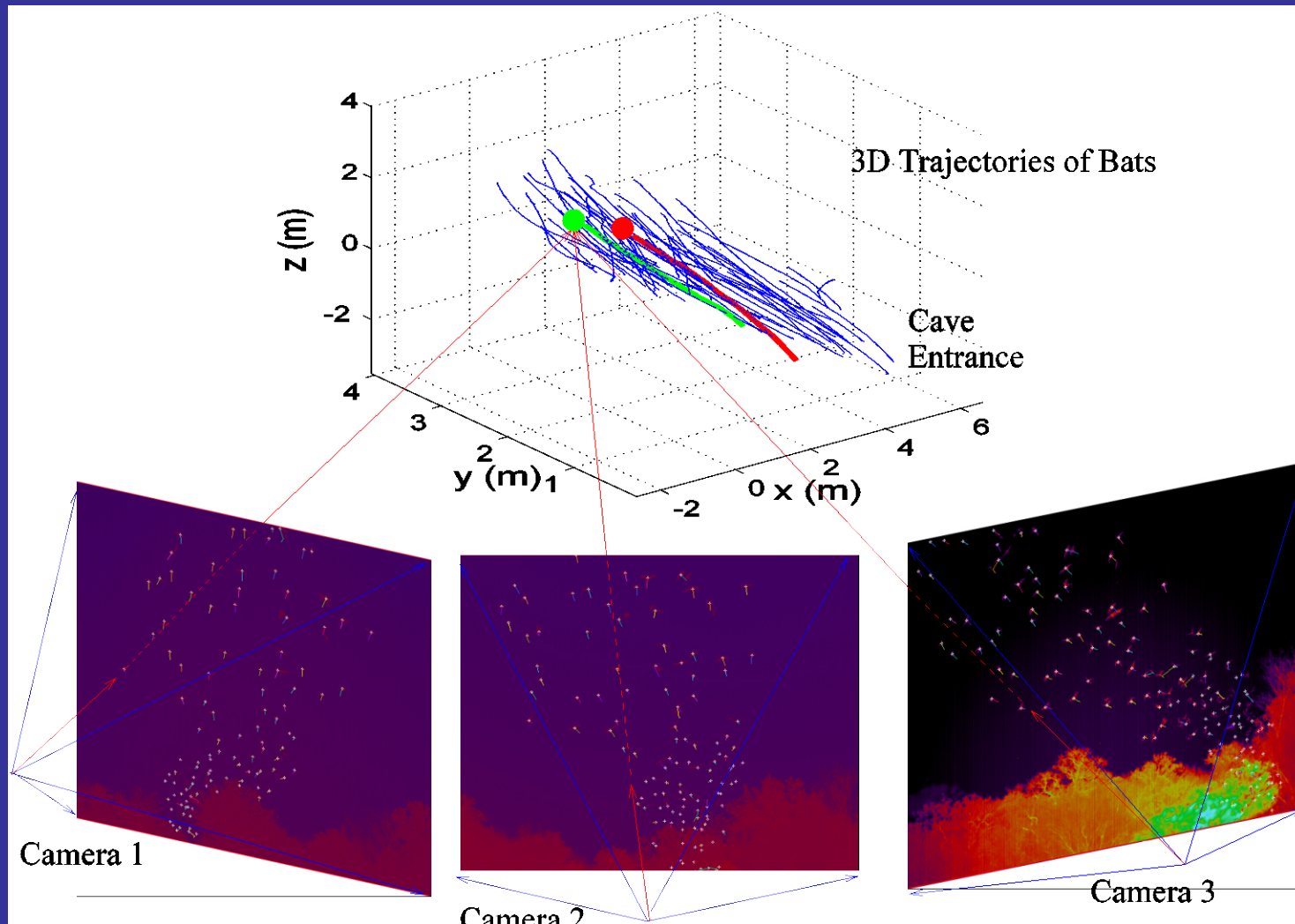
Multiple moving objects in 2D -> 2D tracks

- ❖ What about tracking multiple objects seen in multiple views?

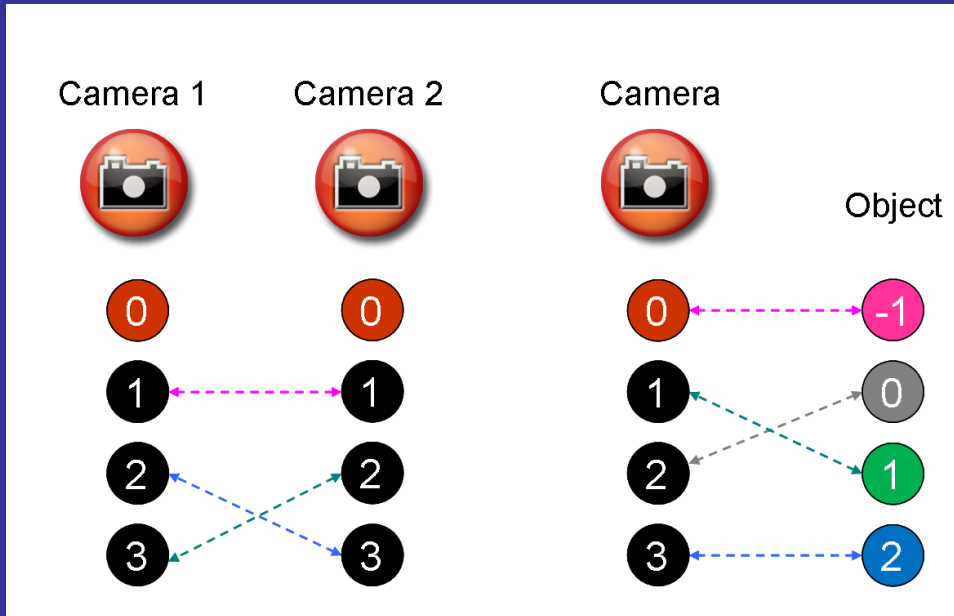
Sequence of multiview 2D measurements

-> Multiple 3D object tracks

3D Trajectories of Multiple Objects from Multiple Views, i.e., Multiple Cameras

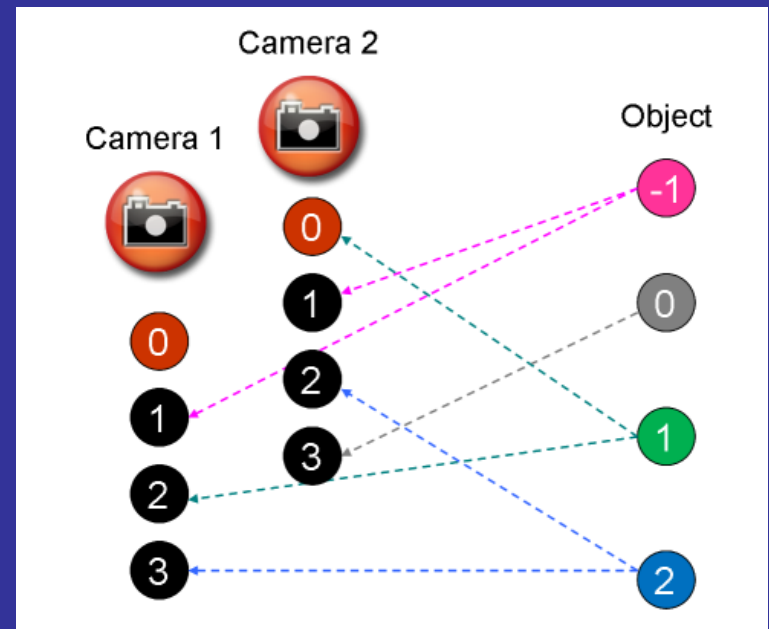


Data Association

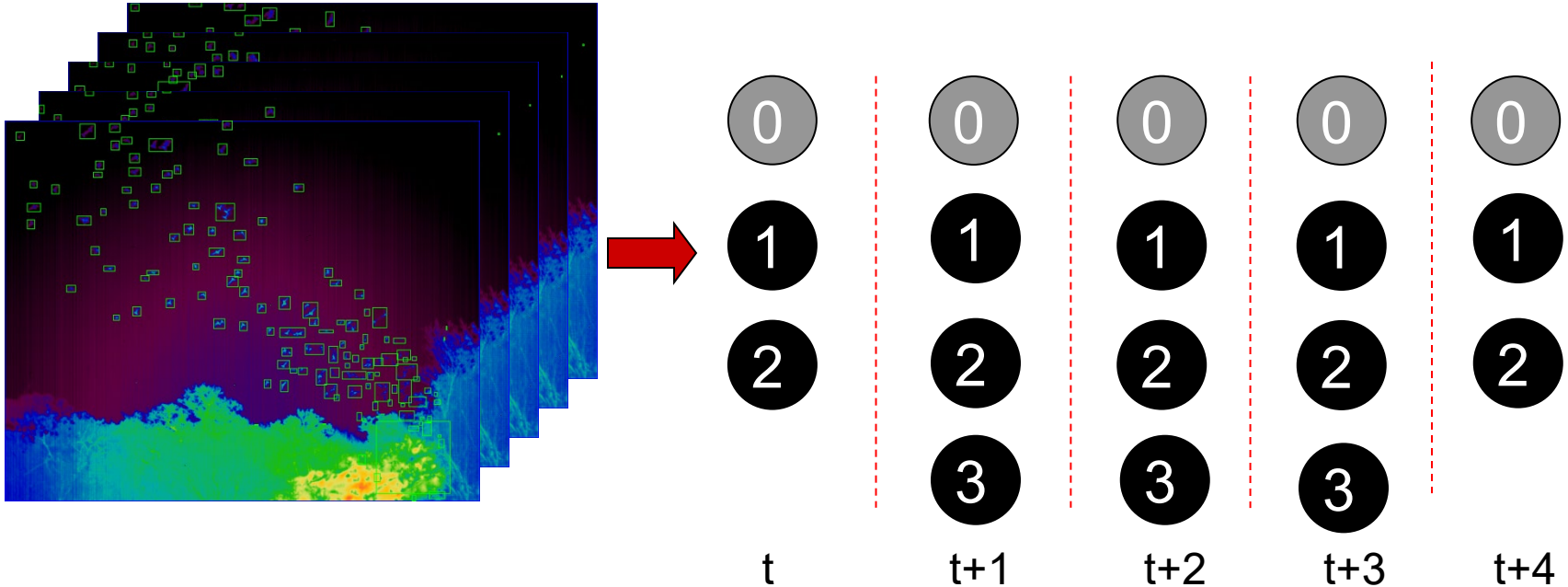


**Spatial
Association**

Temporal Association



Data Association Problem



2 $z_{t,i}$ Measurement / Observation

0 $z_{t,0}$ Missing detection

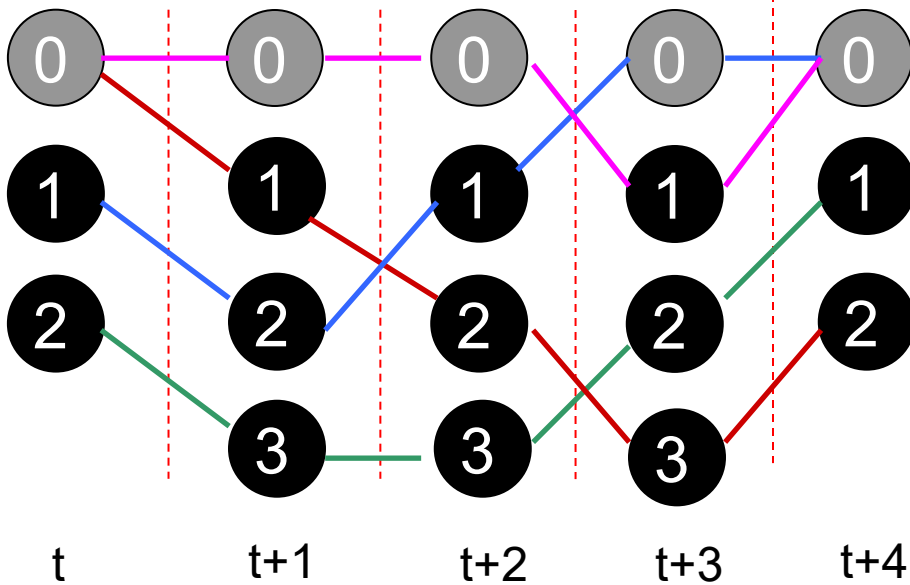


$$Z_{i_1 i_2 i_3 i_4 i_5} = \{z_{1,i_1}, z_{2,i_2}, z_{3,i_3}, z_{4,i_4}, z_{5,i_5}\}$$

Track τ_i with cost $c_{i_1 i_2 i_3 i_4 i_5}$

$$c_{i_1 i_2 i_3 i_4 i_5} = -\ln p(Z_{i_1 i_2 i_3 i_4 i_5} | \tau_i)$$

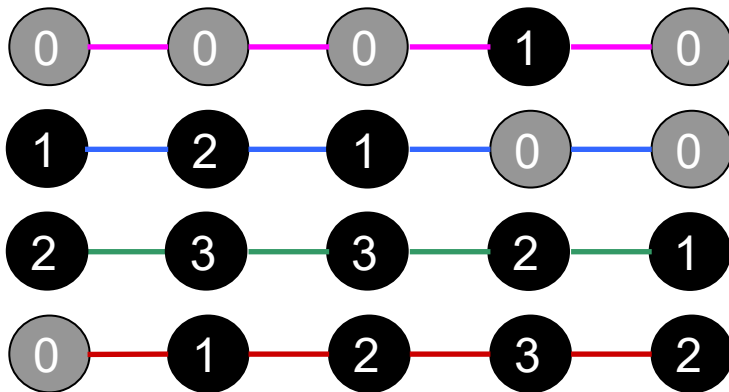
Data Association Problem



ω_i Partition hypothesis

$$C_{\omega_i} = -\ln p(Z | \omega_i)$$

$$= c_{00010} + c_{01232} + c_{12100} + c_{23321}$$



False alarm

Track termination

Track continuation

Track initiation

Multidimensional Assignment

$$c = \min \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \cdots \sum_{i_T=0}^{n_T} c_{i_1 i_2 \dots i_T} x_{i_1 i_2 \dots i_T} \longrightarrow \text{Linear Cost Function}$$

s.t.

$$\sum_{i_2=0}^{n_2} \sum_{i_3=0}^{n_3} \cdots \sum_{i_T=0}^{n_T} x_{i_1 i_2 \dots i_T} = 1; i_1 = 1, 2, \dots, n_1$$

$$\sum_{i_1=0}^{n_1} \sum_{i_3=0}^{n_3} \cdots \sum_{i_T=0}^{n_T} x_{i_1 i_2 \dots i_T} = 1; i_2 = 1, 2, \dots, n_2$$

⋮

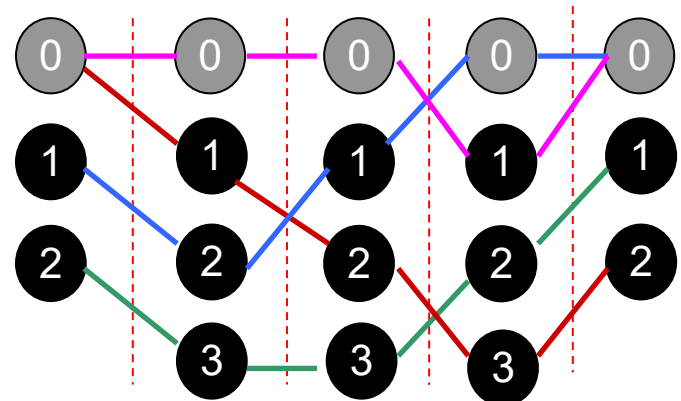
$$\sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \cdots \sum_{i_{T-1}=0}^{n_{T-1}} x_{i_1 i_2 \dots i_T} = 1; i_N = 1, 2, \dots, n_T.$$

$$x_{i_1 i_2 \dots i_T} = \begin{cases} 1, & \text{if } z_{1,i_1}, z_{2,i_2}, \dots, z_{T,i_T} \text{ forms a track} \\ 0, & \text{otherwise} \end{cases}$$

1. Exclusive and Exhaustive

2. No same origin

One-to-one correspondence



Two Technical Approaches for Multi-Object Multi-View Tracking

Tracking objects in 3D space with 2D measurements

- **Reconstruction-Tracking** Method

First reconstruct 3D positions from multiple views, then apply tracking approach (feature-to-feature fusion)

1. Find the correspondence across **views**
2. Find the correspondence across **time**

Solution:

1. **Triangulation**
2. **3D Kalman filtering & 3D Data Association**

Two Technical Approaches for Multi-Object Multi-View Tracking

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- **Tracking-Reconstruction** Method

First apply 2D tracking in each view independently, then reconstruct 3D trajectories through track-to-track associations (track-to-track fusion)

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Two Technical Approaches for Multi-Object Multi-View Tracking

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1. Find the correspondence across **time**
2. Find the correspondence across **views**

Solution by [Wu et al., ICCV 2009](#)

1. **Iterative GRASP**
2. **Information Fusion**

Multi-object Multi-view Tracking

– A Multidimensional Assignment Formulation (1)

Given N calibrated and synchronized cameras that share overlapping fields of view and n_s measurements in the field of view of camera s , the state $\mathbf{x}^{(t)}$ (3D coordinates) of an object of interest at time t can be assumed to evolve in time according to the equations:

$$\mathbf{x}^{(t+1)} = \mathbf{A}\mathbf{x}^{(t)} + \mathbf{v}^{(t)}$$

$$z_{s,i_s}^{(t)} = \mathbf{H}_s \mathbf{x}^{(t)} + \mathbf{w}^{(t_s)} ; s = 1, \dots, N; i_s = 1, \dots, n_s$$

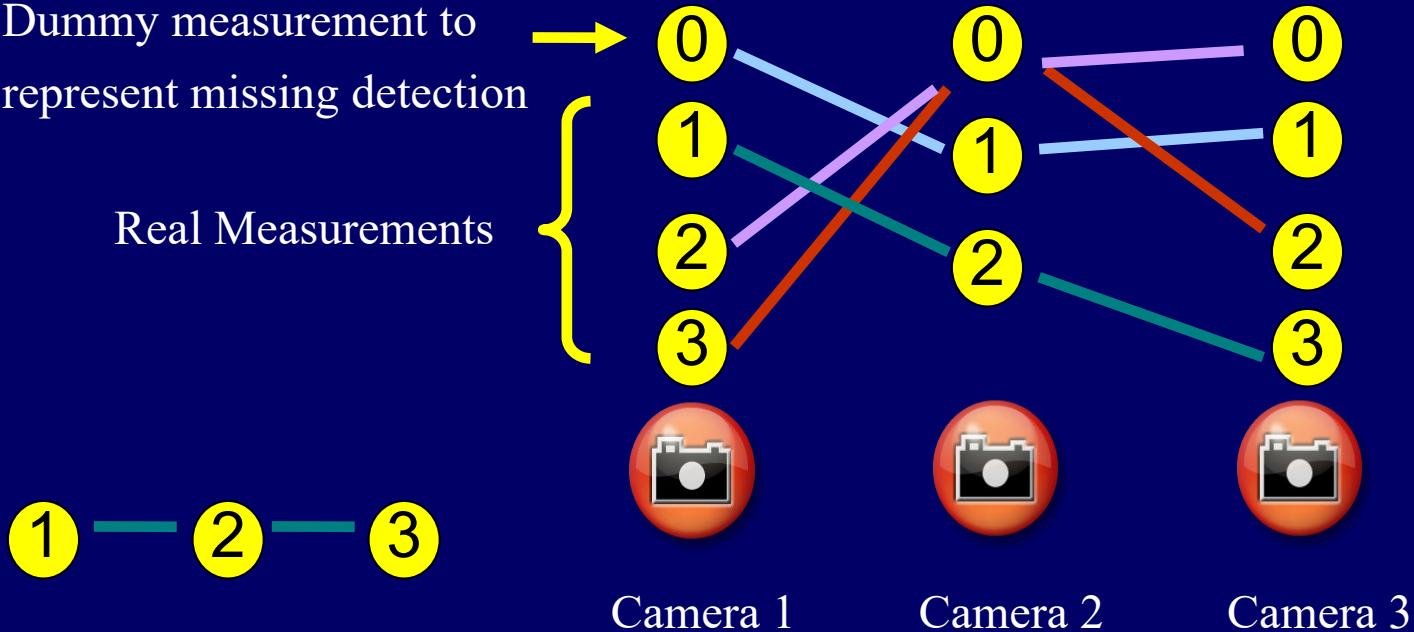
where $\mathbf{v}^{(t)}$ and $\mathbf{w}^{(t_s)}$ are independent zero-mean Gaussian noise processes with respective covariances $\mathbf{Q}(t)$ and $\mathbf{R}_s(t)$, \mathbf{A} is the state transition matrix, and \mathbf{H}_s the projection matrix for camera s .

Multi-object Multi-view Tracking

– A Multidimensional Assignment Formulation (2)

Dummy measurement to represent missing detection

Real Measurements



Cost for this 3-tuple:

$$c_{z_{1,1}z_{2,2}z_{3,3}} = -\ln \frac{p(z_{1,1}z_{2,2}z_{3,3} | a)}{p(z_{1,1}z_{2,2}z_{3,3} | \%)}$$

← Likelihood the measurements describe some object

← Likelihood the measurements are all false positives

%=dummy object

Multi-object Multi-view Tracking

– A Multidimensional Assignment Formulation (3)

The likelihood that measurements $z_{1,i_1}, z_{2,i_2}, \dots, z_{N,i_N}$ ($=Z_{i_1 i_2 \dots i_N}$) describe object state x_a is given as

$$p(Z_{i_1 i_2 \dots i_N} | x_a) = \prod_{s=1}^N \{ [1 - P_{D_s}]^{1-u(i_s)} \times [P_{D_s} p(z_{s,i_s} | x_a)]^{u(i_s)} \}$$

$$u(i_s) = \begin{cases} 0 & \text{if } i_s = 0, \\ 1 & \text{otherwise} \end{cases}$$

$$p(z_{s,i_s} | x_a) = \mathbf{N}(z_{s,i_s}; H_s x_a, R_s)$$

where P_{D_s} is the detection rate; $z_{s,0}$ means object is not detected in camera s

Multi-object Multi-view Tracking

– A Multidimensional Assignment Formulation (4)

The likelihood that measurements $z_{1,i_1}, z_{2,i_2}, \dots, z_{N,i_N}$ ($Z_{i_1 i_2 \dots i_N}$) are unrelated to any object is given as

$$p(Z_{i_1 i_2 \dots i_N} | \%) = \prod_{s=1}^N \left[\frac{1}{\Phi_s} \right]^{u(i_s)}$$

where Φ_s is the volume of FOV in camera s

We now can define the cost of associating N-tuple $Z_{i_1 i_2 \dots i_N}$ to object a at time t is as the negative log-likelihood ratio:

$$c_{i_1 i_2 \dots i_N} = -\ln \frac{p(Z_{i_1 i_2 \dots i_N} | a)}{p(Z_{i_1 i_2 \dots i_N} | \%)}$$

Multi-object Multi-view Tracking

– A Multidimensional Assignment Formulation (5)

We do not know the true state x_a in the likelihood Equation

$$p(z_{s,i_s} | x_a) = \mathbf{N}(z_{s,i_s}; H_s x_a, R_s)$$

so we replace it by $\hat{x}_a = \arg \min_{x_a} \sum_{s=1}^n d(z_{s,i_s}, H_s x_a)$

where d is Euclidean distance between $H_s x_a$, the object position projected onto the image s , and the corresponding measurement z_{s,i_s}

This is a 3D reconstruction problem.

Multi-object Multi-view Tracking

– A Multidimensional Assignment Formulation (6)

Assuming that such associations are independent, our goal is to find the most likely set of N-tuples that minimizes the linear cost function

$$\begin{aligned} c &= \min \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \cdots \sum_{i_N=0}^{n_N} c_{i_1 i_2 \dots i_N} x_{i_1 i_2 \dots i_N} \\ \text{s.t.} \\ \sum_{i_2=0}^{n_2} \sum_{i_3=0}^{n_3} \cdots \sum_{i_N=0}^{n_N} x_{i_1 i_2 \dots i_N} &= 1; i_1 = 1, 2, \dots, n_1 \\ \sum_{i_1=0}^{n_1} \sum_{i_3=0}^{n_3} \cdots \sum_{i_N=0}^{n_N} x_{i_1 i_2 \dots i_N} &= 1; i_2 = 1, 2, \dots, n_2 \\ &\vdots \\ \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \cdots \sum_{i_{N-1}=0}^{n_{N-1}} x_{i_1 i_2 \dots i_N} &= 1; i_N = 1, 2, \dots, n_N. \end{aligned}$$

NP Hard

where we use binary variable $x_{i_1 i_2 \dots i_N}$ to indicate if $Z_{i_1 i_2 \dots i_N}$ is associated with a candidate object or not.

Multi-object Multi-view Tracking

Greedy Randomized Adaptive Search Procedure (GRASP)

Multistart random process [Feo 1989, Renata 2003]

Within each iteration,

- Construction Phase

Randomly construct a feasible greedy solution

- Local Search Phase

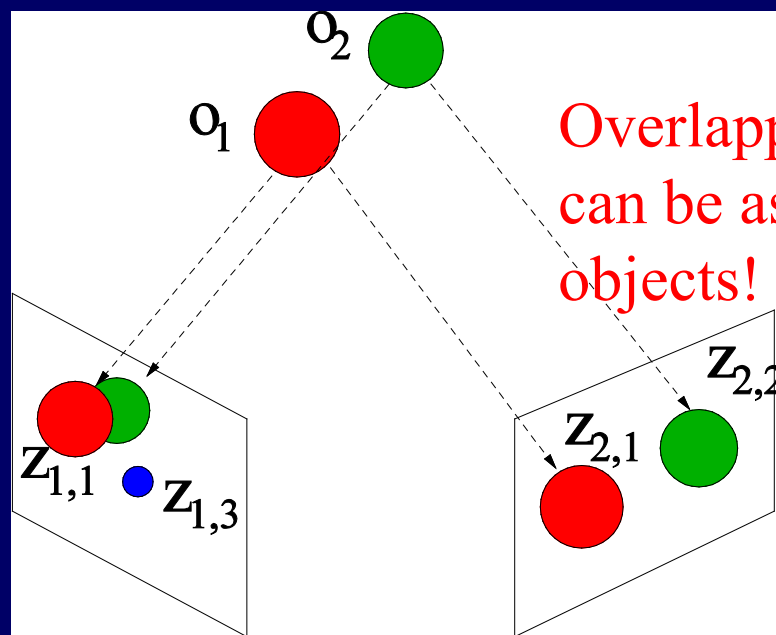
Improve the feasible solution by local search in the neighborhood

1	1	1
2	2	2

Multi-object Multi-view Tracking

— Iterative Greedy Randomized Adaptive Search Procedure (1)

The constraints in the multidimensional assignment formulation imply the **one-to-one** correspondence between measurements and objects, which is not desirable in the multi-view tracking scenario because of occlusion.



Multi-object Multi-view Tracking

— Iterative Greedy Randomized Adaptive Search Procedure (2)

Define all possible N-tuples as $F = Z_1 \times Z_2 \times \dots \times Z_N$, Z_i which is the set of all measurements in camera s plus a “dummy” measurement that represents a missing detection event.

We divide the set of the suboptimal assignments Z found by GRASP into two subsets:

- Confirmed associations:

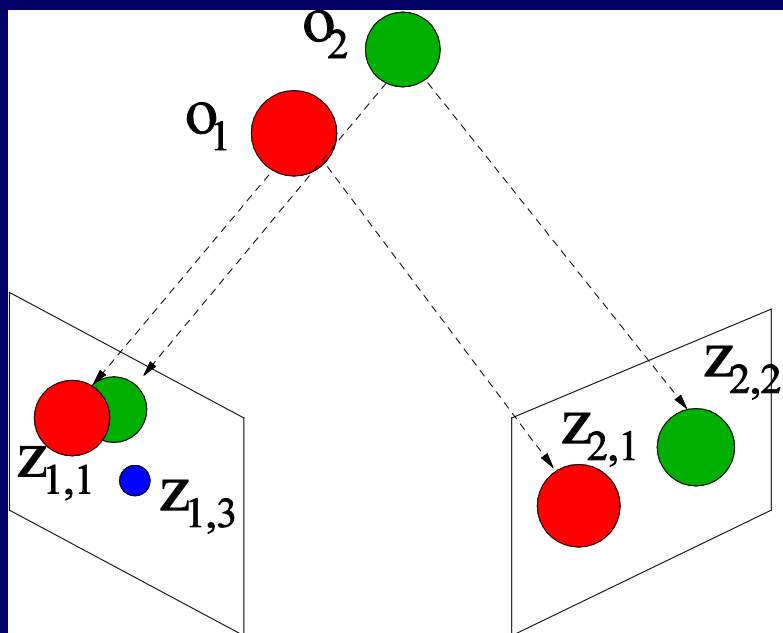
$$M_c = \{Z_{i_1 i_2 \dots i_N} \mid x_{i_1 i_2 \dots i_N} = 1; i_1 \neq 0; \dots; i_N \neq 0\}$$

- Suspicious associations (dummy measurements & objects):

$$M_s = Z \setminus M_c$$

Multi-object Multi-view Tracking

— Iterative Greedy Randomized Adaptive Search Procedure (3)



Iteration 1:

$$F = \{(z_{1,1} z_{2,0}), (z_{1,1} z_{2,1}), (z_{1,1} z_{2,2}), (z_{1,3} z_{2,0}), (z_{1,3} z_{2,1}), (z_{1,3} z_{2,2}), (z_{1,0} z_{2,1}), (z_{1,0} z_{2,2})\}$$

$$Z = \{(z_{1,1} z_{2,1}), (z_{1,3} z_{2,0}), (z_{1,0} z_{2,2})\}$$

$$M_c = \{(z_{1,1} z_{2,1})\}, M_s = \{(z_{1,3} z_{2,0}), (z_{1,0} z_{2,2})\}$$

Iteration 2:

$$F = \{(z_{1,1} z_{2,0}), (z_{1,1} z_{2,2}), (z_{1,3} z_{2,0}), (z_{1,3} z_{2,1}), (z_{1,3} z_{2,2}), (z_{1,0} z_{2,1}), (z_{1,0} z_{2,2})\}$$

$$Z = \{(z_{1,1} z_{2,2}), (z_{1,3} z_{2,0}), (z_{1,0} z_{2,1})\}$$

$$M_c = \{(z_{1,1} z_{2,2})\}, M_s = \{(z_{1,3} z_{2,0}), (z_{1,0} z_{2,1})\}$$

Multi-object Multi-view Tracking

— Iterative Greedy Randomized Adaptive Search Procedure (4)

IGRASP

Building Phase

Initialization by computing the costs for all possible associations in set F

Solving Phase

For $i = 1, \dots, \text{maxiter}$,

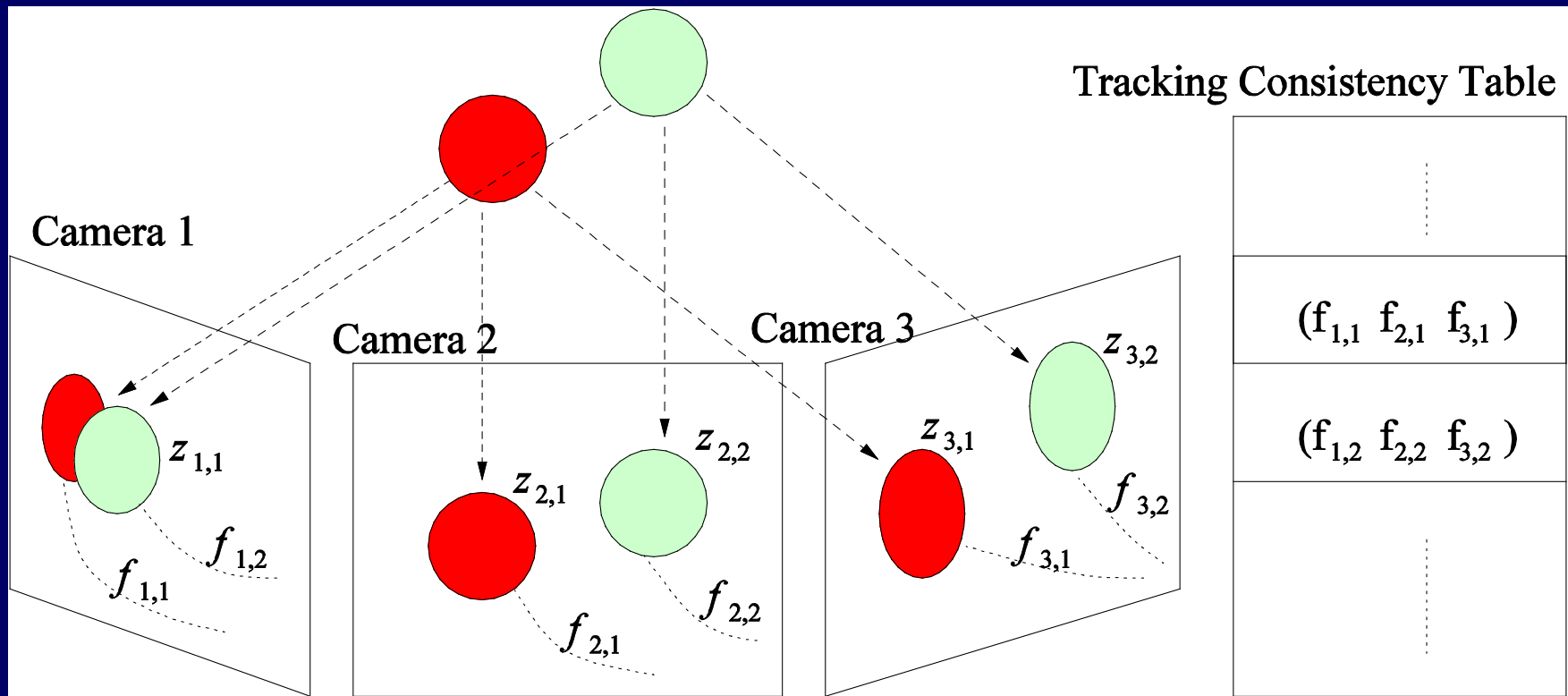
1. Formulate multidimensional assignment problem on set F
2. Solve the problem by GRASP
3. Partition the computed solution into confirmed set M_c and suspicious set M_s
4. If set M_c is empty, terminate;
Else $F = F \setminus M_c$

End

Output the final suboptimal solution.

Multi-object Multi-view Tracking

— Fusion of Information from Multiple Views (1)



$f_{i,j} = j^{\text{th}}$ 2D Kalman tracker in view i ; $z_{i,j} = j^{\text{th}}$ measurement in view i

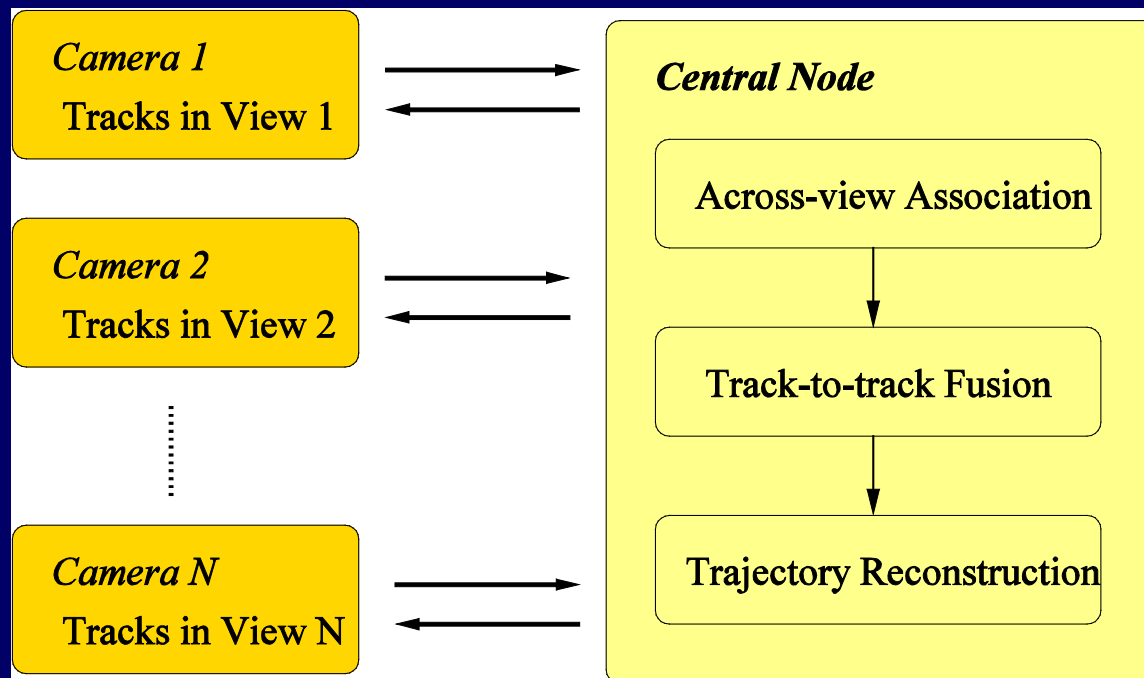
Multi-object Multi-view Tracking

— Fusion of Information from Multiple Views (2)

Tracking is performed at each camera.

Tracks and measurements are sent to a central node for processing.

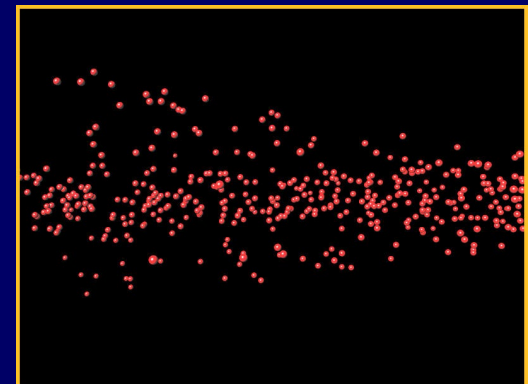
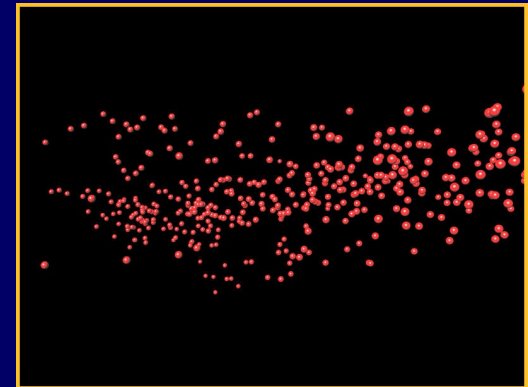
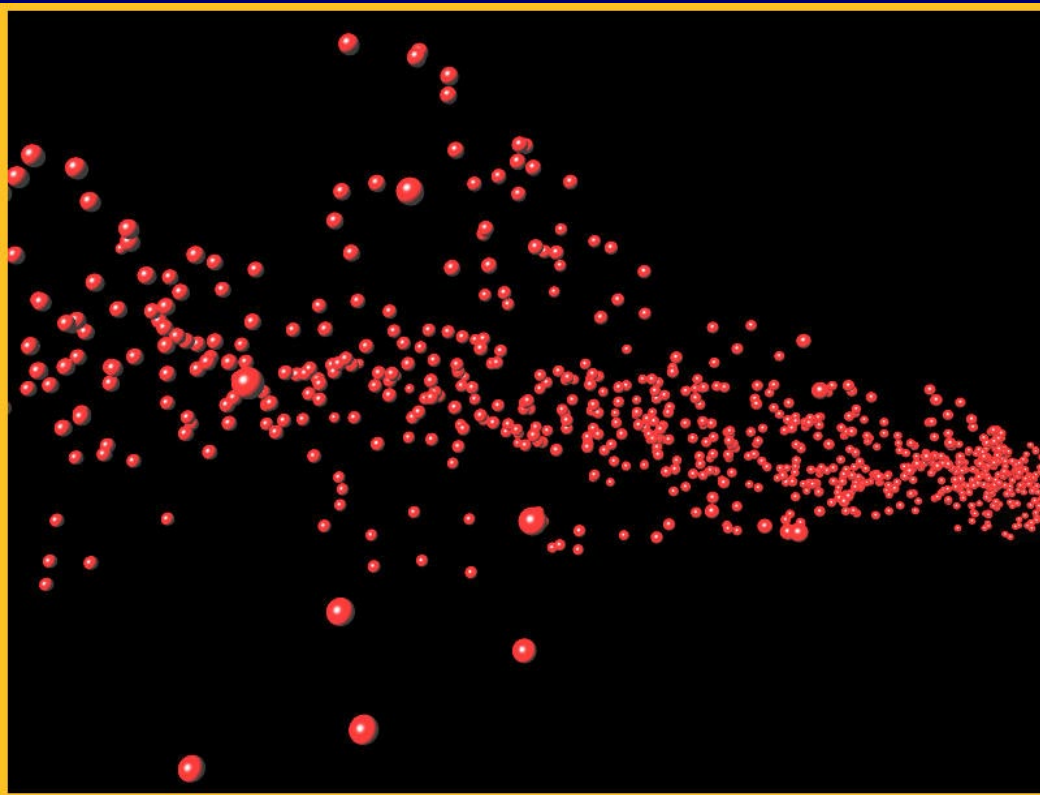
Each camera tracker adjusts its across-time associations based on the fusion result it receives from the central node



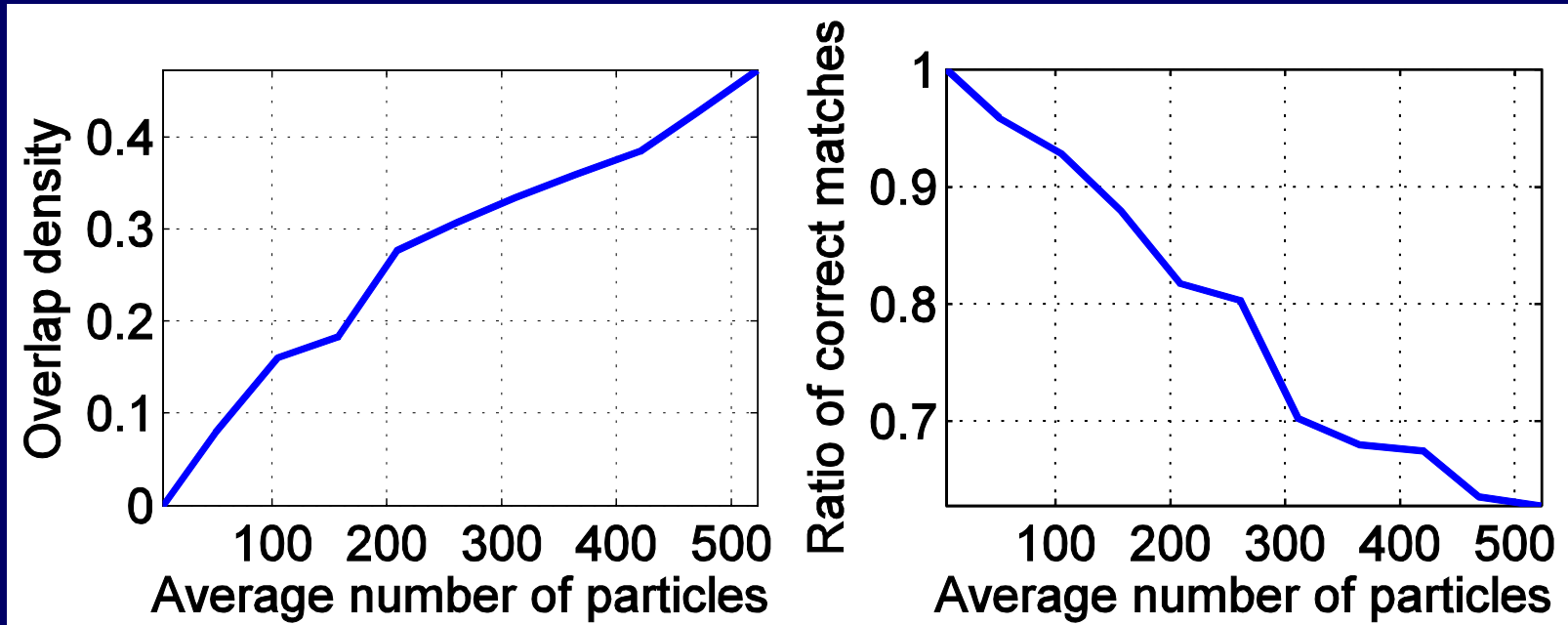
Experiment – Validation of across-view association (1)

Synthetic Data:

- randomly generate spherical particles of radius 28cm to move in $20 \times 5 \times 5 \text{m}^3$ space at a fixed speed of 2m/s
- 10 datasets with increasing emergence rates between 1 and 100 particles/sec



Experiment – Validation of across-view association (2)

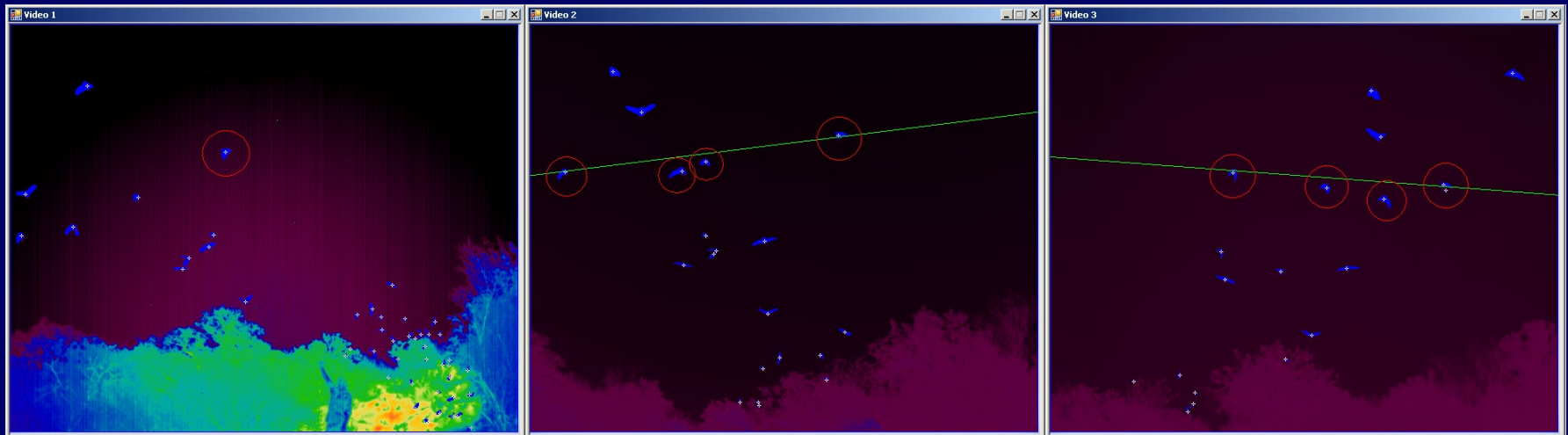


Overlap density: the ratio of number of overlapping particle projections over the total number of particles

Ratio of correct matches: number of correct tuples found by IGRASP over the ground truth

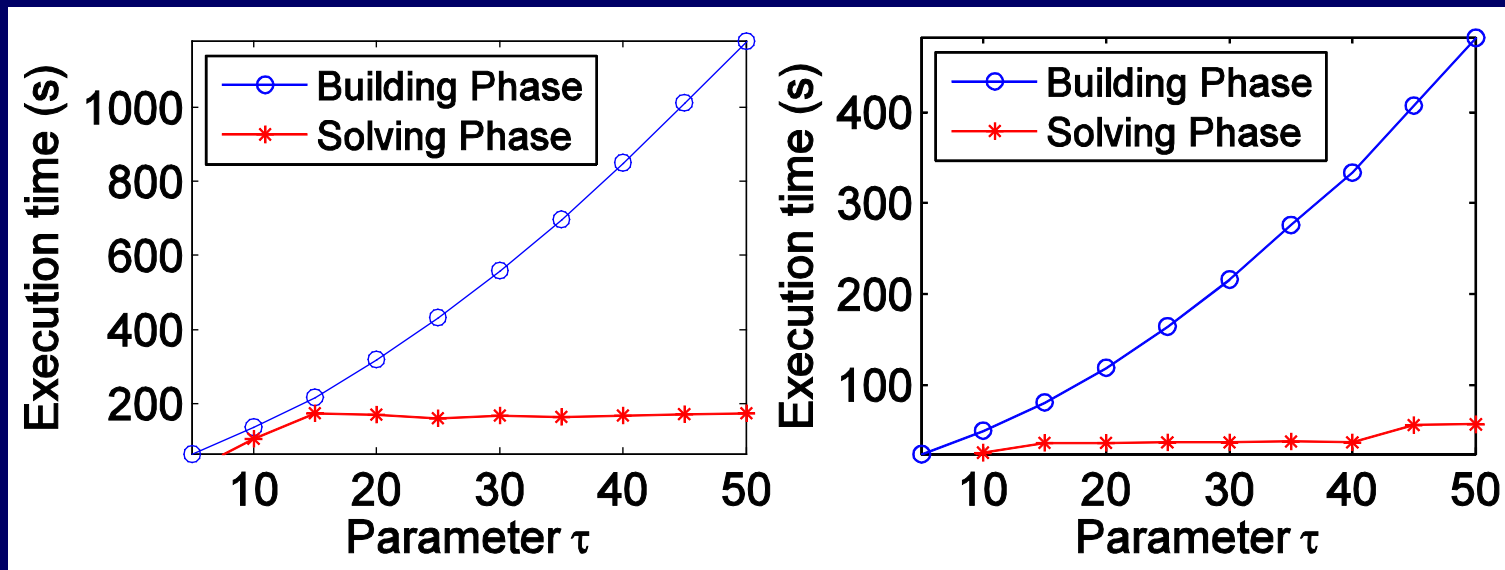
Experiment – Validation of across-view association (3)

- **Make the problem as sparse as possible!!!** Evaluate the candidate tuples that lie within the neighborhood of corresponding **epipolar lines**

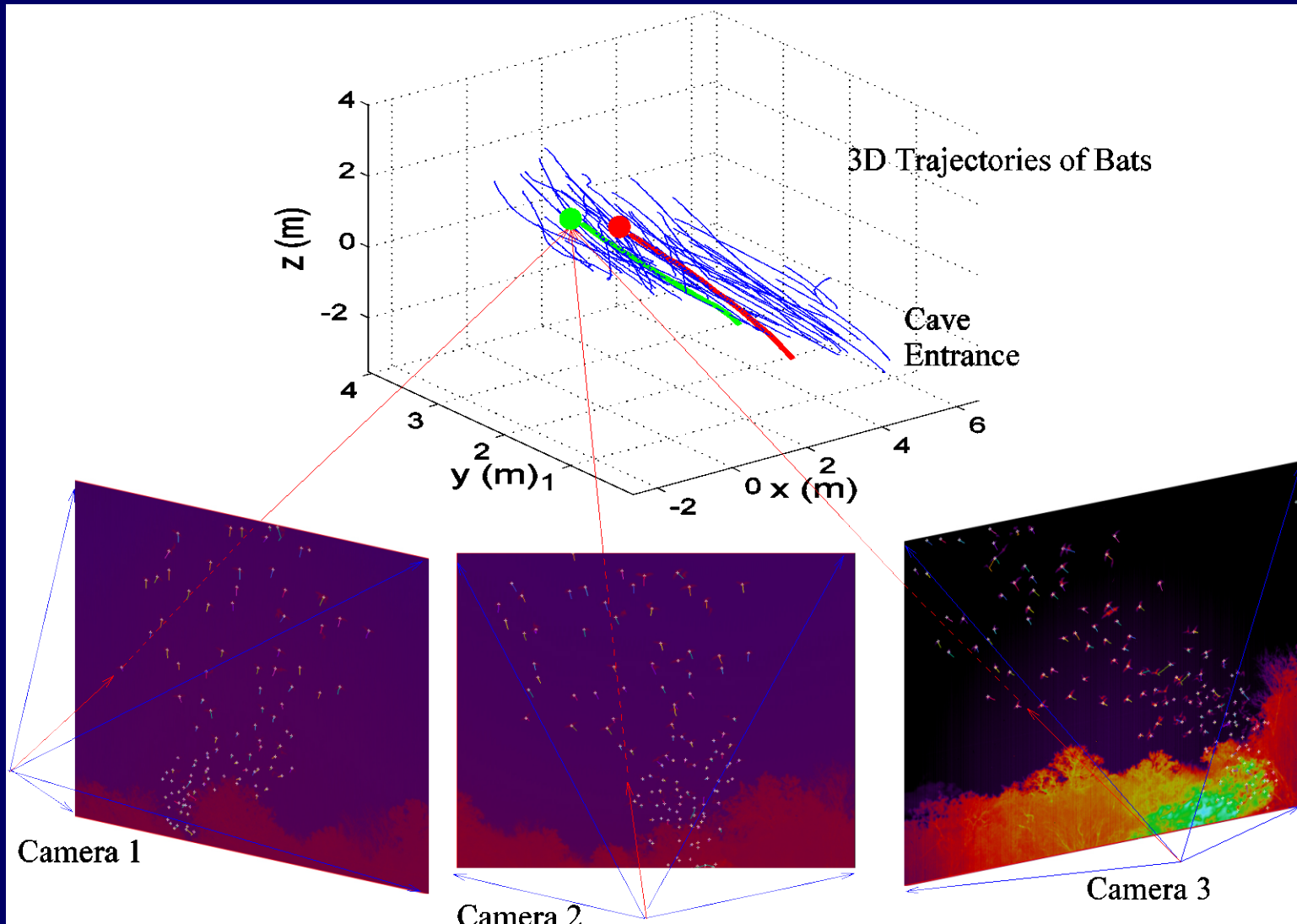


Experiment – Validation of across-view association (4)

- **Make the problem as sparse as possible!!!** Evaluate the candidate tuples that lie within the neighborhood of corresponding **epipolar lines**



Experiment – Infrared Thermal Video Analysis (1)



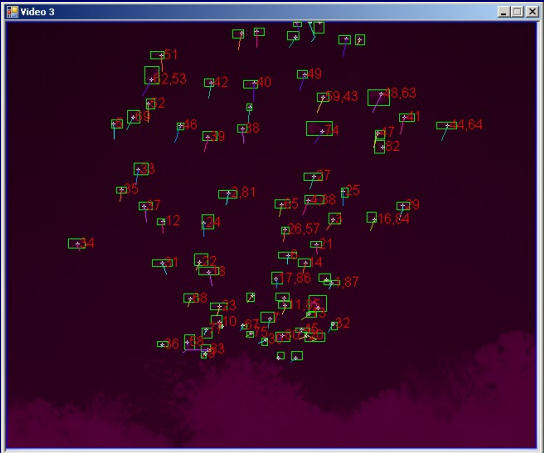
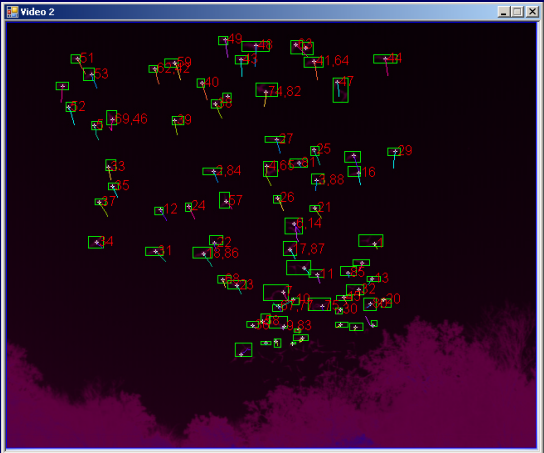
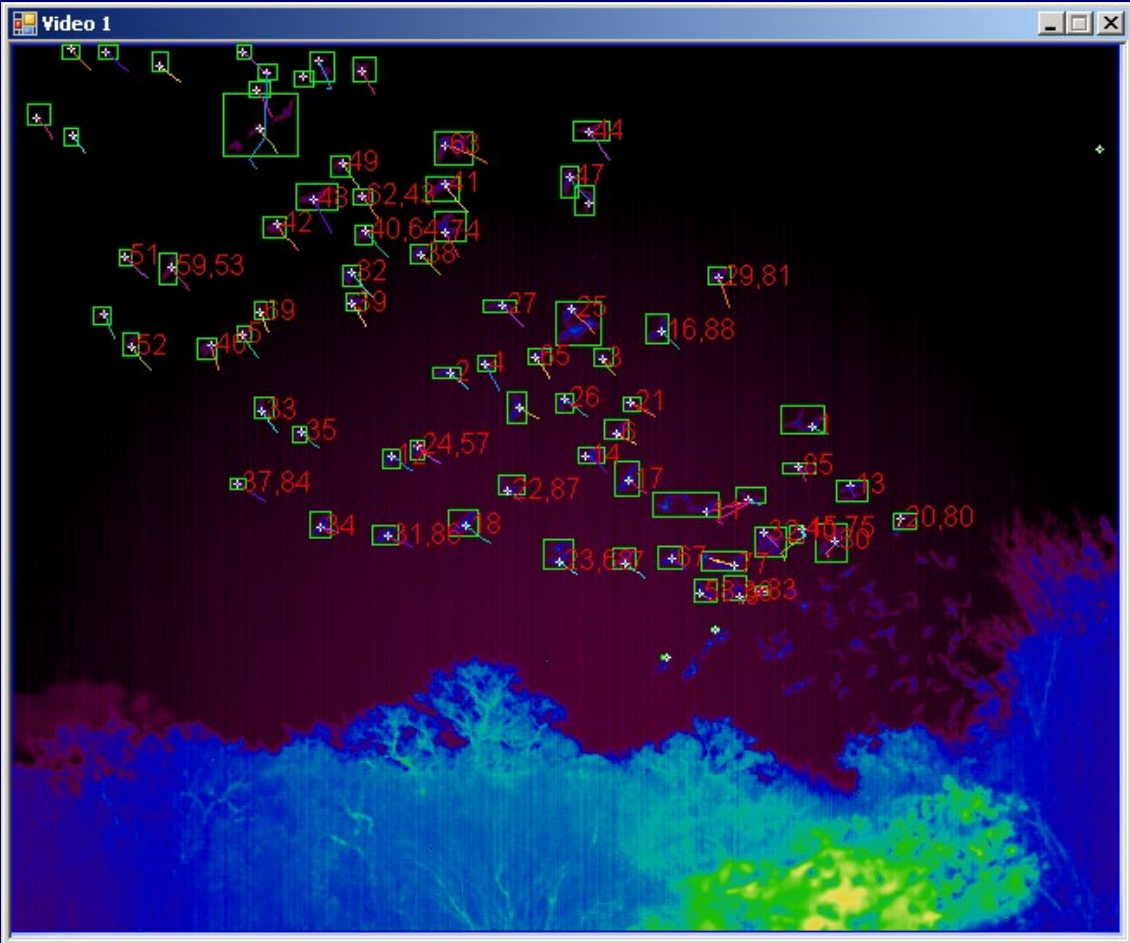
Experiment – Infrared Thermal Video Analysis (2)

- 4 test sequences with different density levels of the column of emerging bats; each sequence has 100 frames.

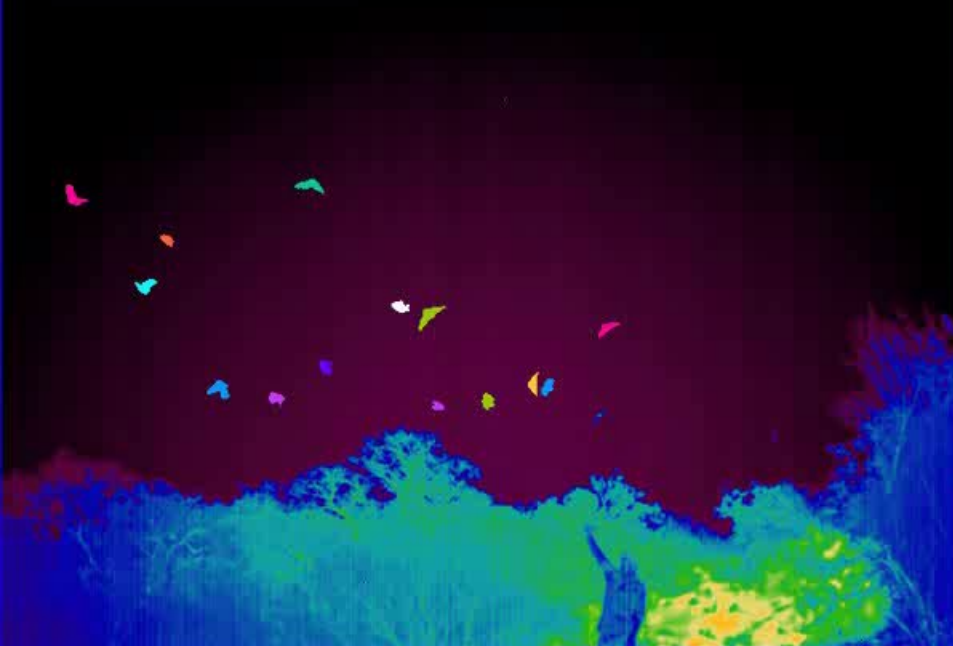
Number of Bat/frame	True Number of Bats	Computed Number of Tracks	Number of Occlusion	Number of Recovered Occlusions
20	25	33	56	40
40	50	63	94	54
60	71	90	140	86
100	119	185	368	88

24% correctly interpretation ...
Occlusions happen in more than one view

Experiment – Infrared Thermal Video Analysis (3)



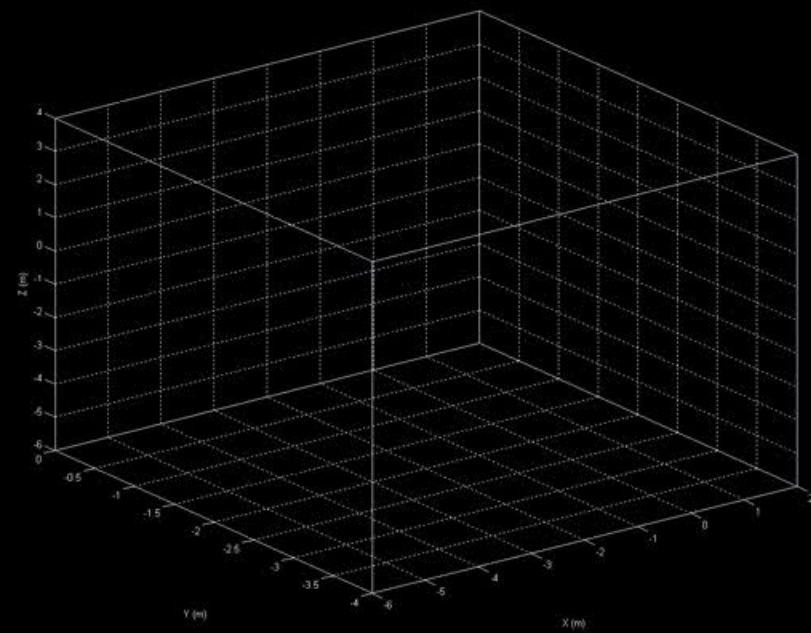
Camera A:Frame5001



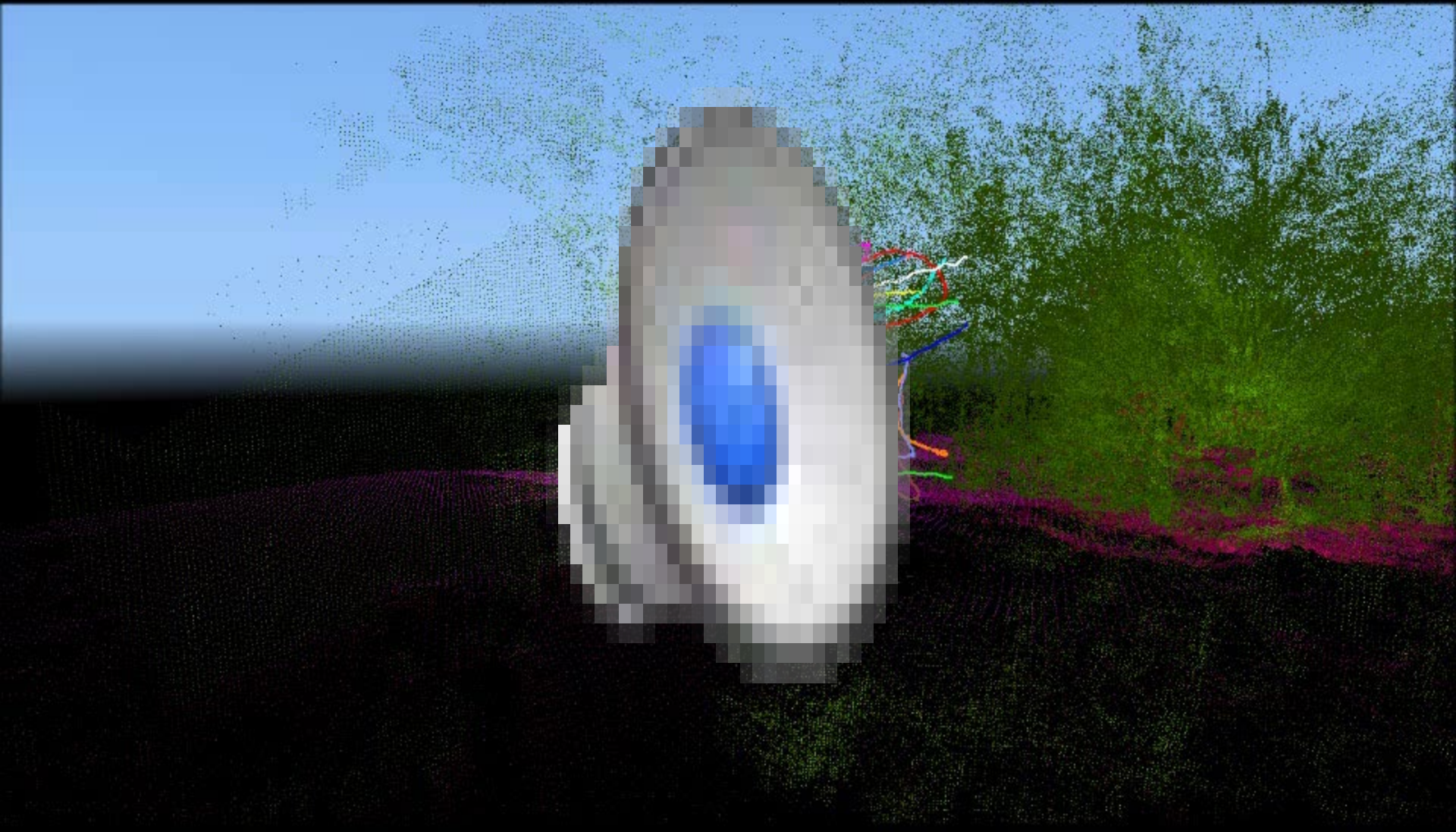
Camera B:Frame5001



Camera C:Frame5001



Multi modal reconstruction



MultiView MultiObject Tracking

State of the Art in 2024

- Deep learning used for detection of objects
- Measurement Association/Track Association still done by traditional algorithms
- Survey (incomplete): [Amosa et al., 2023](#)
- Tracking-reconstruction & reconstruction-tracking paradigms applied to people tracking: [Yang et al., 2022](#)

Learning Outcomes: Be able to

- ✱ Define the Multi-view Multi-object Tracking problem
- ✱ Explain Difference between Tracking-Reconstruction and Reconstruction-Tracking Methods and possible solutions
- ✱ Discuss how multi-view multi-object tracking systems can be validated experimentally