Computer Science

Image and Video Computing

Photometric Stereo and Shape from Shading

Lecture by Margrit Betke
April 4, 2024

Problem Definitions

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Shape from Shading

Find 3D shape in scene from a single 2D image

Photometric Stereo ≠ binocular stereo

Find 3D shape in scene from a set of 2D images that are taken under different lighting conditions

"stereo" = "solid" in Greek, used to refer to solidity, three-dimensionality

2 Algorithms

Photometric Stereo

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Example:

Find 3D shape in scene from these images of faces

Image Credit: Adrian N. Evans

Photometric Stereo

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3D shape visualized with texture from 1st image

Image Credit: Adrian N. Evans

Approach

Light reflected at surface patch depends on

Approach

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Light reflected at surface patch depends on

- surface orientation
- reflectance properties of surface
- distribution of light sources illuminating surface

Approach

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Light reflected at surface patch depends on

- surface orientation
- reflectance properties of surface
- distribution of light sources illuminating surface

Reconstruction Method:

Determine surface reflectance properties and direction of light source(s)

Compute surface orientation

Connection to Computer Graphics?

Connection to Computer Graphics

Computer Vision: Image E(x,y) given

- Determine surface reflectance properties of object in image
- Determine directions of light source & viewer
- Compute 3D surface orientation of object at each z(x,y)

Computer Graphics: 3D object shape z(x,y) given

- Determine surface reflectance properties of object
- Determine directions of light source and viewer
- Find intersection of ray along viewer direction with surface point z(x,y) ("ray tracing")
- Create realistic-looking image E(x,y) for z(x,y)

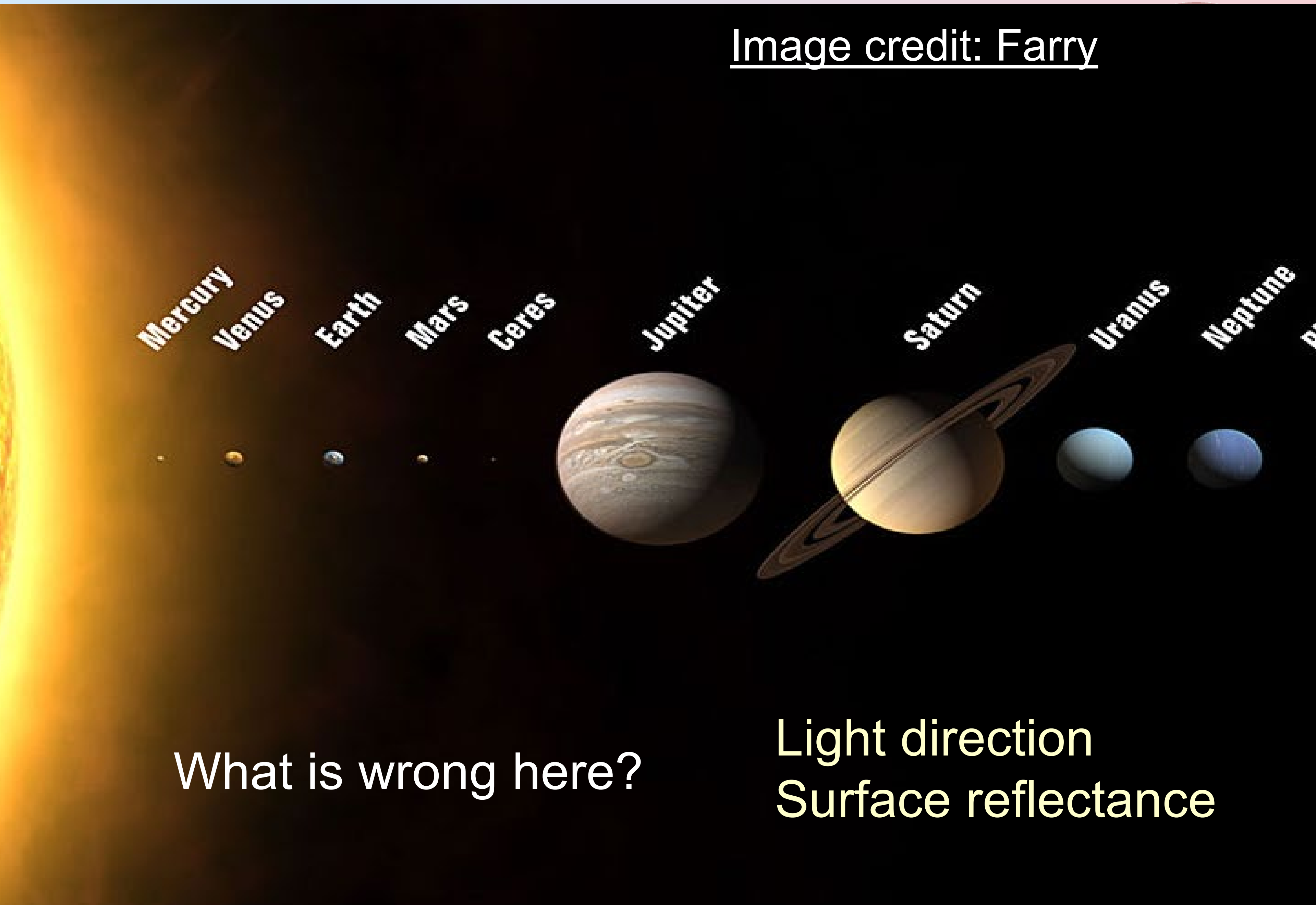

Image credit: Farry

What is wrong here?

Light direction
Surface reflectance

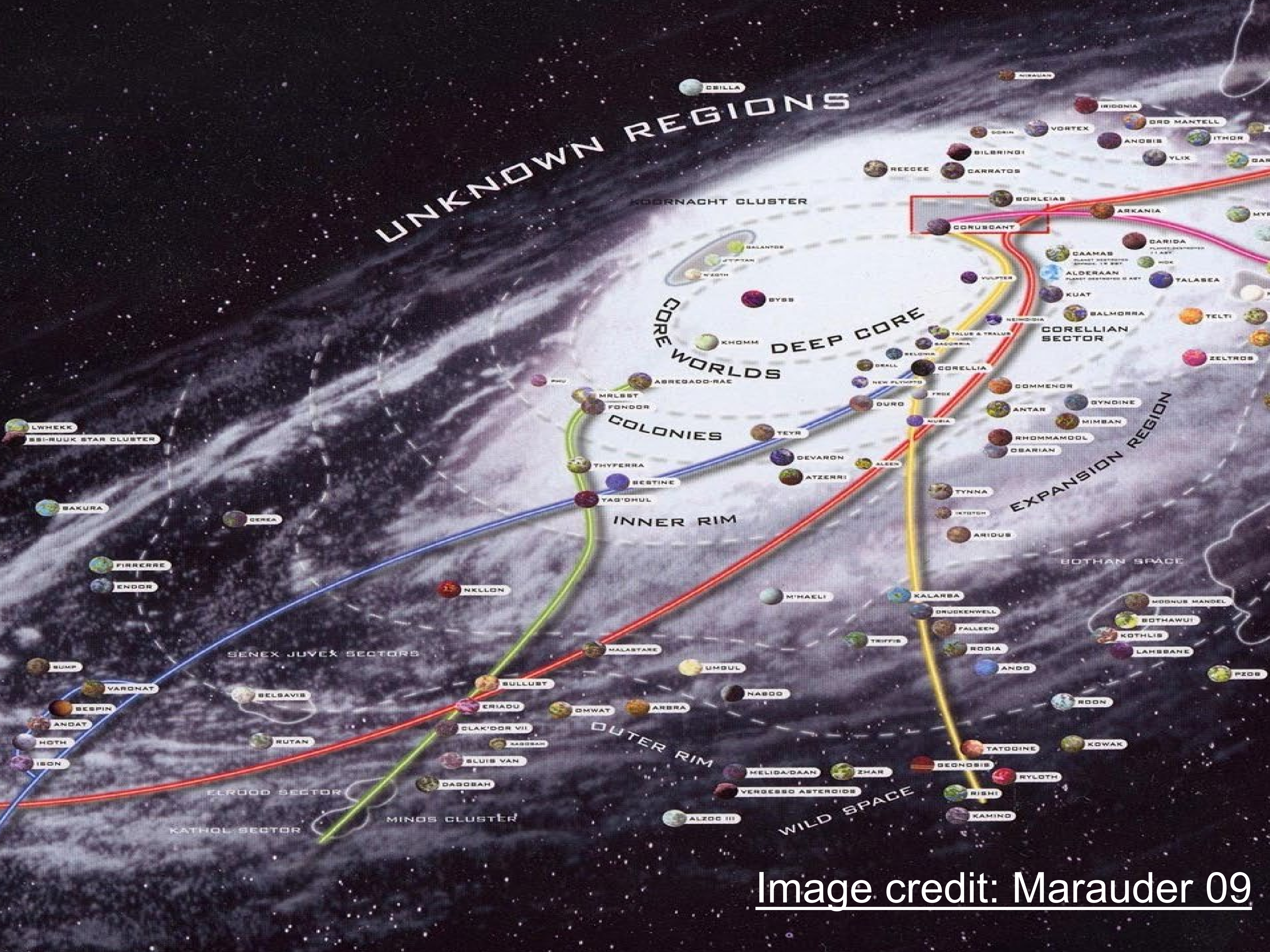

Image credit: Marauder 09

What do we need to consider for image E to look right?

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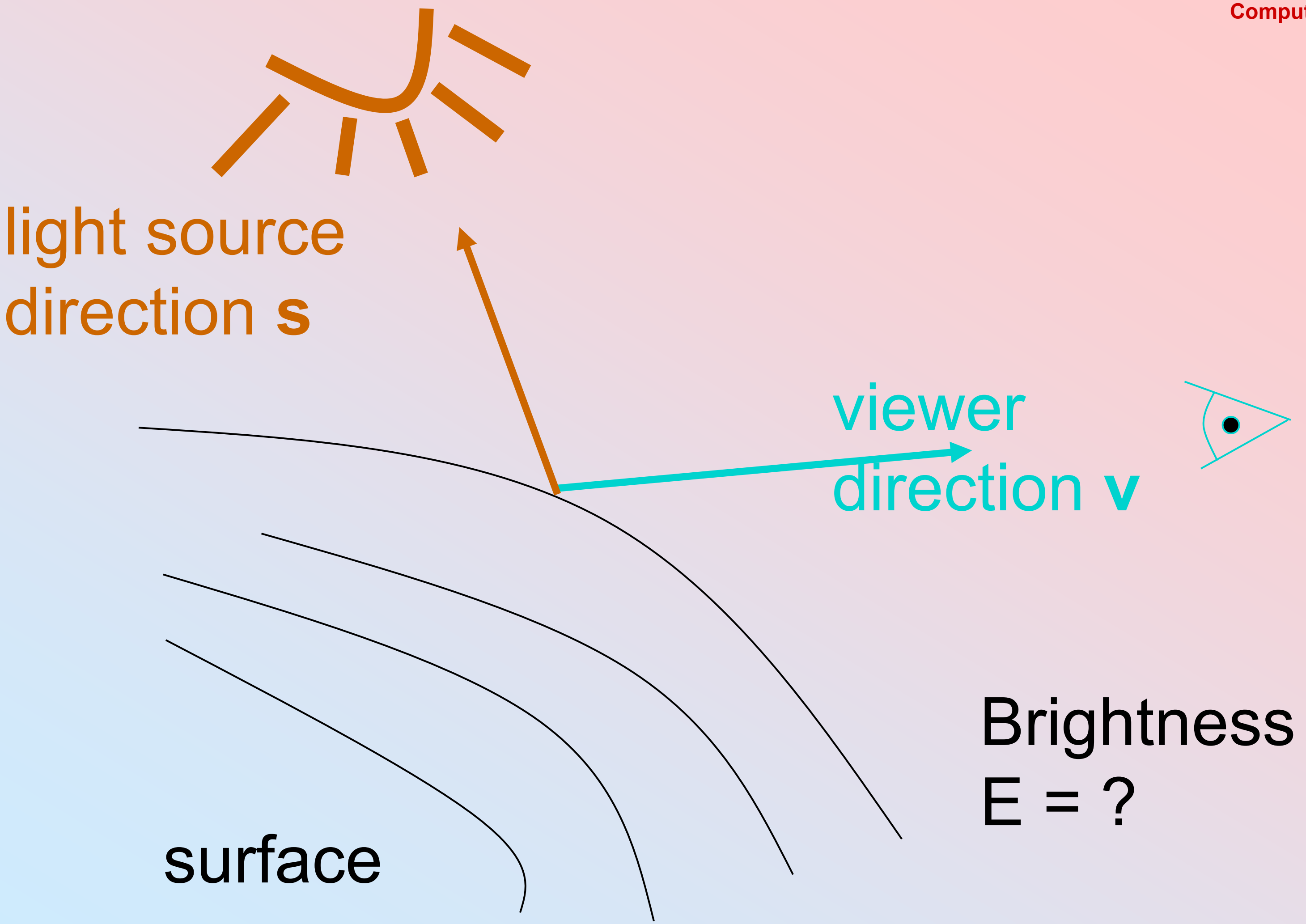
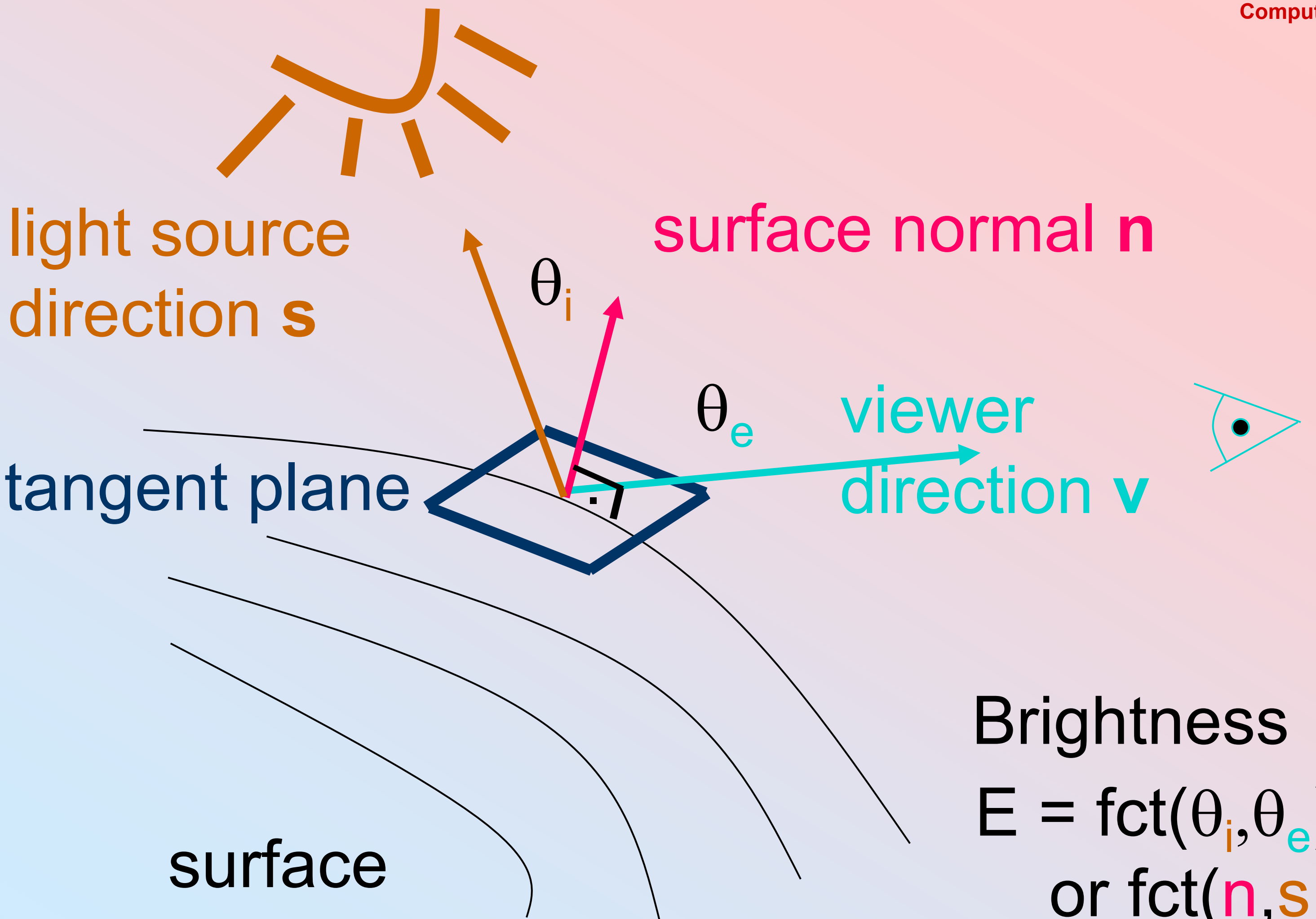


Surface-centered Definition



Brightness

$E = fct(\theta_i, \theta_e)$

or $fct(n, s, v)$

Surface Reflectance

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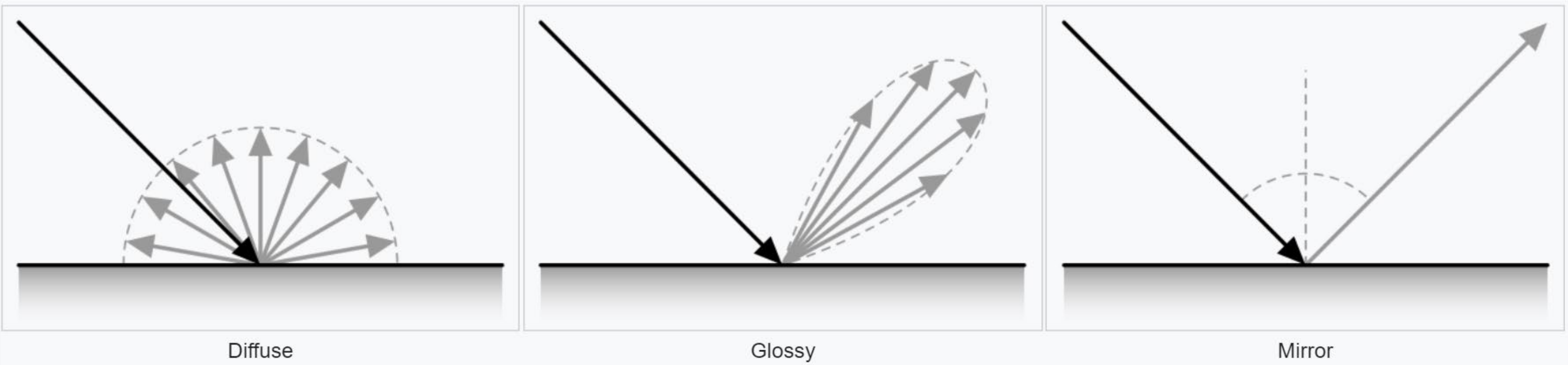
fct (n,s,v) = BRDF =
Bidirectional Reflectance Distribution Function



Three elemental components that can be used to model a variety of light-surface interactions.[8] The incoming light ray is shown in black, the reflected ray(s) modeled by the BRDF in gray.

Image credit: Wikipedia, BRDF

Diffuse reflecting surface = Lambertian

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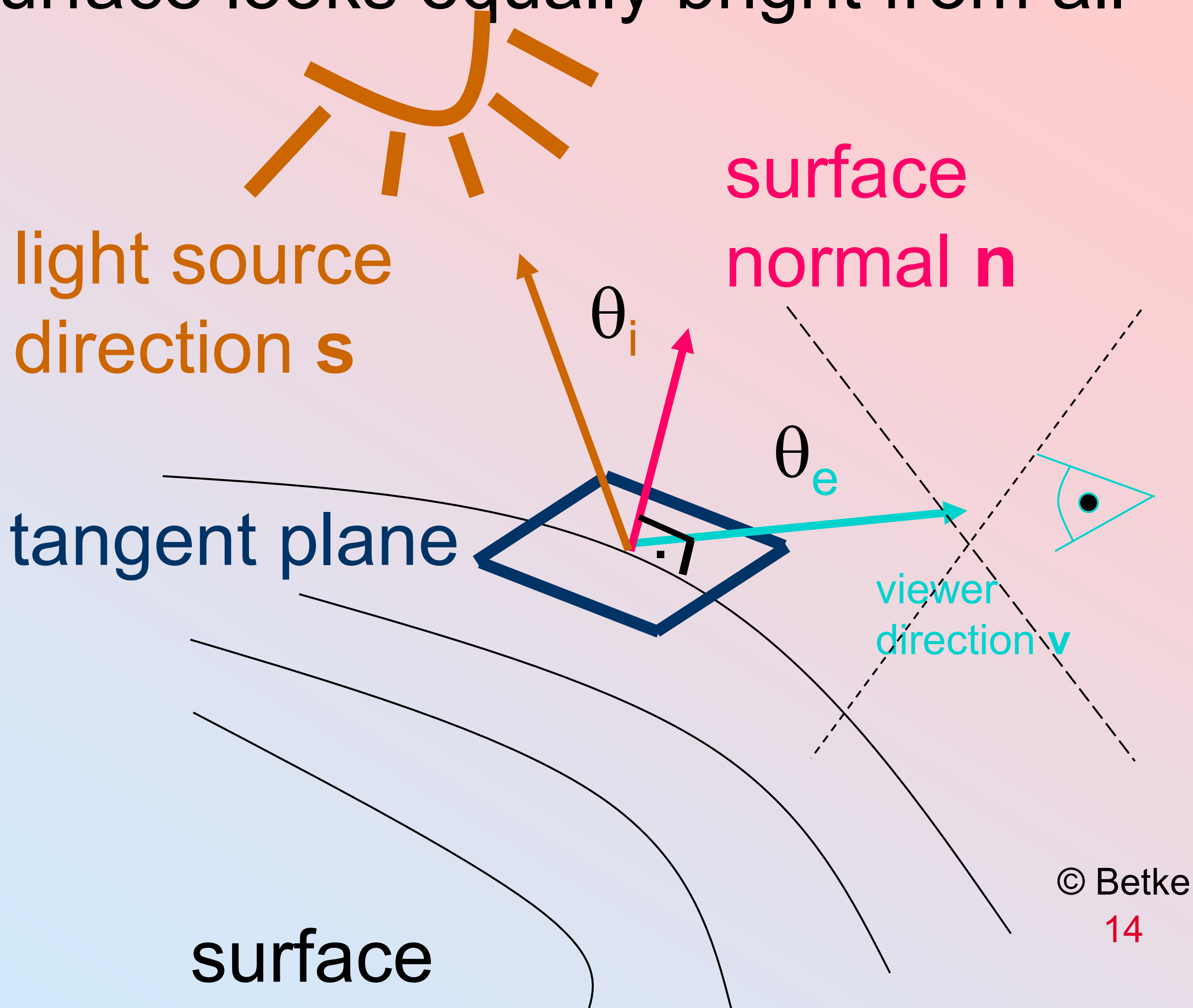
Ideal Lambertian surface looks equally bright from all directions

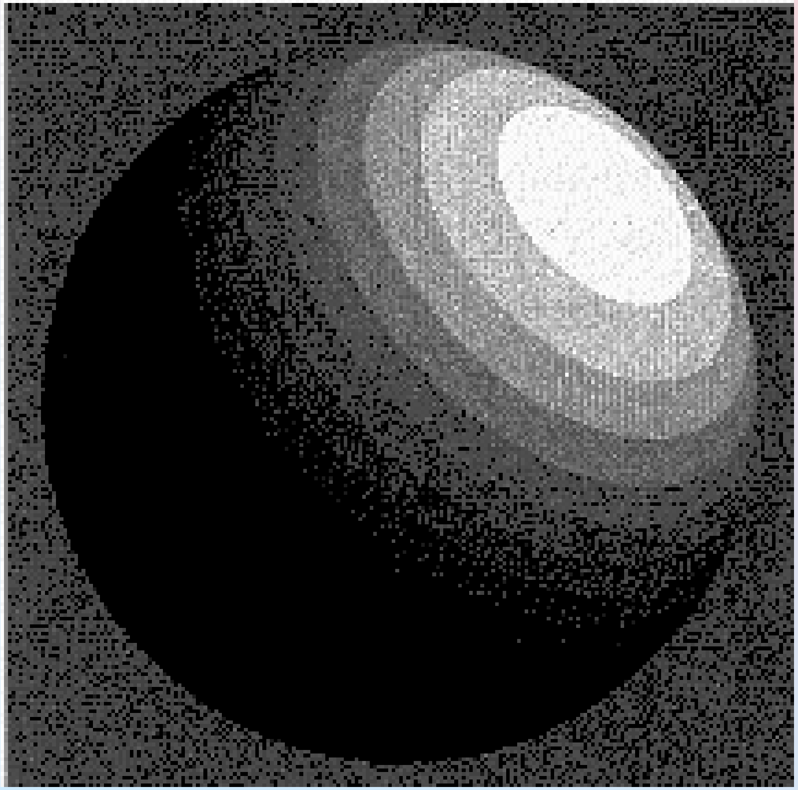
Brightness

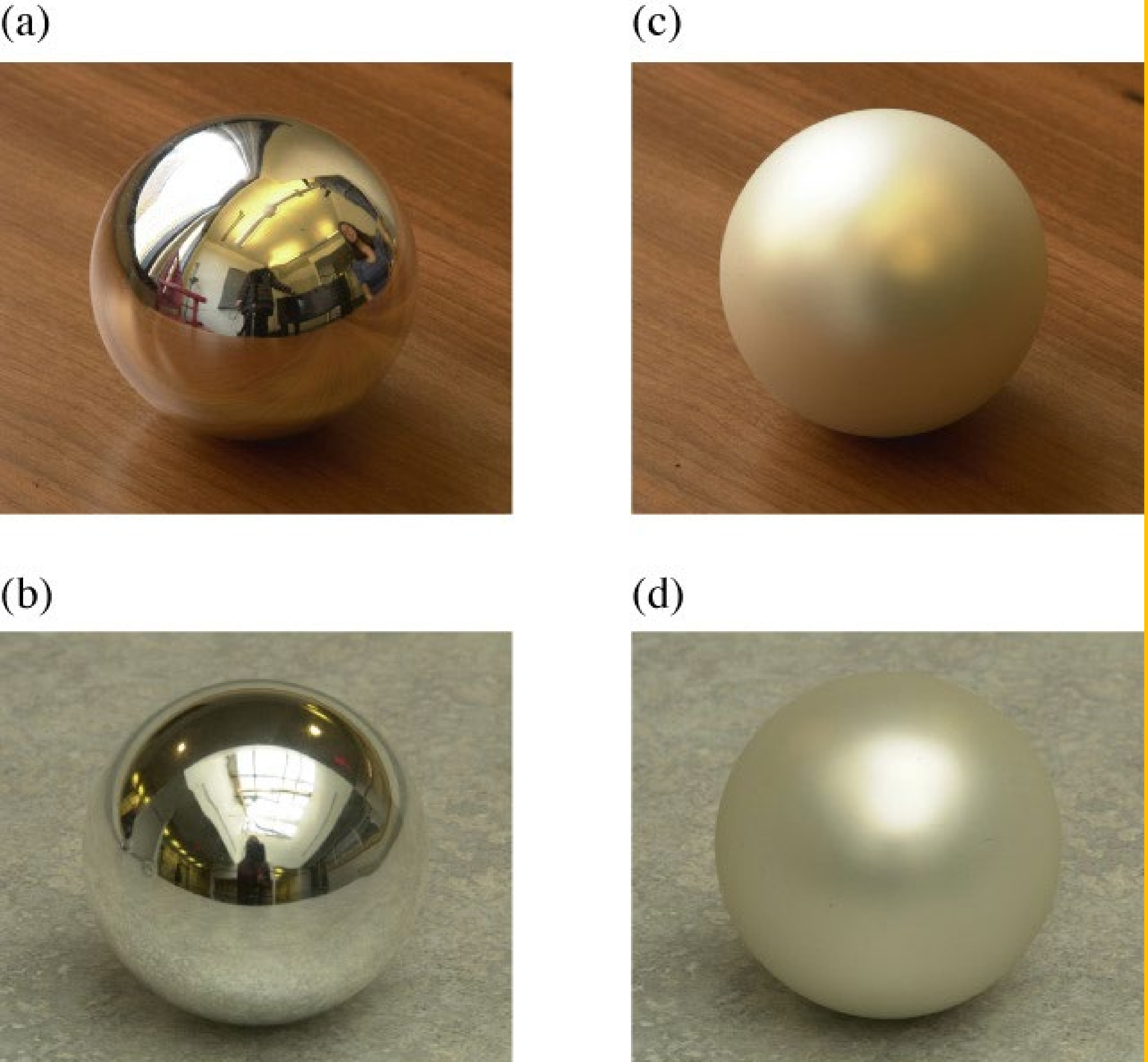
$E = fct(\theta_i, \theta_e)$
or $fct(n, s, v)$

$E = \cos \theta_i = \mathbf{n.s}$



Examples of Lambertian Surfaces





Poll: Which objects are Lamber-tian?

Image credit: Fleming, 2013

(a)

(c)

Specular

(not quite ideal) Lambertian

(b)

(d)

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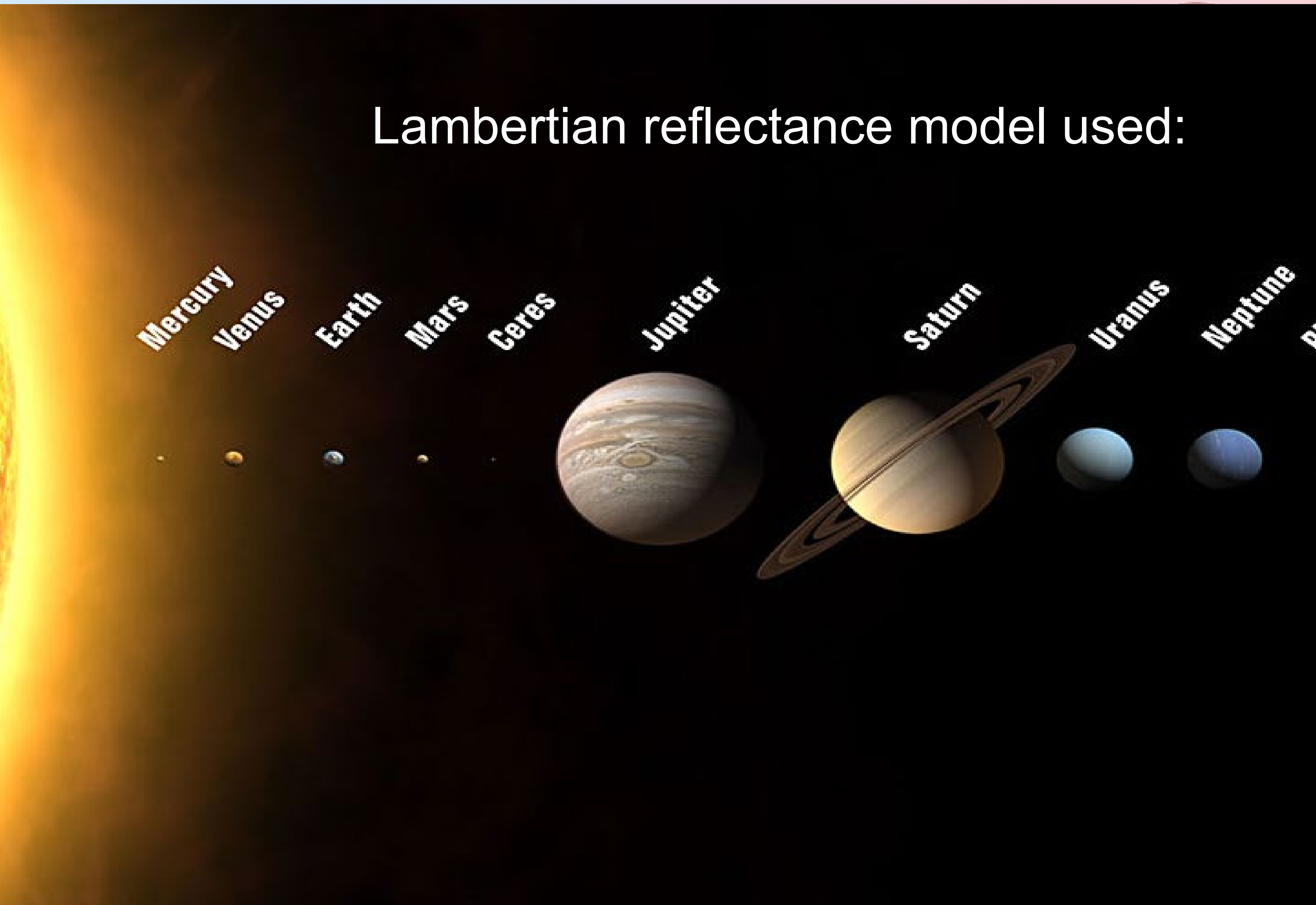
Image credit: Fleming, 2013

Lambertian reflectance model used:



Reflectance Properties of Moon

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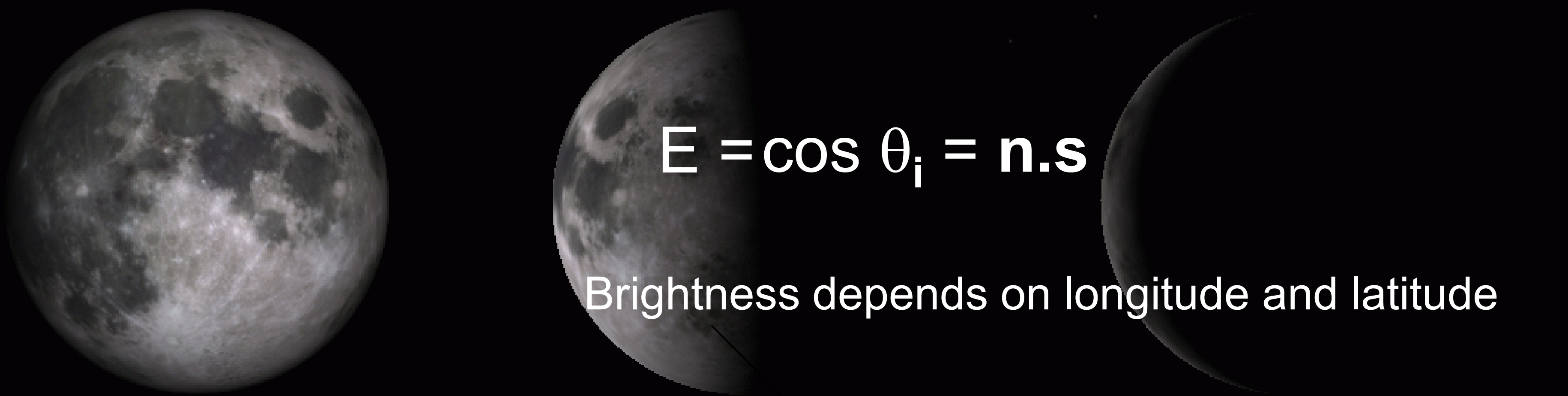
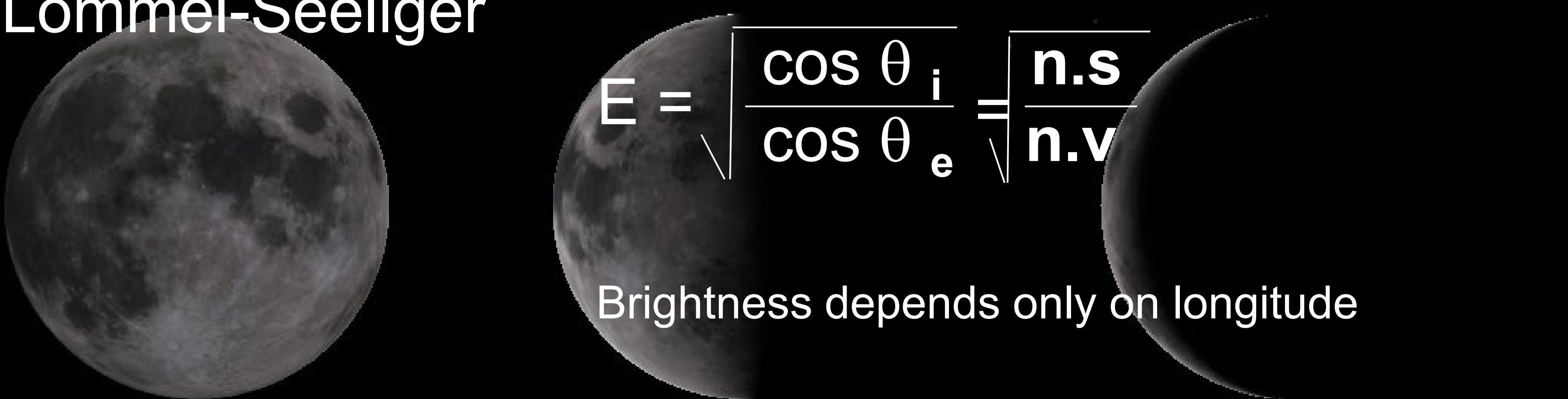
Lambertian? No, moon does not look like sphere, but like flat disk



$$E = \cos\theta_i = \mathbf{n.s}$$

Brightness depends on longitude and latitude

Lommel-Seeliger



$$E = \sqrt{\frac{\cos\theta_i}{\cos\theta_e}} = \sqrt{\frac{\mathbf{n.s}}{\mathbf{n.v}}}$$

Brightness depends only on longitude

Surface Orientation

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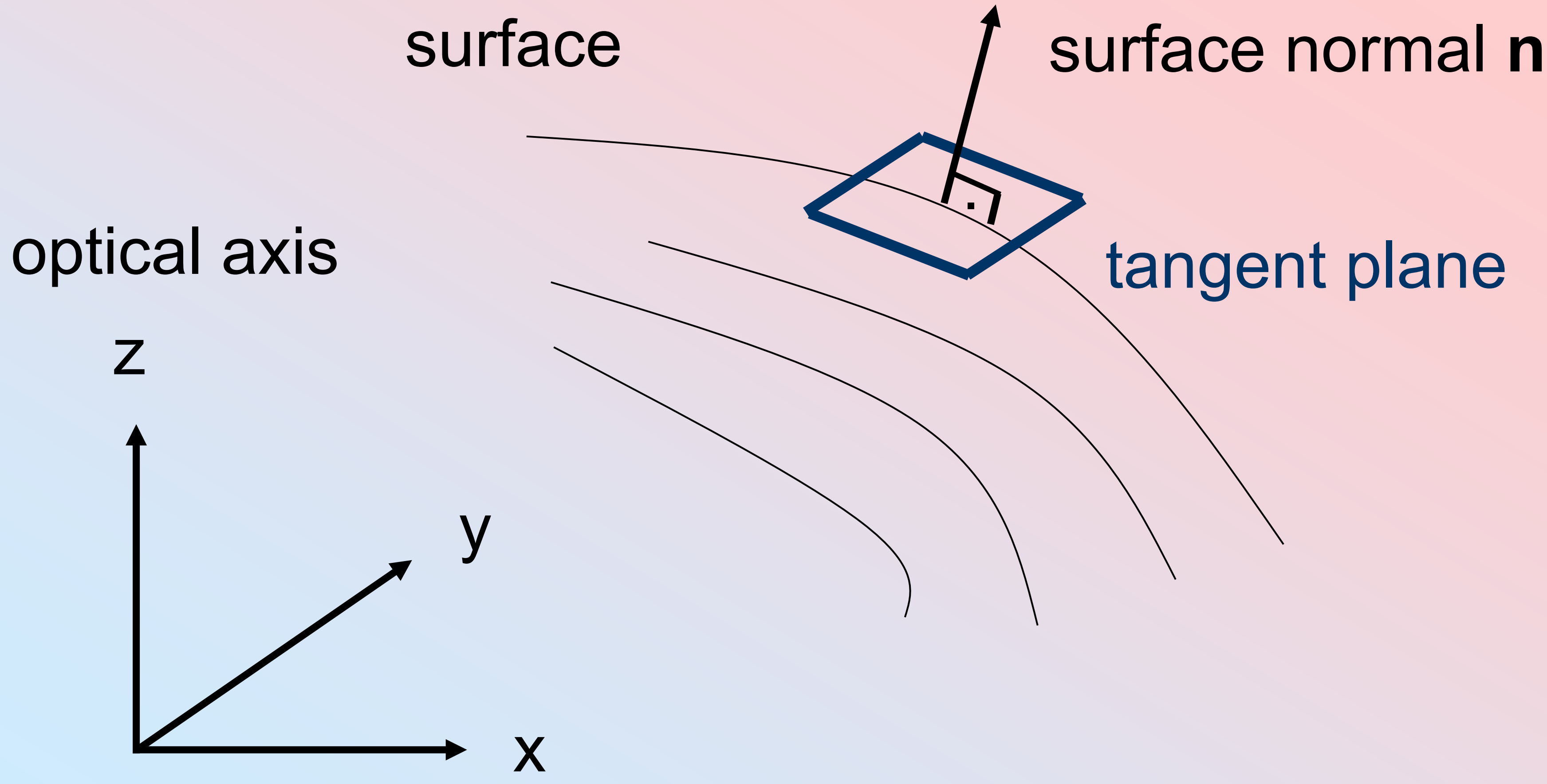
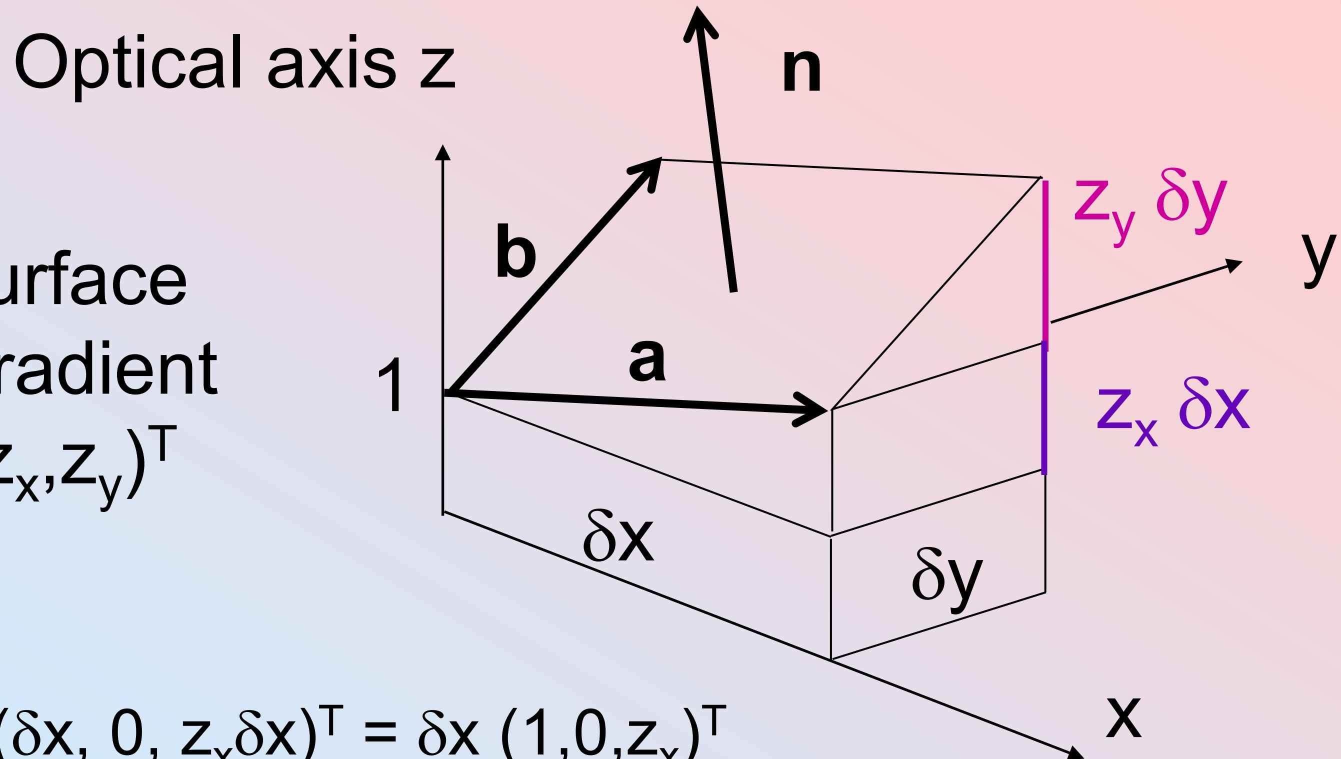


Surface Orientation



Surface Gradient $(z_x, z_y)^T$

a $=(\delta x, 0, z_x\delta x)^T = \delta x\,(1,0,z_x)^T$

b $=(0, \delta y, z_y\delta y)^T = \delta y\,(0,1,z_y)^T$

n || (**a** x **b**)

n $= (-z_x, -z_y, 1)^T = (-p,-q,1)^T$

Reflectance Map

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Two different projections can create maps of the surface gradients on "Gaussian" (or unit) sphere:

Stereographic plane:

Whole sphere is projected

Includes occluding boundary of sphere

Reflectance map: -- we'll use this projection

Upper hemisphere of sphere is projected

Isobrightness lines extend to infinity

Reflectance Map

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A reflectance map R(p,q) is a function that gives scene radiance as a function of surface orientation.

Scene radiance = light reflected from surface patch and measured by camera.
Given at a pixel (which is the image of the center of the patch) as a normalized gray value [0..1].

Surface orientation $n=(-p,-q,1)^T$

$$\hat{n}=(-p,-q,1)/\mathrm{sqrt}(p^2+q^2+1)$$

Reflectance map of matte surface

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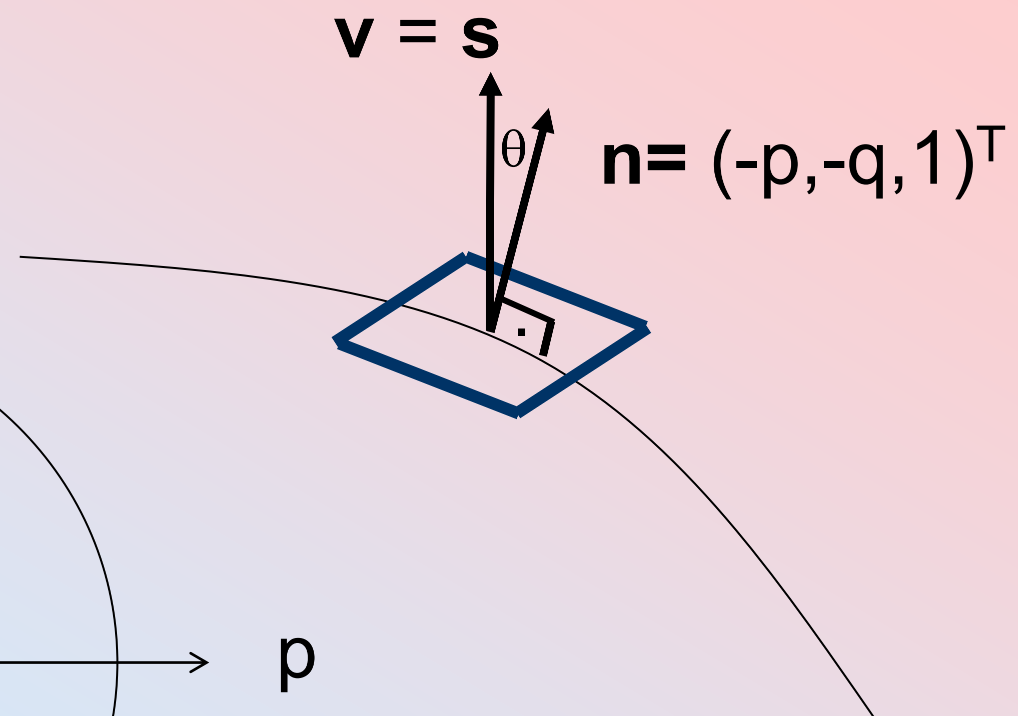
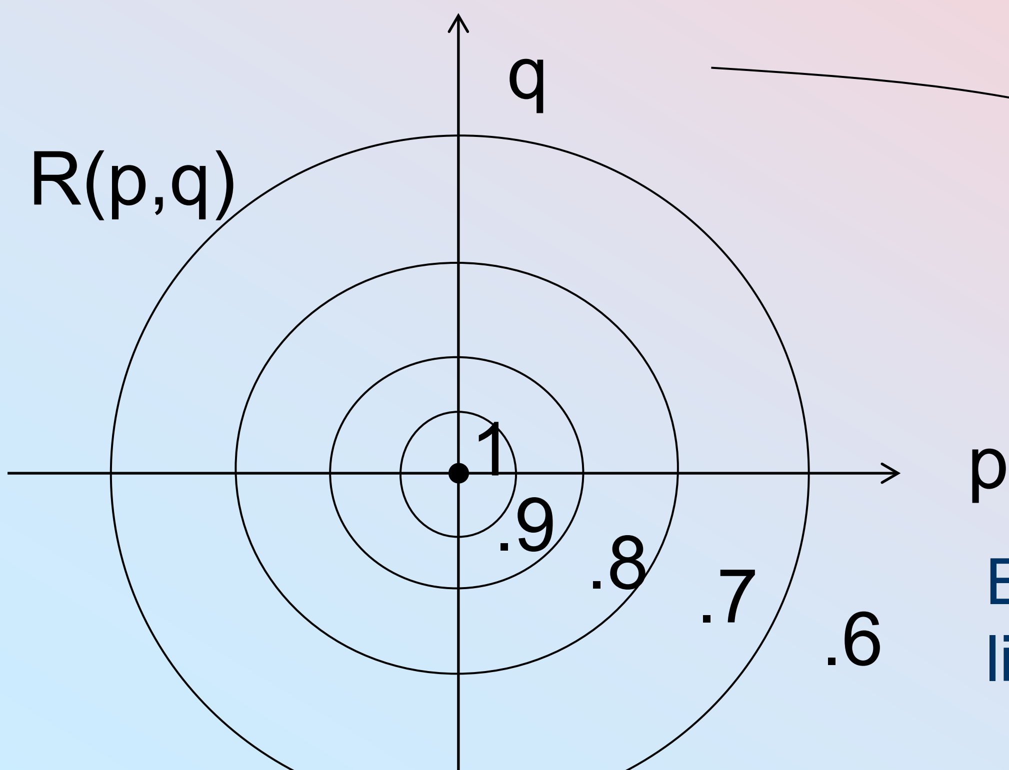
Ideal Lambertian surface looks equally bright from all directions

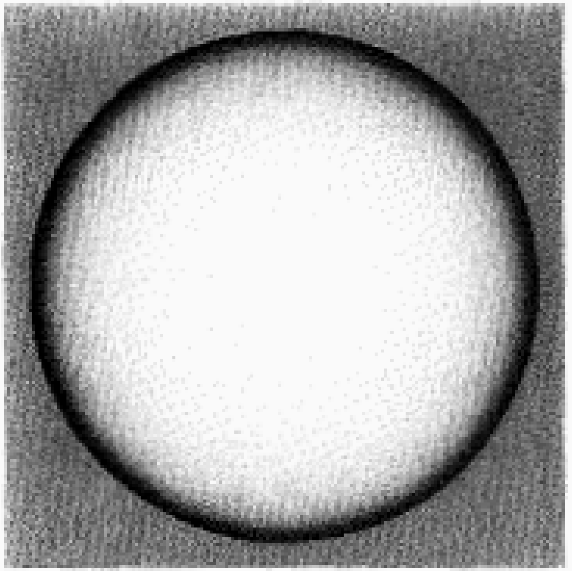
$\cos\theta = \hat{\mathbf{n}}.\hat{\mathbf{s}}$





Example:
light source near viewer

R(p,q) of Lambertian Surface: Here: Light Source near Viewer



Nalwa, 1993

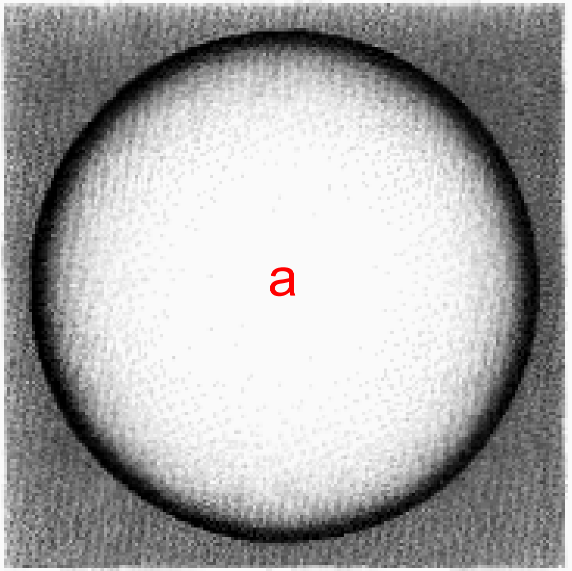
Where on the sphere are all points such that

(a) E= 1 ?

(b) E= 0.2 ?

(c) E= 0.7 ?

R(p,q) of Lambertian Surface: Here: Light Source near Viewer



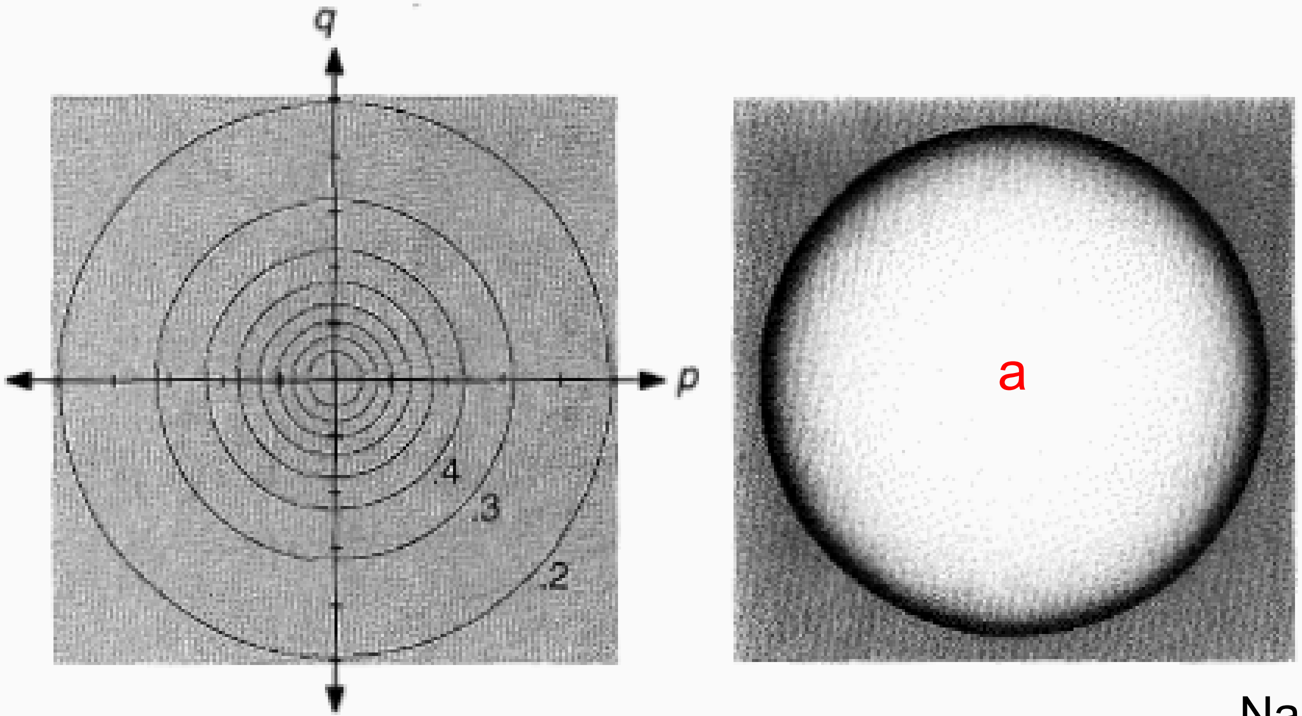
Nalwa, 1993

Where on the sphere are all points such that

(a) E= 1 because brightest spot on sphere is facing viewer = source direction

$\cos\theta = \cos 0^{o} = 1$

R(p,q) of Lambertian Surface: Here: Light Source near Viewer



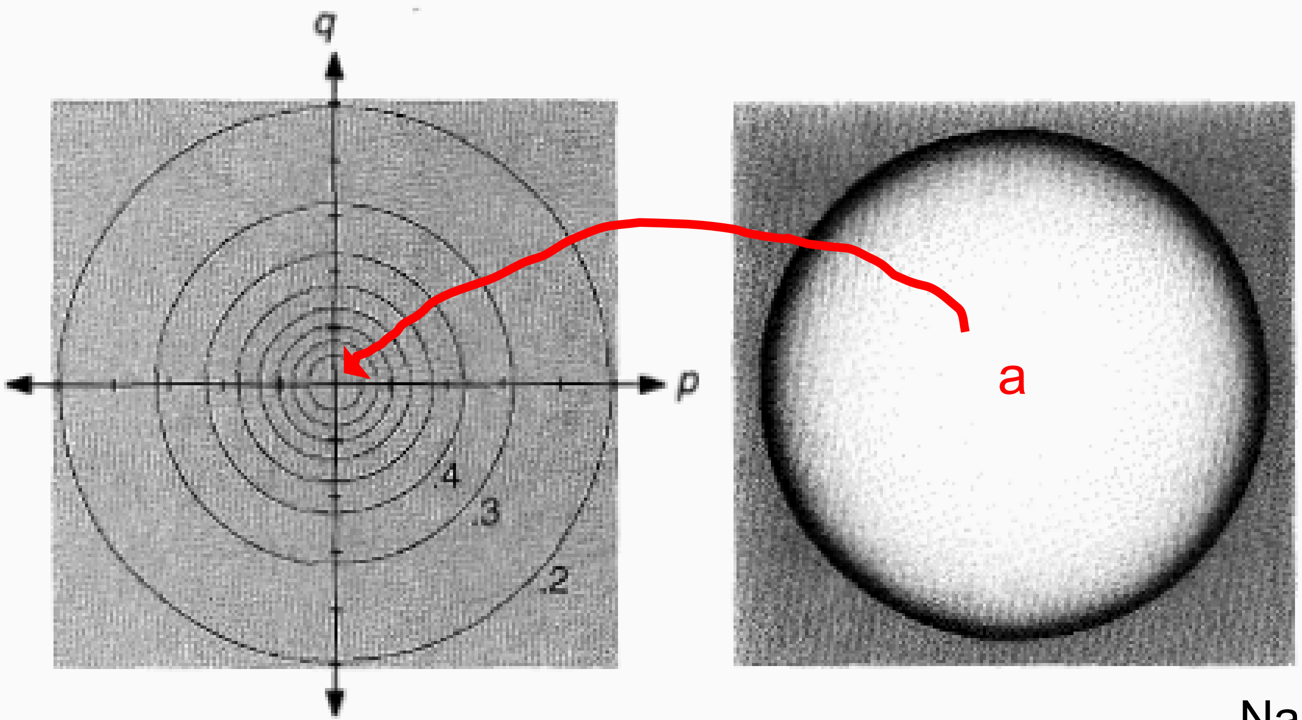
Nalwa '93

Where on the sphere are all points such that

(a) E= 1 because brightest spot on sphere is facing viewer = source direction

$\cos\theta = \hat{\mathbf{n}}.\hat{\mathbf{s}} = (-p, -q, 1)/\mathrm{sqrt}(p^2+q^2+1).(0,0,1) = \cos 0^o = 1$

R(p,q) of Lambertian Surface: Here: Light Source near Viewer



Nalwa '93

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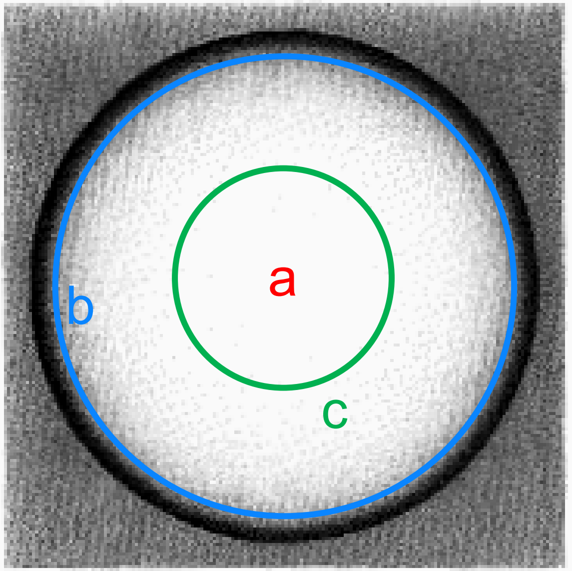
(a) E= 0.2 ?

(b) E= 0.7 ?

R(p,q) of Lambertian Surface: Here: Light Source near Viewer



Nalwa ‘93

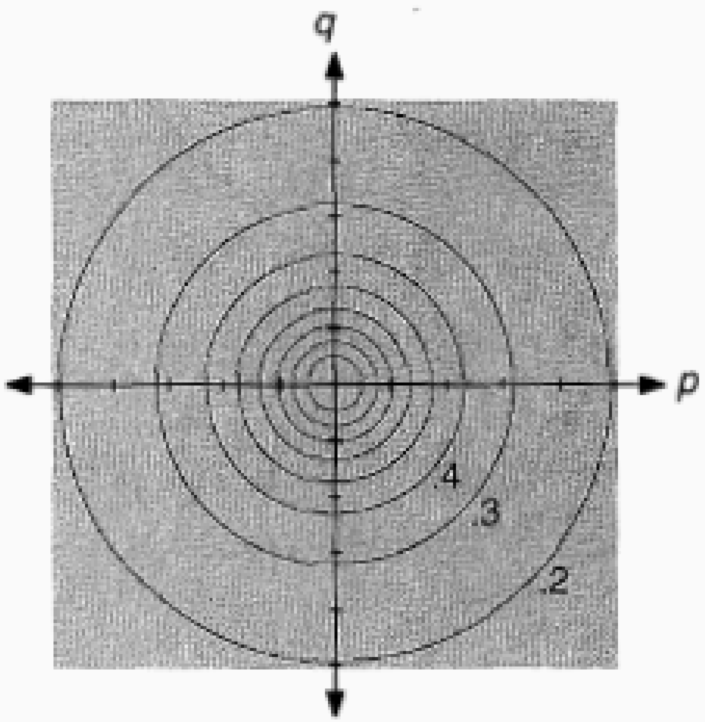
Where on the sphere are all points such that

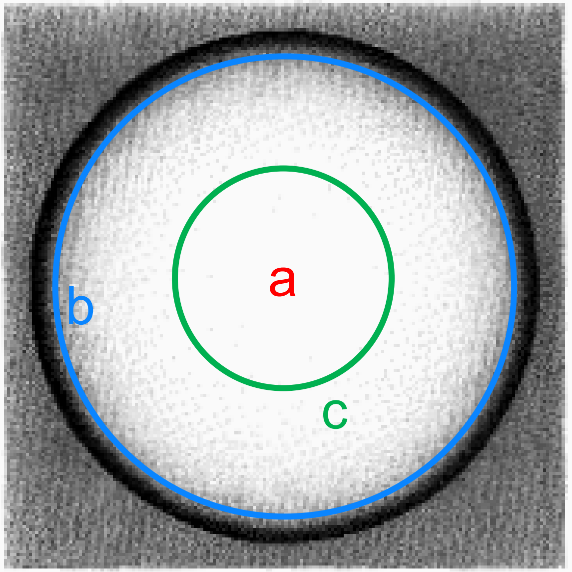
(a) E= 1 because $\cos(0^o)=1$

(b) E= 0.2 $\cos(78.5^o)=0.2$

(c) E= 0.7 $\cos(45^o)\sim 0.7$

R(p,q) of Lambertian Surface: Here: Light Source near Viewer





Nalwa '93

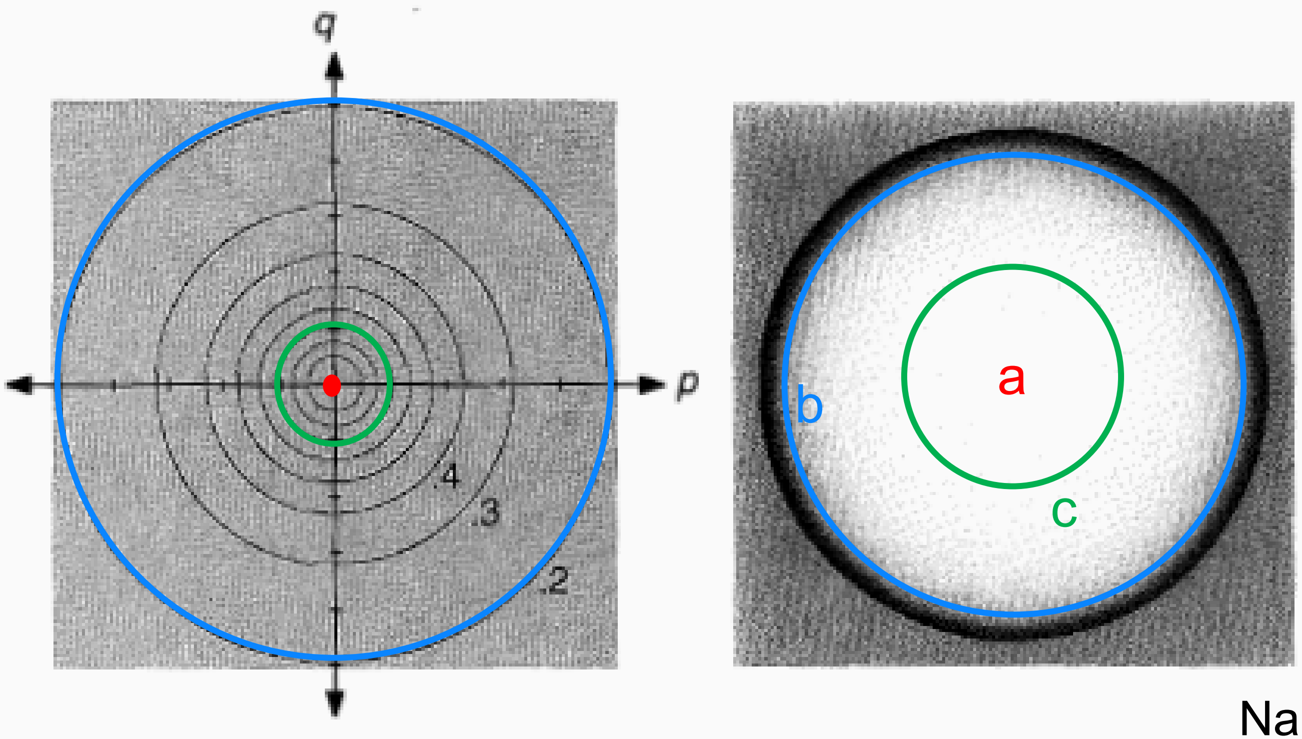
Where on the sphere are all points such that

(a) E= 1 because cos(0°)=1

(b) E= 0.2 cos(78.5°)=0.2

(c) E= 0.7 cos(45°)~0.7

R(p,q) of Lambertian Surface: Here: Light Source near Viewer



Nalwa '93

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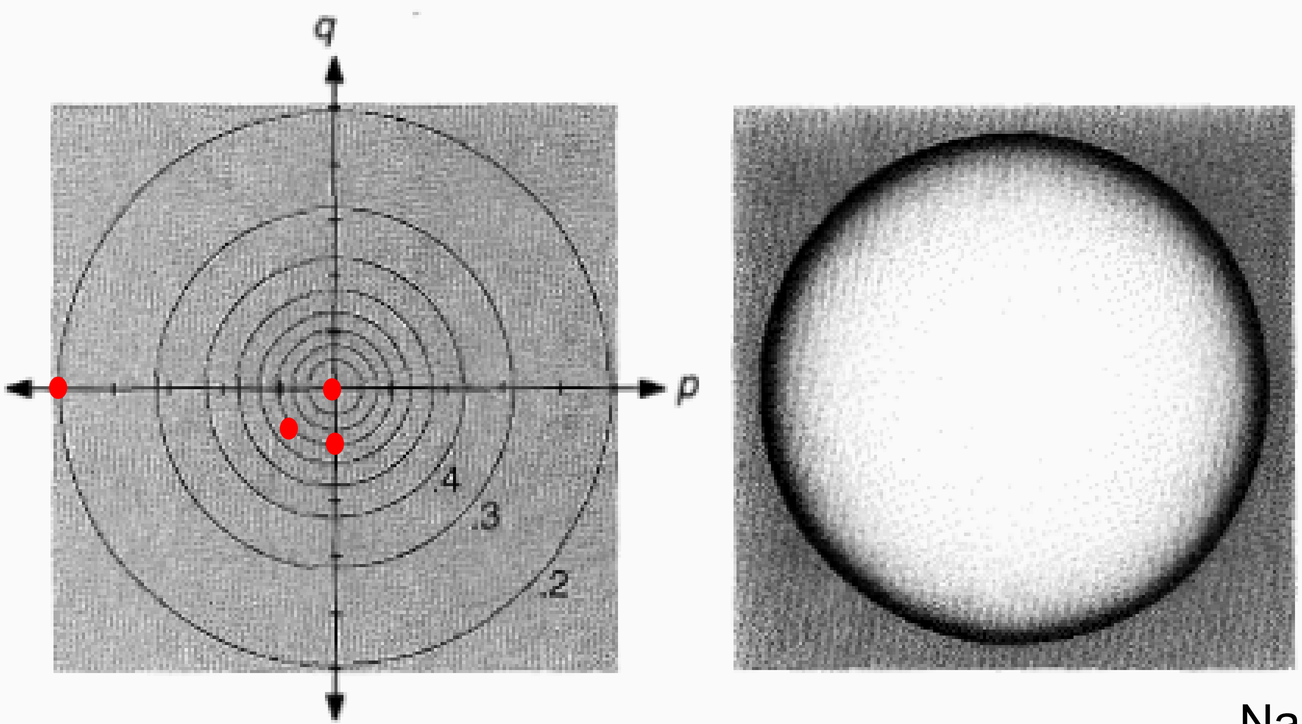
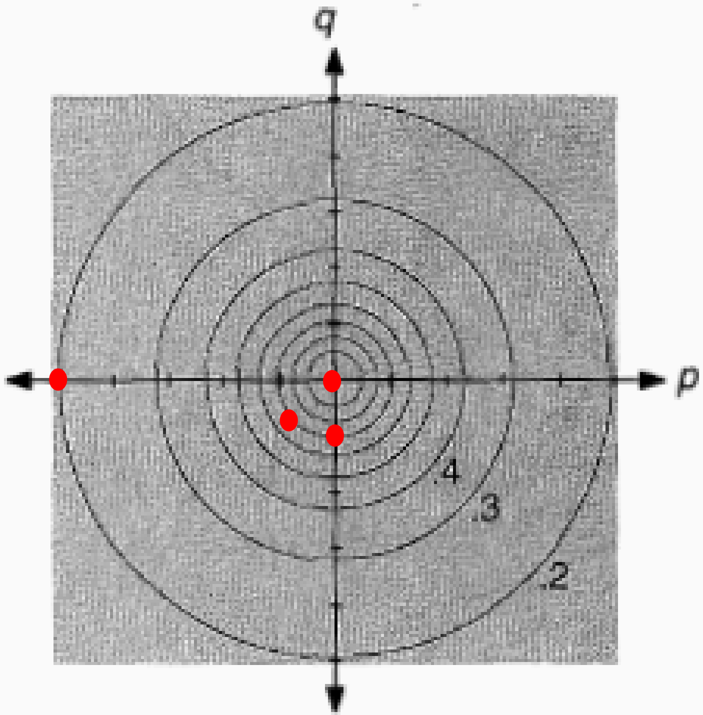
R(p,q) of Lambertian Surface:
Here: Light Source near Viewer

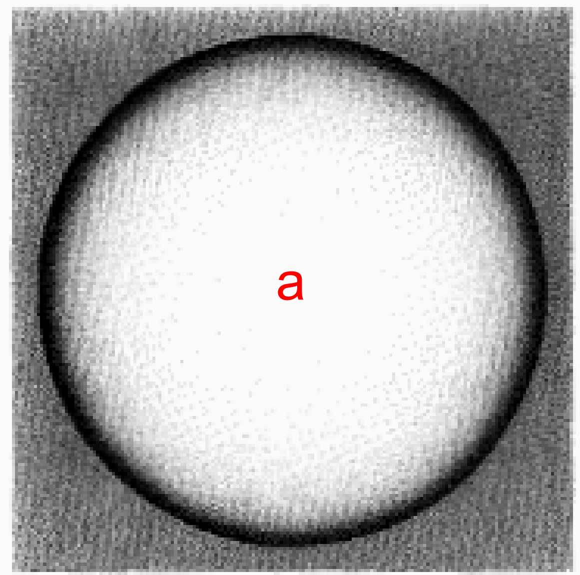


Nalwa '93

Where on the sphere is | How bright?
(a) p=0, q=0 | E=
(b) p=-5, q=0 | E=
(c) p=0, q=-1 | E=
(d) p=-0.707, q=-0.707 | E=

R(p,q) of Lambertian Surface: Here: Light Source near Viewer





Nalwa '93

Where on the sphere is

(a) p=0, q=0

(b) p=-5, q=0?

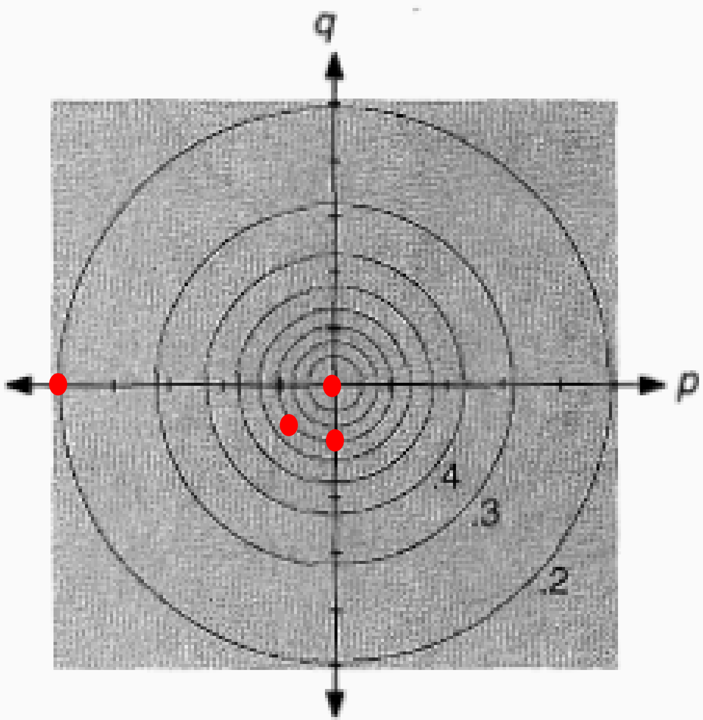
(c) p=0, q=-1?

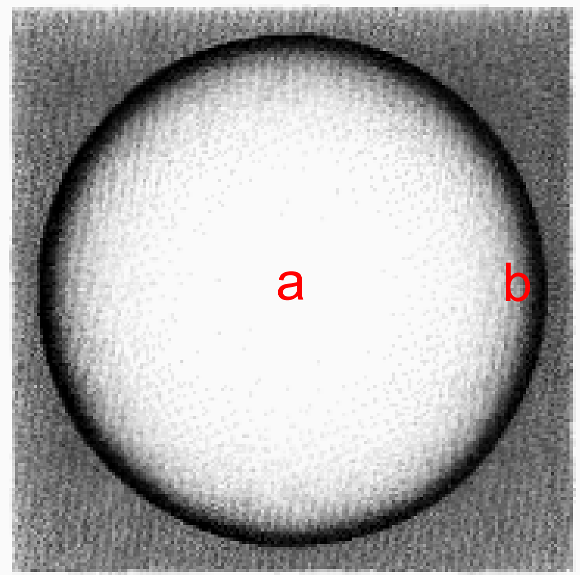
(d) p=-0.707, q=-0.707?

How bright?

$E=\hat{n}.s=(0,0.1)(0,0,1)=1$

R(p,q) of Lambertian Surface: Here: Light Source near Viewer





Nalwa '93

Where on the sphere is

(a) p=0, q=0

(b) p=-5, q=0

(c) p=0, q=-1?

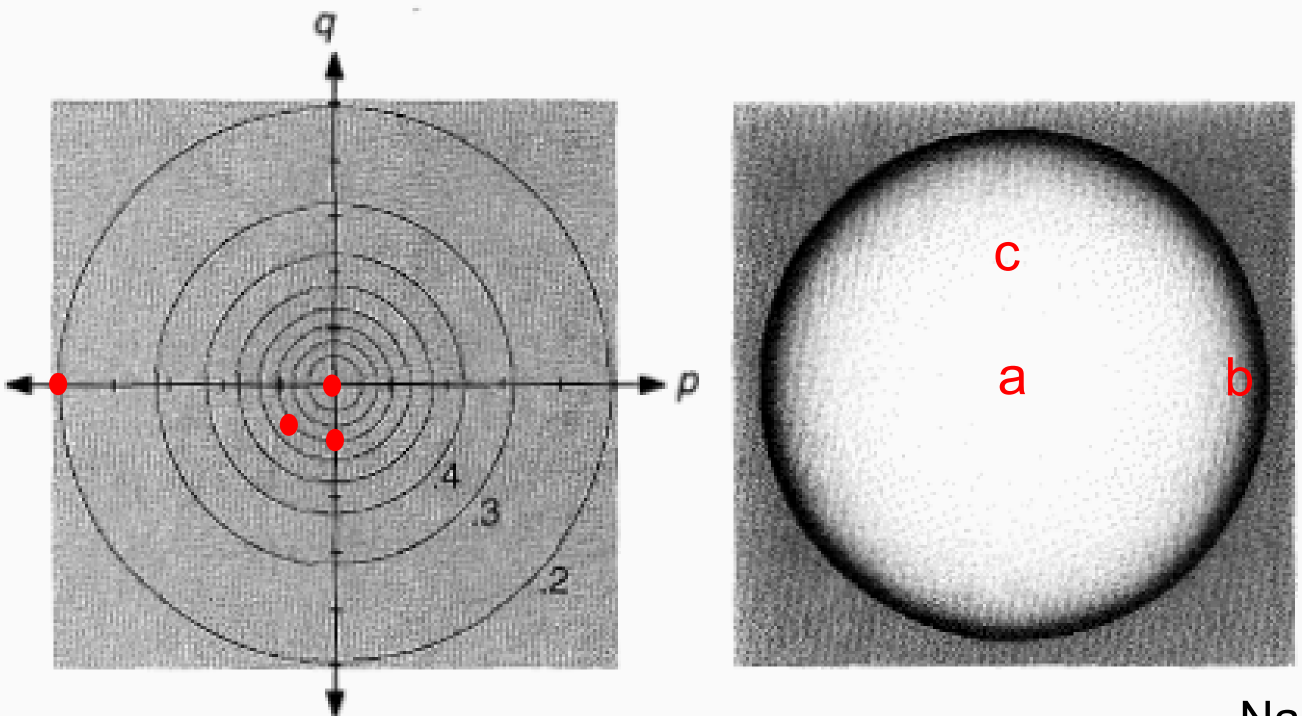
(d) p=-0.707, q=-0.707?

How bright?

$E=\hat{n}.s=(0,0.1)(0,0,1)=1$

$E=\hat{n}.s=(5,0,1)/\mathrm{sqrt}(25+1).(0,0,1)=0.2$

R(p,q) of Lambertian Surface: Here: Light Source near Viewer



Nalwa ‘93

Where on the sphere is

(a) p=0, q=0

(b) p=-5, q=0

(c) p=0, q=-1

(d) p=-0.707, q=-0.707

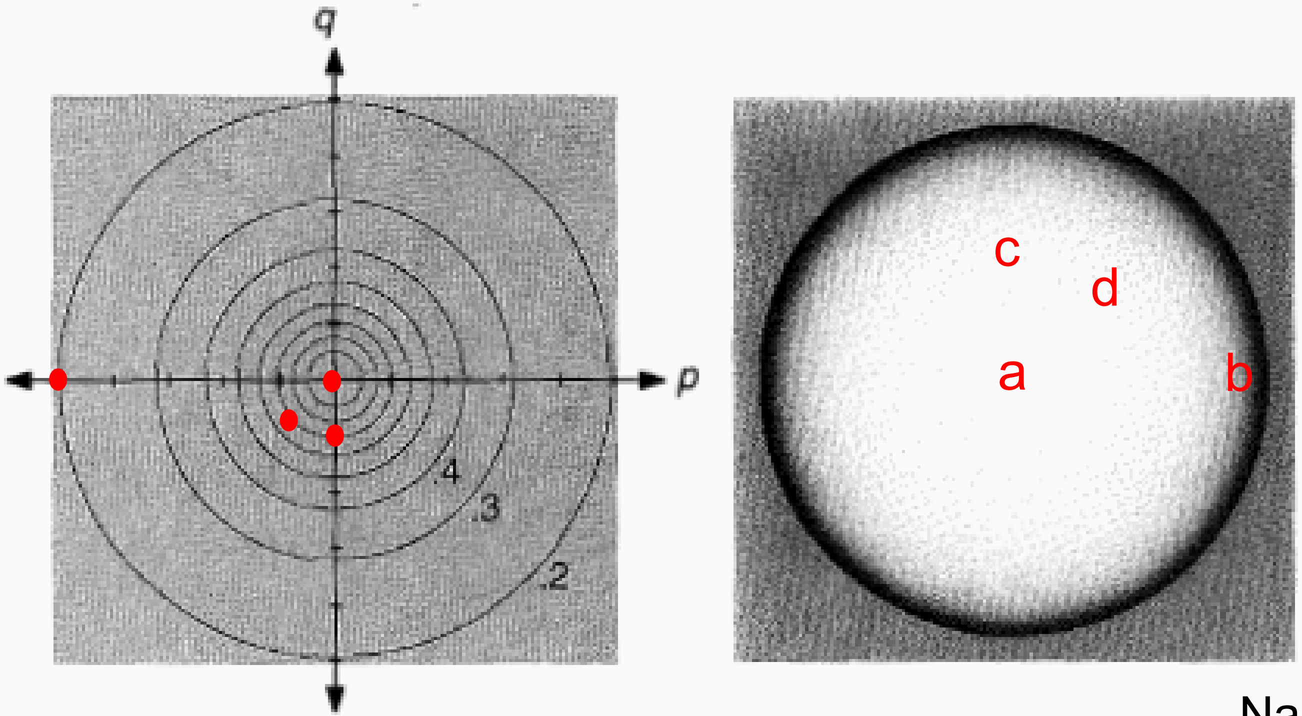
How bright?

$E=\hat{n}.s=(0,0.1)(0,0,1)=1$

$E=\hat{n}.s=(5,0,1)/\mathrm{sqrt}(25+1).(0,0,1)=0.2$

$E=\hat{n}.s=(0,1,1)/\mathrm{sqrt}(1+1).(0,0,1)\sim 0.707$

R(p,q) of Lambertian Surface: Here: Light Source near Viewer



Nalwa '93

Where on the sphere is

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(b) p=-5, q=0

(c) p=0, q=-1

(d) p=-0.707, q=-0.707

How bright?

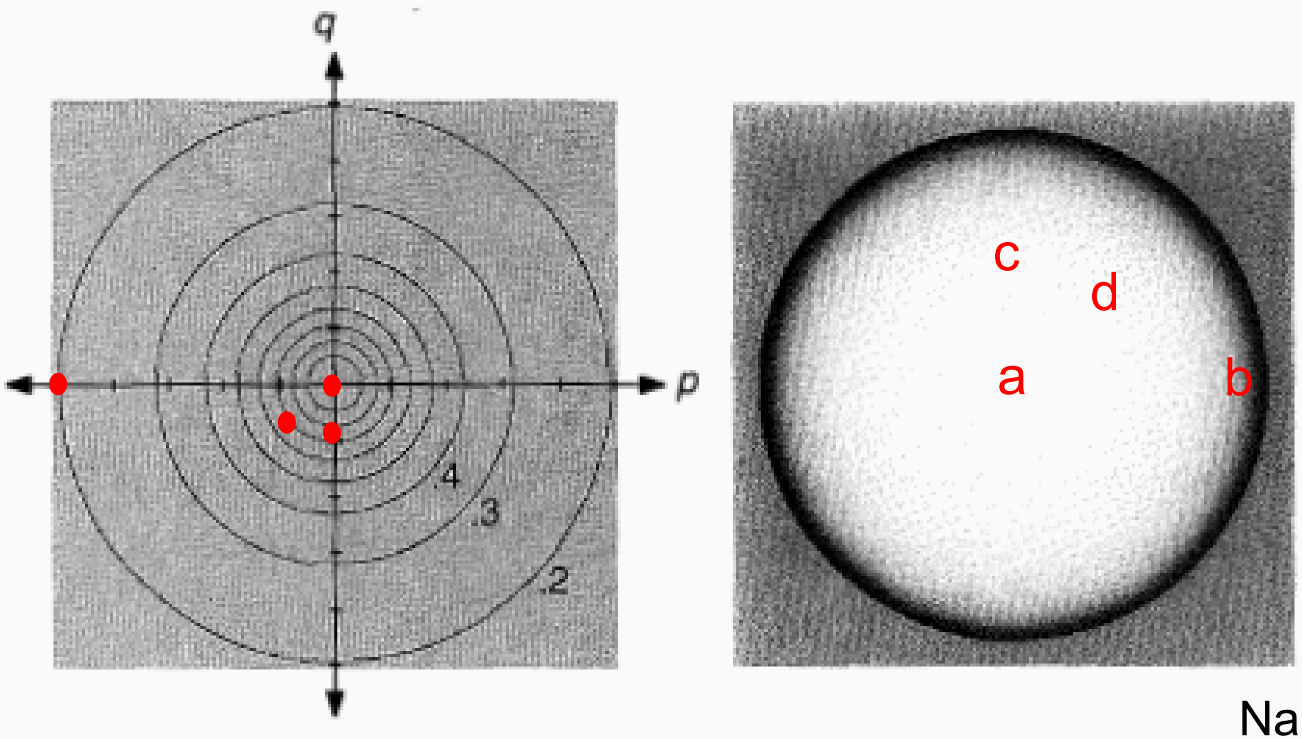
$E=\hat{n}.s=(0,0.1)(0,0,1)=1$

$E=\hat{n}.s=(5,0,1)/sqrt(25+1).(0,0,1)=0.2$

$E=\hat{n}.s=(0,1,1)/sqrt(1+1).(0,0,1)\sim 0.707$

$E=\hat{n}.s=(1/sqrt(2),1/sqrt(2),1)/sqrt(2*0.5+1).s=0.707$

R(p,q) of Lambertian Surface:
Here: Light Source near Viewer



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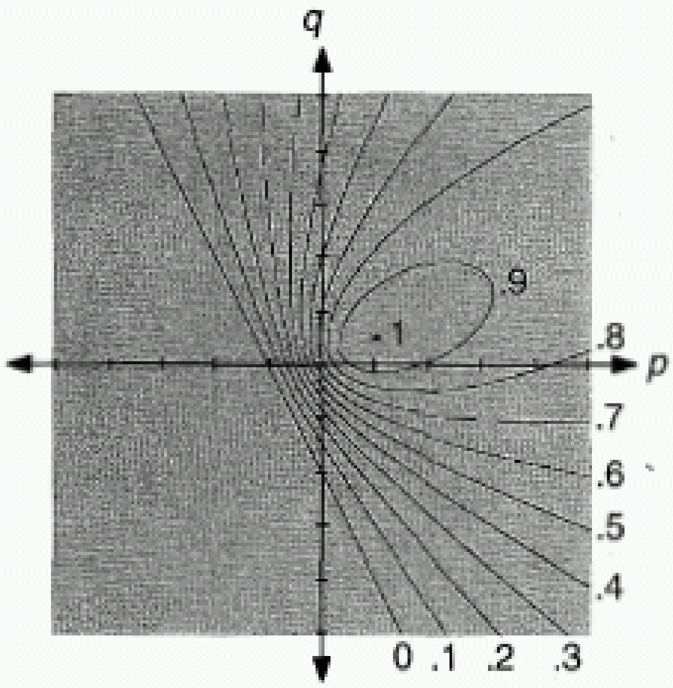
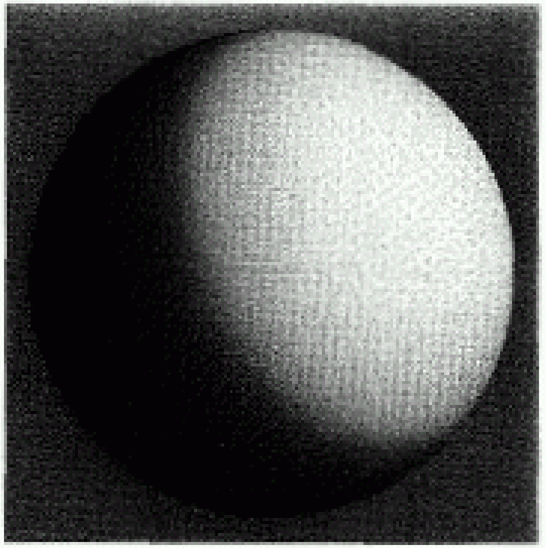
Where on the sphere is	How bright?
(a) p=0, q=0	E= 1
(b) p=-5, q=0	E= 0.2
(c) p=0, q=-1	E= 0.707
(d) p=-1, q=-1	E= 0.707

R(p,q) of Lambertian Surface:
Here: Light Source top right





Nalwa '93

Where is the brightest spot on the sphere?

R(p,q) of Lambertian Surface: Here: Light Source top right

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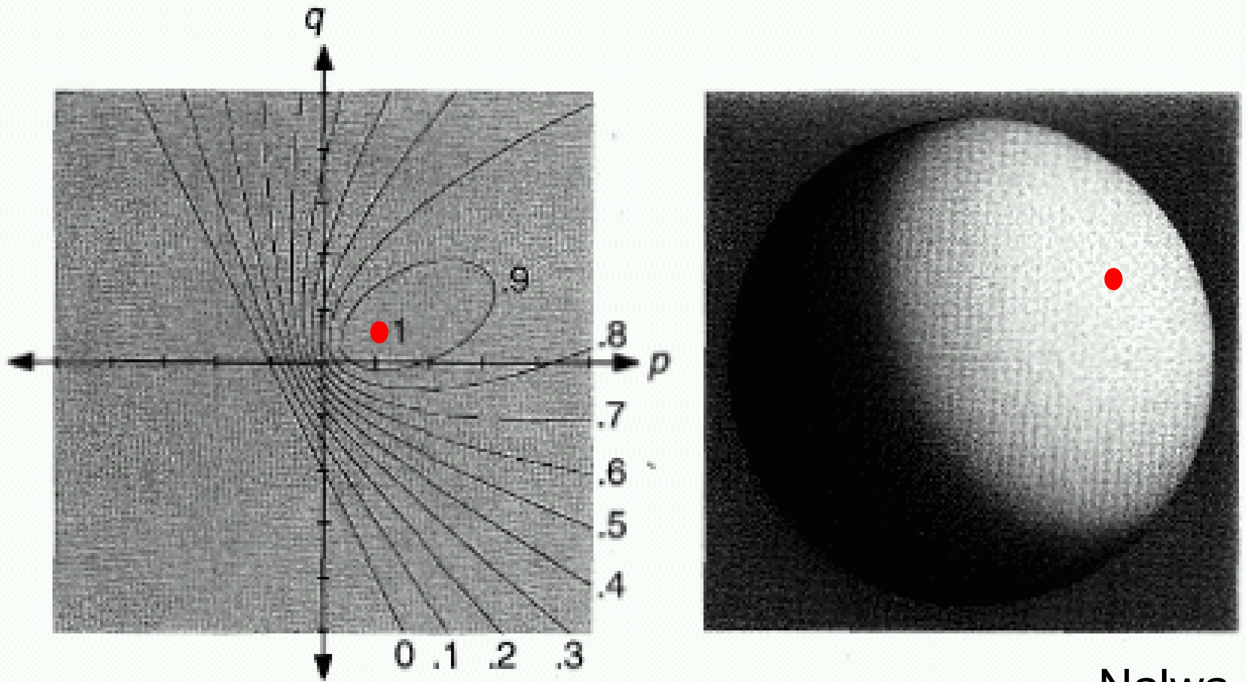

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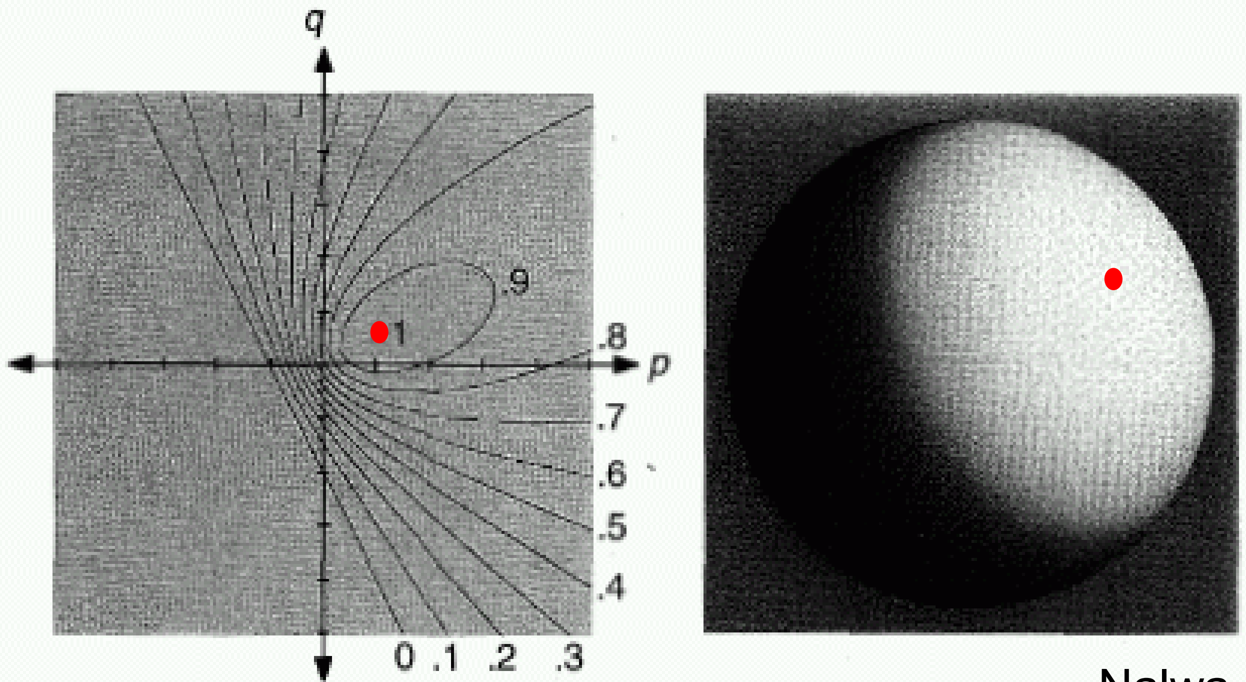
$\mathbf{s}=(-1,-0.5,1)^T$

Where is the brightest spot on the sphere?

$E=1=\hat{\mathbf{n}}.\hat{\mathbf{s}}=(p^2+q^2+1^2)/(p^2+q^2+1^2)$

R(p,q) of Lambertian Surface: Here: Light Source top right



Nalwa '93

$\mathbf{s}=(-1,-0.5,1)^T$

Where are the points on the sphere with brightness E=0.707?

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R(p,q) of Lambertian Surface: Here: Light Source top right

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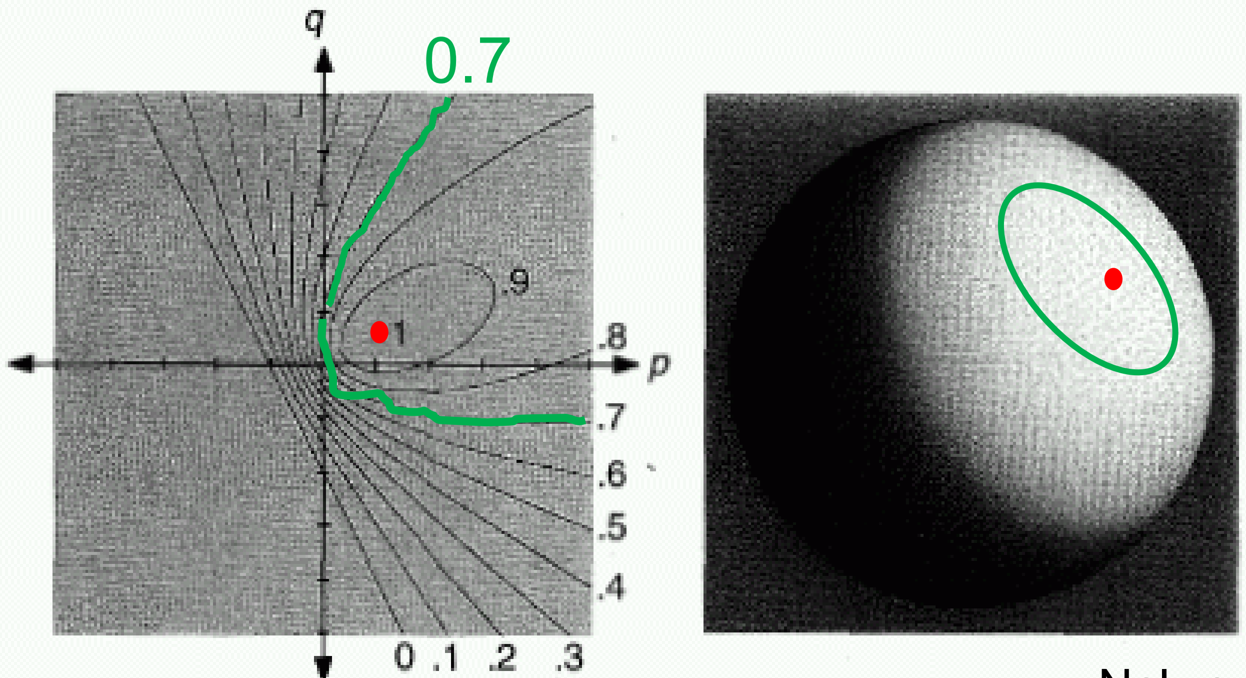


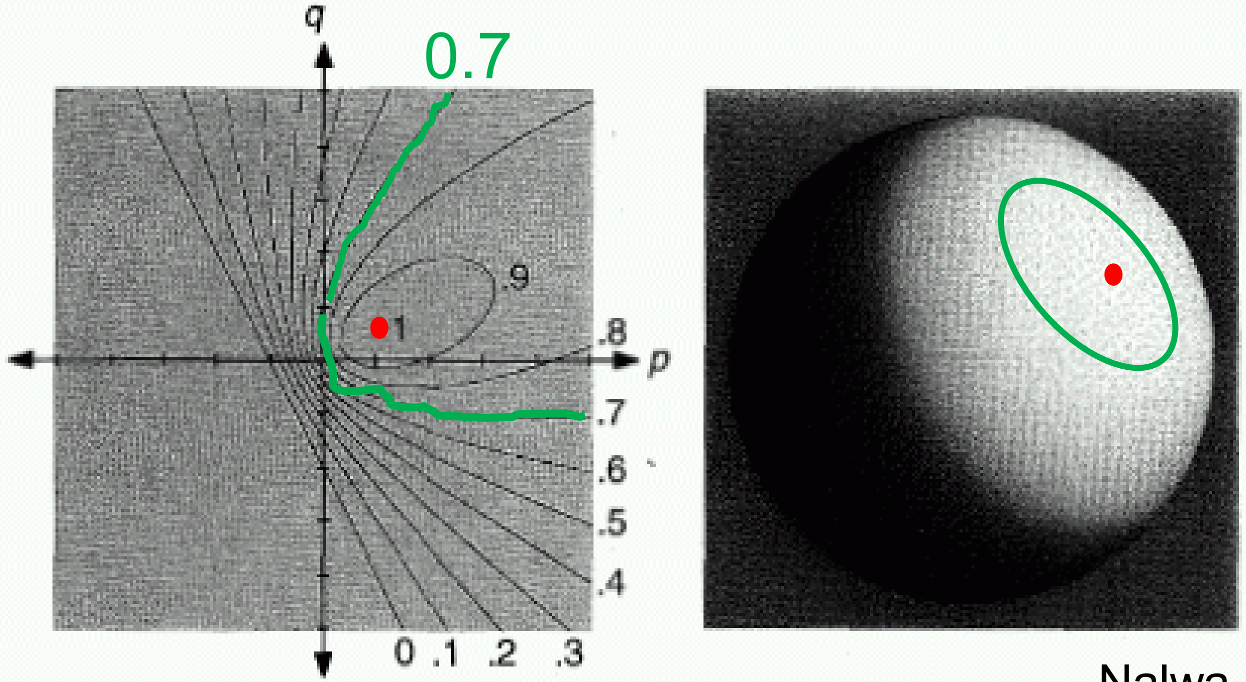
Nalwa '93

$\mathbf{s}=(-1,-0.5,1)^T$

Where are the points on the sphere with brightness E=0.707?

$$E=\hat{\mathbf{n}}.\hat{\mathbf{s}}=(-p,-q,1)/\mathrm{sqrt}(p^2+q^2+1^2).\mathbf{s}=\cos(45^o)$$

R(p,q) of Lambertian Surface: Here: Light Source top right



Nalwa '93

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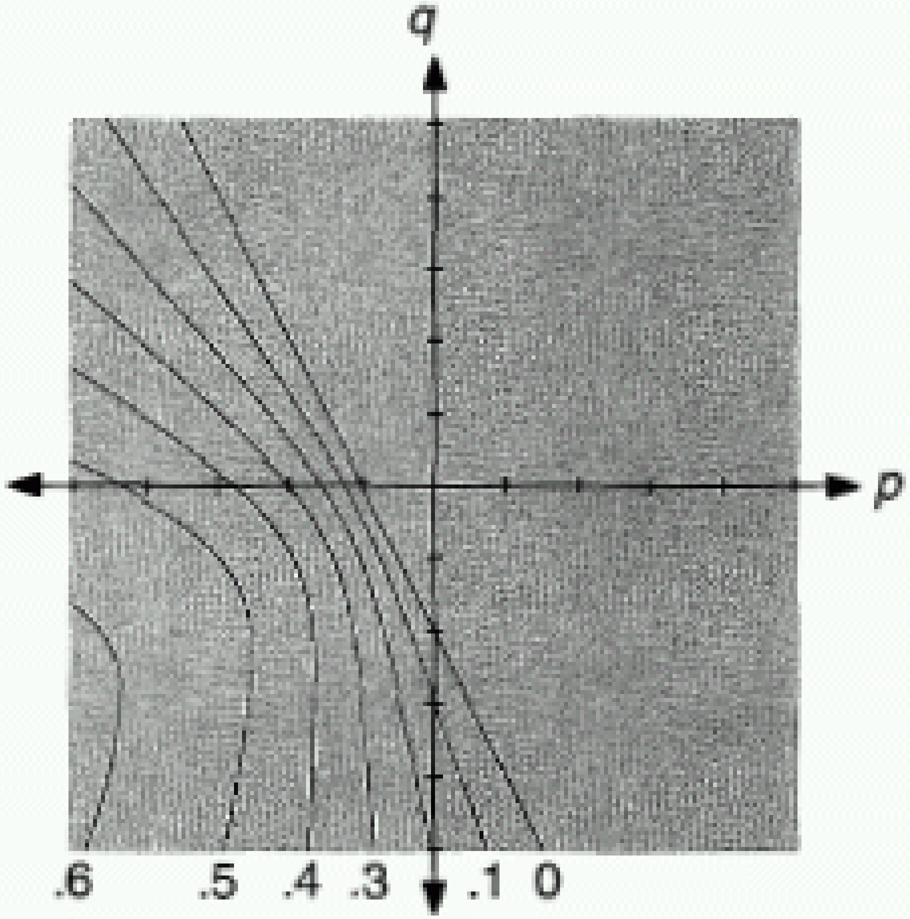
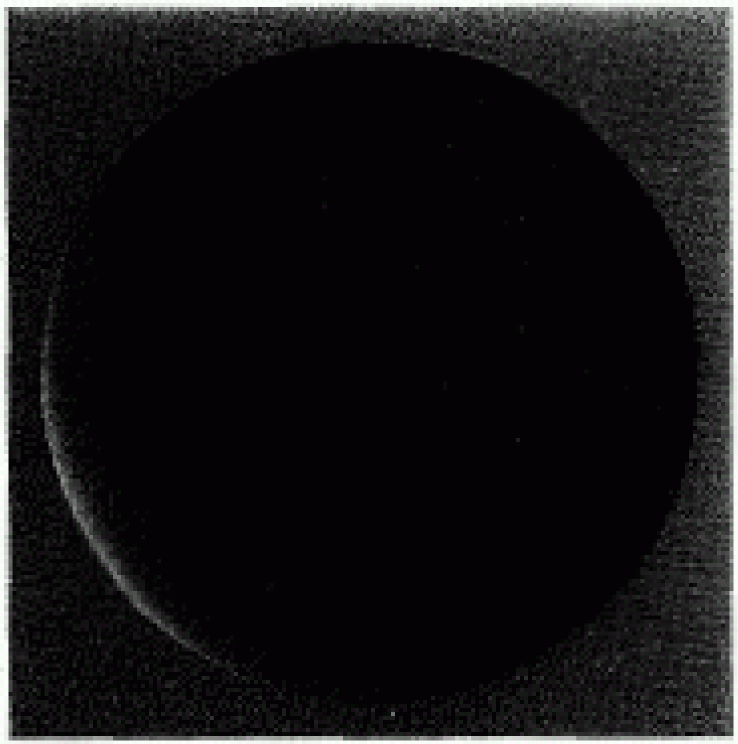
$E=\hat{\mathbf{n}}.\hat{\mathbf{s}}=(-p,-q,1)/\mathrm{sqrt}(p^2+q^2+1^2).\mathbf{s}=\cos(45^o)$

R(p,q) of Lambertian Surface





Nalwa '93

Reflectance Maps

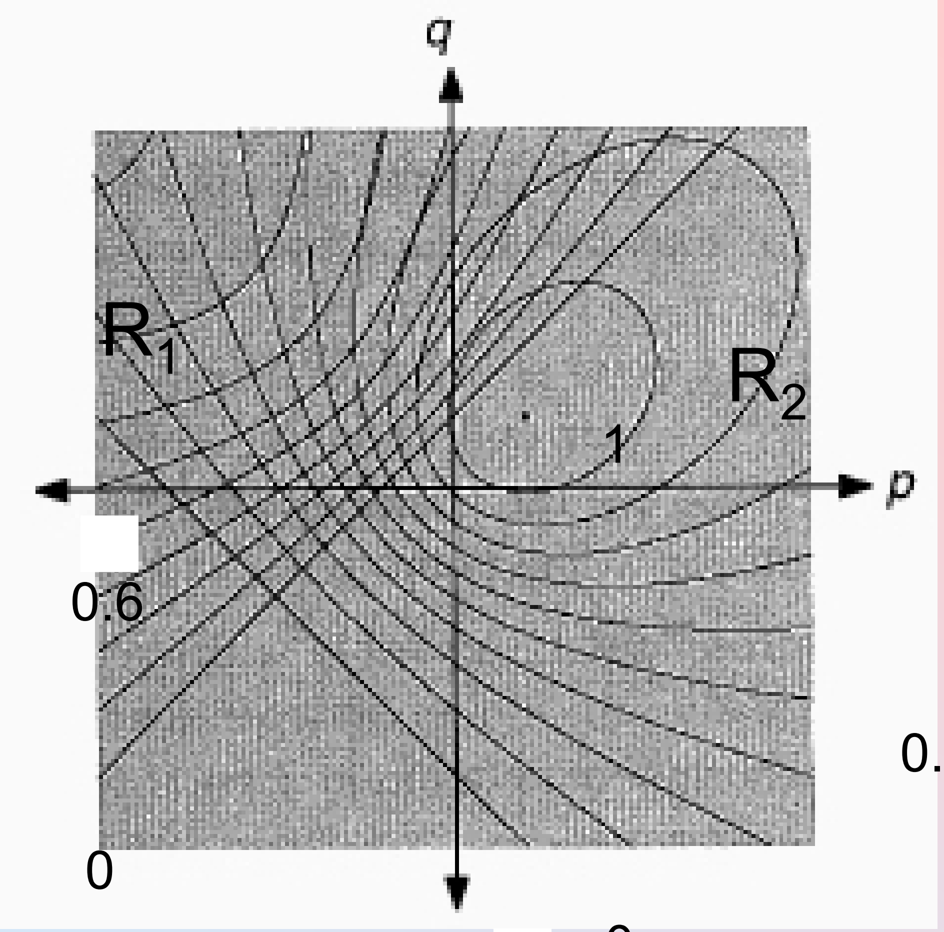
Computer Science

- How to obtain reflectance maps? Library or own experiment.
- One reflectance map per light source direction.
- For teaching purposes, we used a sphere in the previous slides. The goal is not to reconstruct the surface of a sphere but the unknown surface of a planet or face etc. The reflectance maps are valid for any object with Lambertian surface reflectance properties.
- Algorithms use the reflectance maps by looking up p and q. Input: brightness E, Output: $n=(-p,-q,1)^T$

Photometric Stereo

Computer Science

Goal: Given images E_1 and E_2 under 2 lighting conditions (p_1,q_1) and (p_2,q_2), find surface orientation $\mathbf{n} = (-p,-q,1)^T$, i.e., find p & q.



2 nonlinear equations:

$E_1 = R_1(p,q)$

$E_2 = R_2(p,q)$

If $(p_1,q_1) = (p_2,q_2)$
infinite number of solutions
else 0, 1, or 2 solution(s)

Better, use *N* images & least-squares method

LSM for Photometric Stereo

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Number of images = source directions = *i*

Gray value at a specific pixel in *i*th image: E_i

$$\min_{\mathbf{n}} \sum_{i=1}^{n} = (\mathbf{n}.\mathbf{s}_i - E_i)^2$$

Take derivative with respect to unknown normal ***n*** of surface patch imaged at this pixel

$$2\sum_{i=1}^{n}(\mathbf{n}.\mathbf{s}_i - E_i)\frac{d(\mathbf{n}.\mathbf{s}_i - E_i)}{d\mathbf{n}} = 0$$

How do you take a derivative of a dot product with respect to a vector?

$$\frac{d}{d\mathbf{a}}(\mathbf{a}.\mathbf{b}) = \mathbf{b}$$

Solve this linear equation for surface normal ***n***:

$$\sum_{i=1}^{n}(\mathbf{n}.\mathbf{s}_i - E_i)\mathbf{s}_i = 0$$

$$\sum_{i=1}^{n}(\mathbf{n}.\mathbf{s}_i)\mathbf{s}_i = \sum_{i=1}^{n} E_i\mathbf{s}_i$$

$$\sum_{i=1}^{n}(\mathbf{s}_i\mathbf{s}_i^T)\mathbf{n} = \sum_{i=1}^{n} E_i\mathbf{s}_i$$

outer product

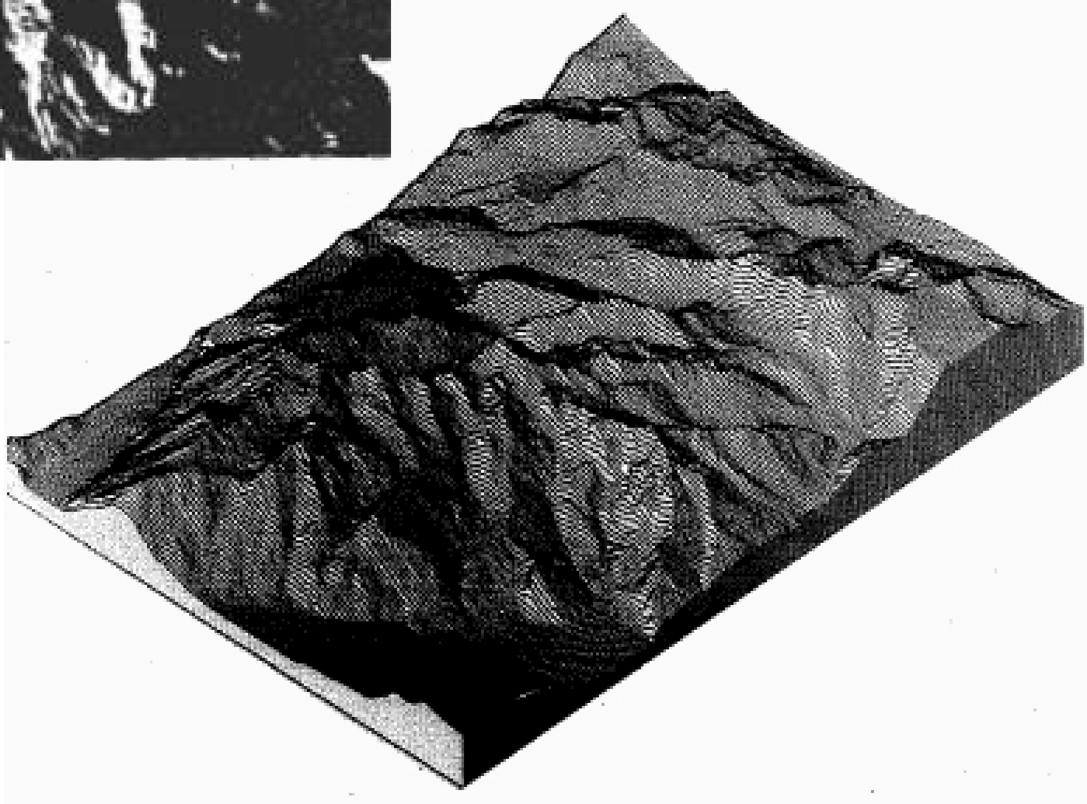
$$S\mathbf{n} = \mathbf{E}$$

$$\mathbf{n} = S^{-1}\mathbf{E}$$

Mars

Viking
Lander I
1977





Shape from Shading

Computer Science

Find 3D shape in scene from a single 2D image

Horn's Algorithm, 1989

Least-squares method:

Minimize sum of squared error

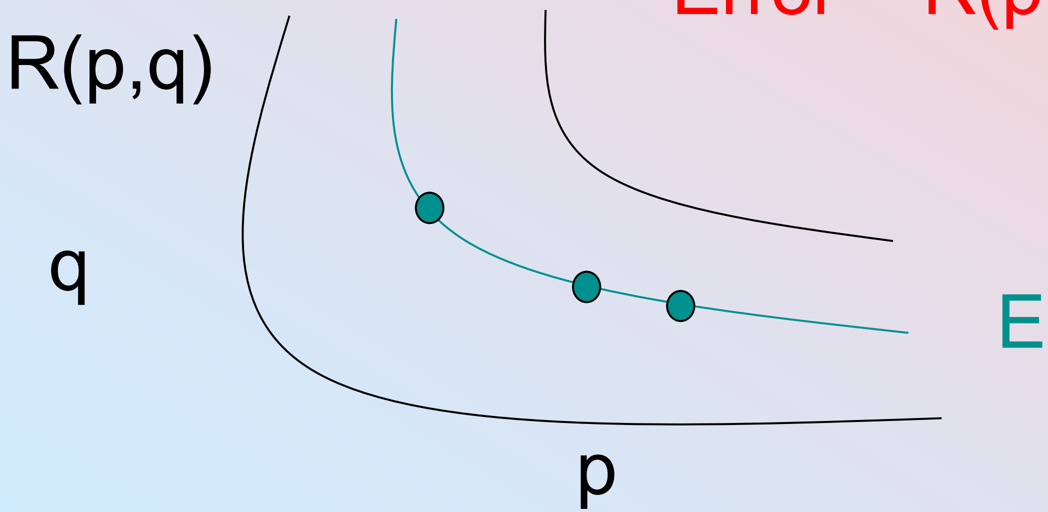
Derivation of error function?

Brightness Constraint

The measured brightness E should not vary much from the brightness R(p,q) at each pixel (i,j).

➔ Values for p and q should lie on the isobrightness curve labeled with E in reflectance map R.

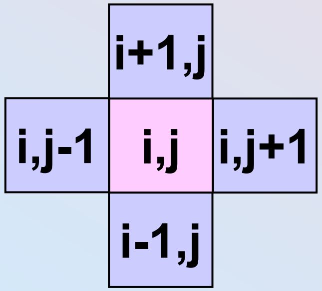
Error = R(p,q) – E



Smoothness Constraint

Computer Science

The surface orientation, defined by surface normal $\mathbf{n}_{ij} = (-p_{ij}, -q_{ij}, 1)^T$, at a pixel (i,j) should not vary much from the surface orientation at neighboring pixels (i+1,j), (i,j+1), (i-1,j), (i,j-1).



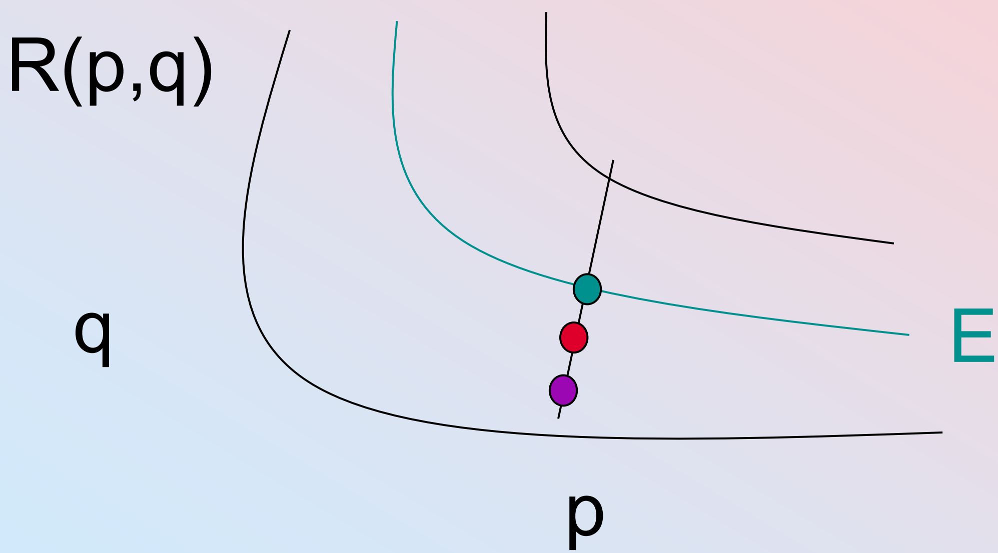
$$\text{Error}_x = p_{ij} - \underline{p}_{ij}$$

average

Combining Constraints

Computer Science

Use compromise of isobrightness solution and average-of-neighbors solution.



Combining Constraints

Computer Science

Combine two error measures, departure from

brightness: $Error_{brightness} \quad (E_{ij} - R(p_{ij},q_{ij}))^2$

smoothness: $Error_{smoothness} \quad (p_{ij} - \underline{p}_{ij})^2 + (q_{ij} - \underline{q}_{ij})^2$

where $\underline{p}$, $\underline{q}$ are local averages:

$$\underline{p} = 1/4\,(p_{i+1,j} + p_{i,j+1} + p_{i-1,j} + p_{i,j-1})$$
$$\underline{q} = 1/4\,(q_{i+1,j} + q_{i,j+1} + q_{i-1,j} + q_{i,j-1})$$

using regularization:

$$Error_{smoothness} + \lambda\, Error_{brightness}$$

Horn's Shape-from-Shading Algorithm

BOSTON UNIVERSITY
Computer Science

Minimization Problem:

$$\text{Error}_{\text{smoothness}} + \lambda\, \text{Error}_{\text{brightness}}$$

$$\min_{p_{kl},\, q_{kl}} \left\{ \Sigma \left((p_{ij} - \underline{p}_{ij})^2 + (q_{ij} - \underline{q}_{ij})^2 \right) + \lambda\, \Sigma \left(E_{ij} - R(p_{ij}, q_{ij}) \right)^2 \right\}$$

Solution: Iterative Scheme

$$p_{ij}^{(n+1)} = \underline{p}_{ij}^{(n)} - \lambda \left(E_{ij} - R(p_{ij}^{(n)}, q_{ij}^{(n)}) \right) \left(\frac{\partial R}{\partial p_{ij}} \right)^{(n)}$$

$$q_{ij}^{(n+1)} = \underline{q}_{ij}^{(n)} - \lambda \left(E_{ij} - R(p_{ij}^{(n)}, q_{ij}^{(n)}) \right) \left(\frac{\partial R}{\partial q_{ij}} \right)^{(n)}$$

Initialization of Algorithm

Computer Science

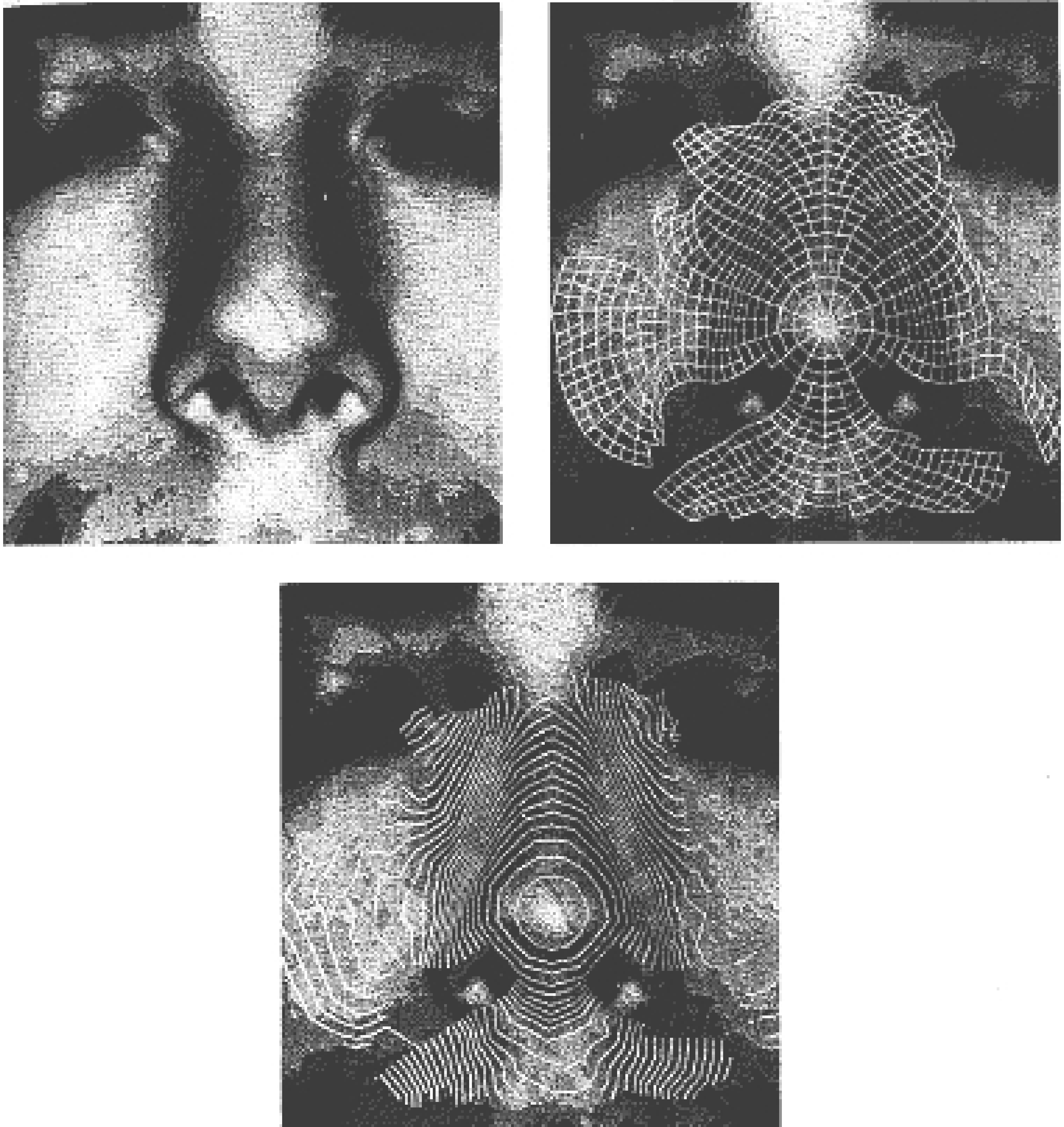
All non-boundary points:

$p_{ij}^{(0)} = 0$ and $q_{ij}^{(0)} = 0$

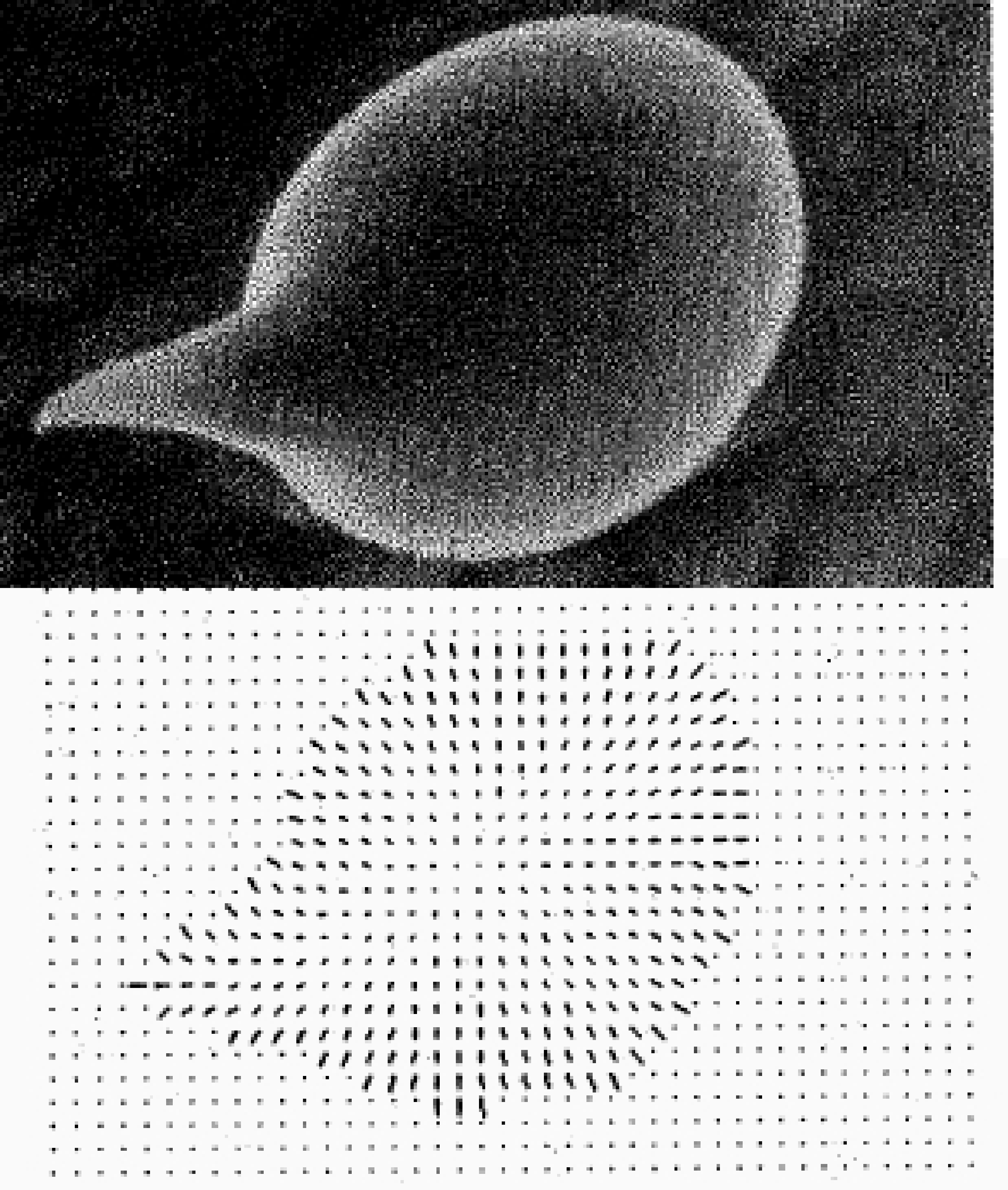
Silhouette boundaries with smooth edges:

Surface orientation is perpendicular to viewer's line of sight (optical axis of camera) and to silhouette.

All such surface orientations project onto circle in p-q-space.



Horn ‘86

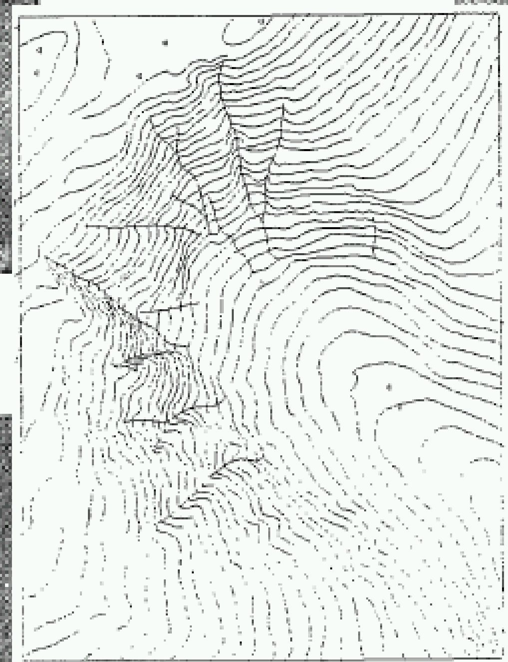


Microscope image and needle diagram of reconstructed surface of droplet of flower of *Cannabis sativa* plant

Horn ‘86

Input image

Ground truth 3D terrain



2^{nd} image, not input

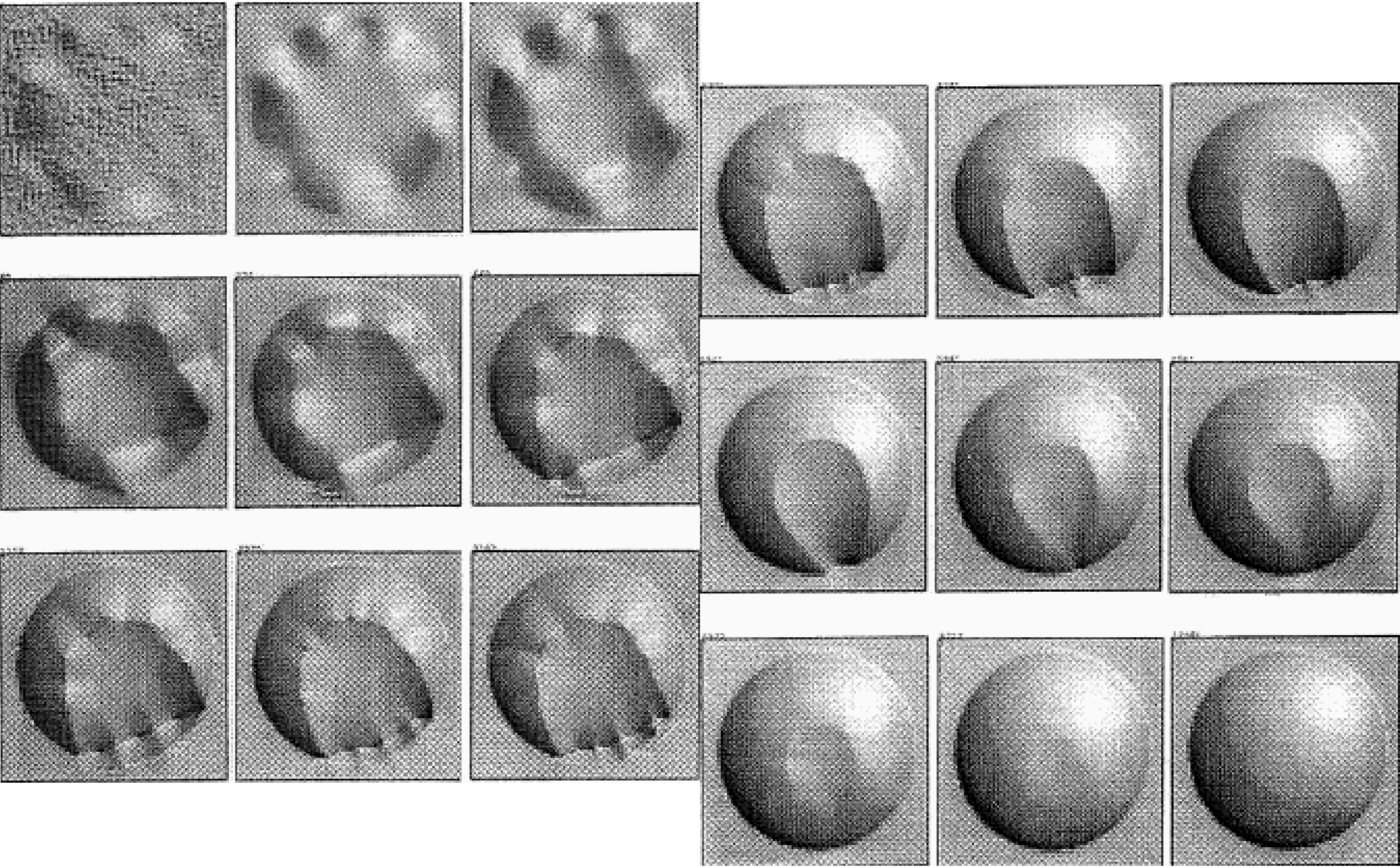
Output image

Pro-cessed output matches image above

Horn ‘90

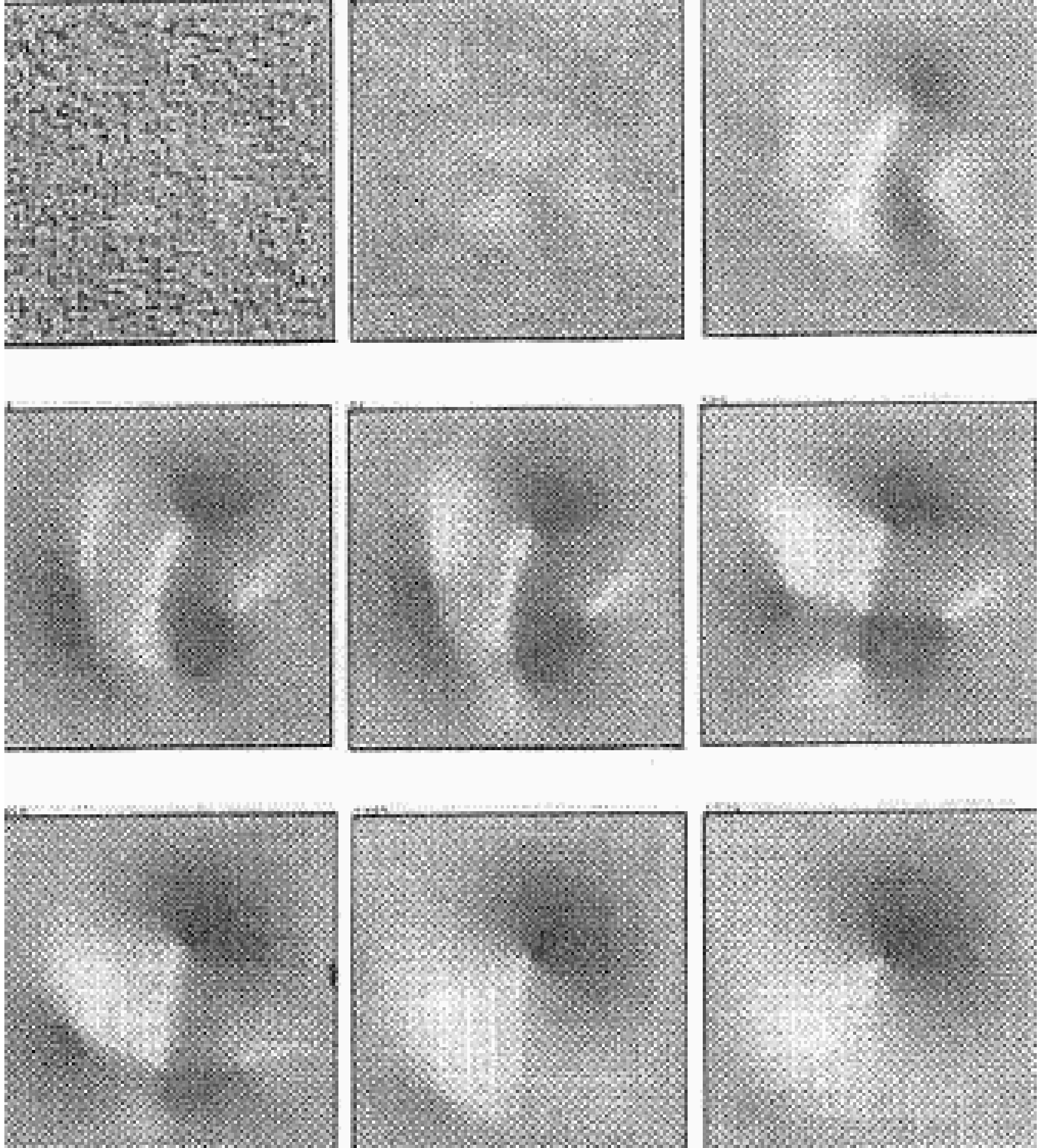
Horn '90

Shape-from-Shading iterative algorithm:



Spherical cap on plane

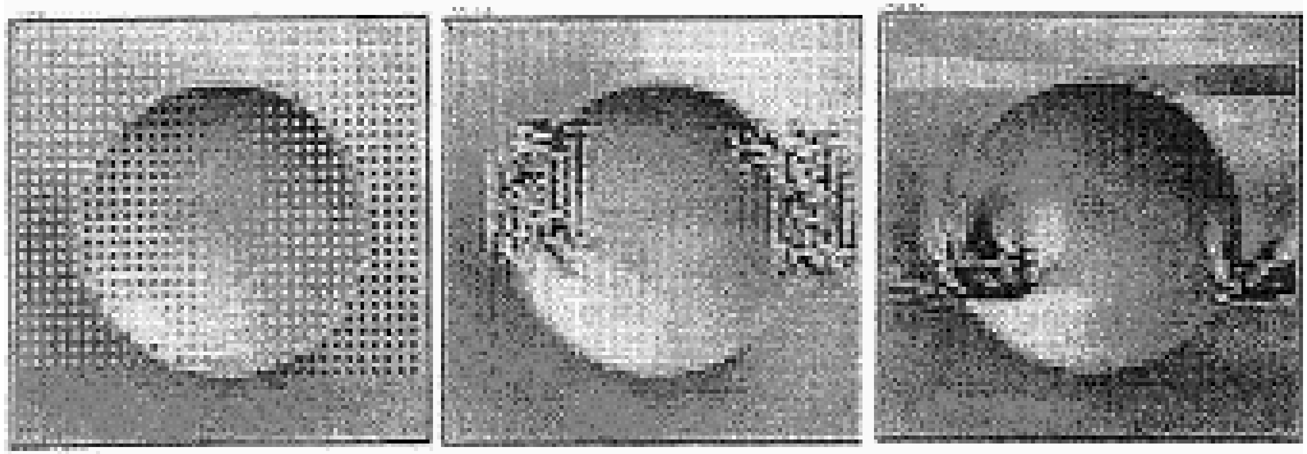
Horn '90



Conical singularity

Horn ‘90

Too rapid reduction of penalty term in smoothness



Various programming errors

Propagation of instability at edge of image when penalty term set to zero

Learning Objectives:

Computer Science

- *Shape from shading* is a heavily underconstrained problem. Solutions involve iterative schemes & careful attention to implementation & testing methodology.
- *Photometric Stereo* is a problem that can be solved with 2 input images but the more the better (LSM). Ensure sufficiently different light source directions.
- Connection to Computer Graphics: The *Lambertian surface reflectance model* is convenient (but sometimes not applied properly).
- Tools learned: Modeling reflectance properties, with BRDF, $E=\cos\theta_i$, reflectance map R(p,q), LSM, iterative method