Image and Video Computing

Photometric Stereo and Shape from Shading

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Problem Definitions

Shape from Shading
Find 3D shape in scene from a single 2D image

Photometric Stereo ≠ binocular stereo
Find 3D shape in scene from a set of 2D images that are taken under different lighting conditions

“stereo” = “solid” in Greek, used to refer to solidity, three-dimensionality

2 Algorithms
Photometric Stereo

Example:
Find 3D shape in scene from these images of faces

Image Credit: Adrian N. Evans
Photometric Stereo

3D shape visualized with texture from 1\textsuperscript{st} image

Image Credit: Adrian N. Evans
Approach

Light reflected at surface patch depends on
Light reflected at surface patch depends on

- surface orientation
- reflectance properties of surface
- distribution of light sources illuminating surface
Approach

Light reflected at surface patch depends on
- surface orientation
- reflectance properties of surface
- distribution of light sources illuminating surface

Reconstruction Method:
Determine surface reflectance properties and direction of light source(s)
Compute surface orientation

Connection to Computer Graphics?
Connection to Computer Graphics

Computer Vision: Image $E(x,y)$ given
- Determine surface reflectance properties of object in image
- Determine directions of light source & viewer
- Compute 3D surface orientation of object at each $z(x,y)$

Computer Graphics: 3D object shape $z(x,y)$ given
- Determine surface reflectance properties of object
- Determine directions of light source and viewer
- Find intersection of ray along viewer direction with surface point $z(x,y)$ (“ray tracing”)
- Create realistic-looking image $E(x,y)$ for $z(x,y)$
What is wrong here?

Light direction
Surface reflectance
What do we need to consider for image E to look right?

light source direction $s$

viewer direction $v$

Brightness $E = ?$
Surface-centered Definition

The brightness \( E \) of a surface can be defined as a function of the incident angle \( \theta_i \) and the emission angle \( \theta_e \) or as a function of the surface normal \( n \), the light source direction \( s \), and the viewer direction \( v \):

\[
E = \text{fct}(\theta_i, \theta_e) \quad \text{or} \quad \text{fct}(n, s, v)
\]
Surface Reflectance

\[ fct(n,s,v) = BRDF = \]

Bidirectional Reflectance Distribution Function

Three elemental components that can be used to model a variety of light-surface interactions. The incoming light ray is shown in black, the reflected ray(s) modeled by the BRDF in gray.

Image credit: Wikipedia, BRDF
Ideal Lambertian surface looks equally bright from all directions.

Brightness:

\[ E = f(\theta_i, \theta_e) \]

or

\[ E = \cos \theta_i = \hat{n} \cdot \hat{s} \]

Hat notation = unit length vector.
Examples of Lambertian Surfaces
Poll: Which objects are Lamber-
tian?

Image credit: Fleming, 2013
Lambertian reflectance model used:
Reflectance Properties of Moon

Lambertian?  No, moon does not look like sphere, but like flat disk

Lommel-Seeliger

E = \cos \theta_i = n.s
Brightness depends on longitude and latitude

E = \sqrt{\frac{\cos \theta_i}{\cos \theta_e}} = \frac{n.s}{n.v}
Brightness depends only on longitude
Surface Orientation

surface

surface normal \( \mathbf{n} \)

tangent plane

optical axis

\( x \)

\( y \)

\( z \)
Surface Orientation

Optical axis z

Surface Gradient \((z_x, z_y)^T\)

\[ a = (\delta x, 0, z_x \delta x)^T = \delta x (1, 0, z_x)^T \]

\[ b = (0, \delta y, z_y \delta y)^T = \delta y (0, 1, z_y)^T \]

\[ n \parallel (a \times b) \]

\[ n = (-z_x, -z_y, 1)^T = (-p, -q, 1)^T \]
Reflectance Map

Two different projections can create maps of the surface gradients on “Gaussian” (or unit) sphere:

Stereographic plane:
  Whole sphere is projected
  Includes occluding boundary of sphere

Reflectance map:  -- we’ll use this projection
  Upper hemisphere of sphere is projected
  Isobrightness lines extend to infinity
A reflectance map $R(p,q)$ is a function that gives scene radiance as a function of surface orientation.

Scene radiance = light reflected from surface patch and measured by camera. Given at a pixel (which is the image of the center of the patch) as a normalized gray value $[0..1]$.

Surface orientation $\mathbf{n} = (-p,-q,1)^T$

$\mathbf{n} = (-p,-q,1)/\sqrt{p^2+q^2+1}$
Reflectance map of matte surface

Ideal Lambertian surface looks equally bright from all directions

$$\cos \theta = \mathbf{n} \cdot \mathbf{s}$$

Example:

- Light source near viewer

$$\mathbf{v} = \mathbf{s}$$

$$\mathbf{n} = (-p, -q, 1)^T$$
Where on the sphere are all points such that

(a) $E=1$  ?
(b) $E=0.2$  ?
(c) $E=0.7$  ?
Where on the sphere are all points such that

(a) $E = 1$ because brightest spot on sphere is facing viewer = source direction

$$\cos \theta = \cos 0^\circ = 1$$
Where on the sphere are all points such that

(a) $E = 1$ because brightest spot on sphere is facing viewer = source direction

$$\cos \theta = \hat{n} \cdot \hat{s} = (-p, -q, 1)/\sqrt{p^2 + q^2 + 1}.(0, 0, 1) = \cos 0^\circ = 1$$
Where on the sphere are all points such that

(a) $E = 1$ because brightest spot on sphere is facing viewer = source direction

$$\cos \theta = \hat{n} \cdot \hat{s} = (-p, -q, 1)/\sqrt{p^2 + q^2 + 1}.(0, 0, 1) = \cos 0^\circ = 1$$

(a) $E = 0.2$ ?
(b) $E = 0.7$ ?
Where on the sphere are all points such that
(a) $E = 1$ because $\cos(0^\circ) = 1$
(b) $E = 0.2$ $\cos(78.5^\circ) = 0.2$
(c) $E = 0.7$ $\cos(45^\circ) \approx 0.7$
Where on the sphere are all points such that

(a) $E = 1$ because $\cos(0^\circ) = 1$
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(b) $E=0.2$ $\cos(78.5^\circ)=0.2$
(c) $E=0.7$ $\cos(45^\circ)\approx0.7$
Where on the sphere is $R(p,q)$ of Lambertian Surface:
Here: Light Source near Viewer

<table>
<thead>
<tr>
<th>Where on the sphere</th>
<th>How bright?</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) $p=0$, $q=0$</td>
<td>$E=$</td>
</tr>
<tr>
<td>(b) $p=-5$, $q=0$</td>
<td>$E=$</td>
</tr>
<tr>
<td>(c) $p=0$, $q=-1$</td>
<td>$E=$</td>
</tr>
<tr>
<td>(d) $p=-0.707$, $q=-0.707$</td>
<td>$E=$</td>
</tr>
</tbody>
</table>
Where on the sphere is $R(p,q)$

(a) $p=0$, $q=0$
(b) $p=-5$, $q=0$
(c) $p=0$, $q=-1$
(d) $p=-0.707$, $q=-0.707$

How bright?

$E = \hat{n} \cdot s = (0,0.1)(0,0,1) = 1$
R(p,q) of Lambertian Surface:
Here: Light Source near Viewer

Where on the sphere is How bright?
(a) p=0, q=0 E=\hat{n}.s=(0,0.1)(0,0,1)=1
(b) p=-5, q=0 E=\hat{n}.s=(5,0,1)/\sqrt{25+1}.(0,0,1)=0.2
(c) p=0, q=-1?
(d) p=-0.707, q=-0.707?
R\((p,q)\) of Lambertian Surface:
Here: Light Source near Viewer

Where on the sphere is

(a) \(p=0, q=0\)
(b) \(p=-5, q=0\)
(c) \(p=0, q=-1\)
(d) \(p=-0.707, q=-0.707\)

How bright?

\[ \hat{E} = \hat{n} \cdot \hat{s} = (0,0.1)(0,0,1) = 1 \]
\[ \hat{E} = \hat{n} \cdot \hat{s} = (5,0,1)/\sqrt{25+1} \cdot (0,0,1) = 0.2 \]
\[ \hat{E} = \hat{n} \cdot \hat{s} = (0,1,1)/\sqrt{1+1} \cdot (0,0,1) \approx 0.707 \]

© Betke
Where on the sphere is
(a) \(p=0, q=0\)
(b) \(p=-5, q=0\)
(c) \(p=0, q=-1\)
(d) \(p=-0.707, q=-0.707\)

How bright?
\[
E = \hat{n} \cdot s = (0,0,1)(0,0,1) = 1
\]
\[
E = \frac{(5,0,1)}{\sqrt{25+1}} \cdot (0,0,1) = 0.2
\]
\[
E = \frac{(0,1,1)}{\sqrt{1+1}} \cdot (0,0,1) \approx 0.707
\]
\[
E = \frac{(1/\sqrt{2},1/\sqrt{2},1)}{\sqrt{2*0.5+1}} \cdot s = 0.707
\]
R(p,q) of Lambertian Surface:
Here: Light Source near Viewer

Where on the sphere is
(a) p=0, q=0
(b) p=-5, q=0
(c) p=0, q=-1
(d) p=-1, q=-1

How bright?
E= 1
E= 0.2
E= 0.707
E= 0.707
Where is the brightest spot on the sphere?

R(p,q) of Lambertian Surface:
Here: Light Source top right

Nalwa '93
Where is the brightest spot on the sphere?

\[ E = 1 = \hat{n} \cdot \hat{s} = \frac{p^2 + q^2 + 1^2}{(p^2 + q^2 + 1^2)} \]

\[ \hat{s} = (-1, -0.5, 1)^T \]
Where are the points on the sphere with brightness $E=0.707$?

$s = (-1, -0.5, 1)^T$

Nalwa ‘93
Where are the points on the sphere with brightness $E=0.707$?

$E = \frac{n \cdot s}{\sqrt{p^2 + q^2 + 1^2}}$

$s = (-1, -0.5, 1)^T$

$s = \cos(45^\circ)$
Where are the points on the sphere with brightness $E=0.707$?

$E = \frac{n \cdot s}{\sqrt{p^2 + q^2 + 1^2}}$

$E = (-p, -q, 1)/\sqrt{p^2 + q^2 + 1^2}$.

$s = (\cos(45^\circ), 0, 1)^T$

$s = (-1, -0.5, 1)^T$
R(p,q) of Lambertian Surface

Nalwa ‘93
Reflectance Maps

- How to obtain reflectance maps? Library or own experiment.
- One reflectance map per light source direction.
- For teaching purposes, we used a sphere in the previous slides. The goal is not to reconstruct the surface of a sphere but the unknown surface of a planet or face etc. The reflectance maps are valid for any object with Lambertian surface reflectance properties.
- Algorithms use the reflectance maps by looking up p and q. Input: brightness E, Output: $n = (-p, -q, 1)^T$. © Betke
Photometric Stereo

Goal: Given images $E_1$ and $E_2$ under 2 lighting conditions $(p_1,q_1)$ and $(p_2,q_2)$, find surface orientation $\mathbf{n} = (-p,-q,1)^T$, i.e., find $p$ & $q$.

2 nonlinear equations:
$E_1 = R_1(p,q)$
$E_2 = R_2(p,q)$

If $(p_1,q_1) = (p_2,q_2)$
   infinite number of solutions
else 0, 1, or 2 solution(s)

Better, use $N$ images & least-squares method
Number of images = source directions = $i$

Gray value at a specific pixel in $i$th image: $E_i$

Solve this linear equation for surface normal $n$:

$$\sum_{i=1}^{n} (n \cdot s_i - E_i) s_i = 0$$

Taking derivative with respect to unknown normal $n$ of surface patch imaged at this pixel:

$$2 \sum_{i=1}^{n} (n \cdot s_i - E_i) \frac{d(n \cdot s_i - E_i)}{dn} = 0$$

How do you take a derivative of a dot product with respect to a vector?

$$\frac{d}{da} (a \cdot b) = b$$

$$Sn = E$$

$$n = S^{-1}E$$
Mars

Viking Lander I
1977
Shape from Shading

Find 3D shape in scene from a single 2D image

Horn's Algorithm, 1989

Least-squares method:
Minimize sum of squared error

Derivation of error function?
The measured brightness $E$ should not vary much from the brightness $R(p,q)$ at each pixel $(i,j)$.

$\Rightarrow$ Values for $p$ and $q$ should lie on the isobrightness curve labeled with $E$ in reflectance map $R$.

$\text{Error} = R(p,q) - E$
The surface orientation, defined by surface normal $\mathbf{n}_{ij} = (-p_{ij},-q_{ij},1)^T$, at a pixel $(i,j)$ should not vary much from the surface orientation at neighboring pixels $(i+1,j)$, $(i,j+1)$, $(i-1,j)$, $(i,j-1)$.

$$\text{Error}_x = p_{ij} - \bar{p}_{ij}$$
Combining Constraints

Use compromise of isobrightness solution and average-of-neighbors solution.

\[ R(p, q) \]
Combining Constraints

Combine two error measures, departure from

brightness: \( \text{Error}_{\text{brightness}} \left( E_{ij} - R(p_{ij}, q_{ij}) \right)^2 \)

smoothness: \( \text{Error}_{\text{smoothness}} \left( p_{ij} - p_{ij} \right)^2 + \left( q_{ij} - q_{ij} \right)^2 \)

where \( p, q \) are local averages:

\[
\begin{align*}
p &= 1/4 \left( p_{i+1,j} + p_{i,j+1} + p_{i-1,j} + p_{i,j-1} \right) \\
q &= 1/4 \left( q_{i+1,j} + q_{i,j+1} + q_{i-1,j} + q_{i,j-1} \right)
\end{align*}
\]

using regularization:

\[
\text{Error}_{\text{smoothness}} + \lambda \text{Error}_{\text{brightness}}
\]
Horn’s Shape-from-Shading Algorithm

Minimization Problem:

$$\text{Error}_{\text{smoothness}} + \lambda \text{Error}_{\text{brightness}}$$

$$\min \{ \sum ((p_{ij} - p_{ij})^2 + (q_{ij} - q_{ij})^2) + \lambda \sum (E_{ij} - R(p_{ij}, q_{ij}))^2 \}$$

$$p_{kl}, q_{kl}$$

Solution: Iterative Scheme

$$p_{ij}^{(n+1)} = p_{ij}^{(n)} - \lambda (E_{ij} - R(p_{ij}^{(n)}, q_{ij}^{(n)})) \frac{\partial R}{\partial p_{ij}}^{(n)}$$

$$q_{ij}^{(n+1)} = q_{ij}^{(n)} - \lambda (E_{ij} - R(p_{ij}^{(n)}, q_{ij}^{(n)})) \frac{\partial R}{\partial q_{ij}}^{(n)}$$
Initialization of Algorithm

All non-boundary points:
\[ p_{ij}^{(0)} = 0 \text{ and } q_{ij}^{(0)} = 0 \]

Silhouette boundaries with smooth edges:
Surface orientation is perpendicular to viewer’s line of sight (optical axis of camera) and to silhouette. All such surface orientations project onto circle in p-q-space.
Microscope image and needle diagram of reconstructed surface of droplet of flower of *Cannabis sativa* plant

Horn ‘86
Input image

Ground truth
3D terrain

Output image

2nd image, not input

Processed output matches image above

Horn '90
Shape-from-Shading iterative algorithm:

Spherical cap on plane
Too rapid reduction of penalty term in smoothness

Various programming errors

Propagation of instability at edge of image when penalty term set to zero
Learning Objectives:

- **Shape from shading** is a heavily underconstrained problem. Solutions involve iterative schemes & careful attention to implementation & testing methodology.

- **Photometric Stereo** is a problem that can be solved with 2 input images but the more the better (LSM). Ensure sufficiently different light source directions.

- Connection to Computer Graphics: The *Lambertian surface reflectance model* is convenient (but sometimes not applied properly).

- Tools learned: Modeling reflectance properties, with BRDF, $E=\cos\theta_i$, reflectance map $R(p,q)$, LSM, iterative method