

## Image and Video Computing

## Photometric Stereo and Shape from Shading

Lecture by Margrit Betke April 4, 2024

## **Problem Definitions**



Computer Science

#### Shape from Shading

Find 3D shape in scene from a single 2D image

#### Photometric Stereo ≠ binocular stereo Find 3D shape in scene from a set of 2D images that are taken under different lighting conditions

"stereo" = "solid" in Greek, used to refer to solidity, three-dimensionality

#### 2 Algorithms





#### Example: Find 3D shape in scene from these images of faces



Image Credit: Adrian N. Evans





#### 3D shape visualized with texture from 1<sup>st</sup> image



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#### Image Credit: Adrian N. Evans





#### Light reflected at surface patch depends on





#### Light reflected at surface patch depends on

- surface orientation
- reflectance properties of surface
- distribution of light sources illuminating surface





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#### **Reconstruction Method:**

Determine surface reflectance properties and direction of light source(s)

Compute surface orientation

**Connection to Computer Graphics?** 

**Connection to Computer Graphics** 



Computer Science

#### Computer Vision: Image E(x,y) given

- Determine surface reflectance properties of object in image
- Determine directions of light source & viewer
- Compute 3D surface orientation of object at each z(x,y)

Computer Graphics: 3D object shape z(x,y) given

- Determine surface reflectance properties of object
- Determine directions of light source and viewer
- Find intersection of ray along viewer direction with surface point z(x,y) ("ray tracing")
- Create realistic-looking image E(x,y) for z(x,y)



#### What is wrong here?

Light direction Surface reflectance



Image credit: Marauder 09

# What do we need to consider for image E to look right?





## **Surface-centered Definition**





## **Surface Reflectance**



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#### fct (n,s,v) = BRDF = Bidirectional Reflectance Distribution Function



Three elemental components that can be used to model a variety of light-surface interactions.<sup>[8]</sup> The incoming light ray is shown in black, the reflected ray(s) modeled by the BRDF in gray.

#### Image credit: Wikipedia, BRDF

# Diffuse reflecting surface = Lambertian



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Ideal Lambertian surface looks equally bright from all directions surface light source normal **n Brightness** θ direction s  $E = fct(\theta_i, \theta_e)$  $\theta_{e}$ or fct(n,s,v) tangent plane viewe  $E = \cos \theta_i = \hat{n}.\hat{s}$ directio Hat notation=unit length vector © Betke 14 surface



## **Examples of Lambertian Surfaces**



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#### (a)



(b)



(c)



(d)



Poll: Which objects are Lambertian?

Image credit: Fleming, 2013

#### (a)



Specular



(c)



(not quite ideal) Lambertian (d)





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Image credit: Fleming, 2013 © Betke

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#### Lambertian reflectance model used:

Ventune Marcury Joins Saluta Fath Cates mite Brant Mars

## **Reflectance Properties of Moon**



Lambertian? No, moon does not look like sphere, but like flat disk



Brightness depends on longitude and latitude

Lommel-Seeliger

$$\mathsf{E} = \sqrt{\frac{\cos \theta_{i}}{\cos \theta_{e}}} = \frac{\mathsf{n.s}}{\mathsf{n.v}}$$

Brightness depends only on longitude



## **Surface Orientation**









Two different projections can create maps of the surface gradients on "Gaussian" (or unit) sphere:

Stereographic plane: Whole sphere is projected Includes occluding boundary of sphere

Reflectance map: -- we'll use this projection Upper hemisphere of sphere is projected Isobrightness lines extend to infinity



A reflectance map R(p,q) is a function that gives scene radiance as a function of surface orientation.

Scene radiance = light reflected from surface patch and measured by camera. Given at a pixel (which is the image of the center of the patch) as a normalized gray value [0..1].

Surface orientation  $n=(-p,-q,1)^T$  $\stackrel{\wedge}{n}=(-p,-q,1)/sqrt(p^2+q^2+1)$ 

## Reflectance map of matte surface



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Where on the sphere are all points such that

- (a) E= 1 ?
- (b) E= 0.2 ?
- (c) E= 0.7 ?



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Where on the sphere are all points such that

(a) E= 1 because brightest spot on sphere is facing viewer = source direction  $\cos \theta = \cos 0^\circ = 1$ 



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(a) E= 0.2 ? (b) E= 0.7 ?



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Where on the sphere are all points such that

- (a) E=1 because  $cos(0^\circ)=1$
- (b) E=0.2  $\cos(78.5^{\circ})=0$
- (c) E= 0.7

cos(78.5°)=0.2 cos(45°)~0.7



Where on the sphere are all points such that

(a) E=1 because  $cos(0^{\circ})=1$ (b) E=0.2  $cos(78.5^{\circ})=0.2$ 

(c) E = 0.7  $\cos(45^{\circ}) \sim 0.7$ 



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Nalwa '93



Where on the sphere are all points such that

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Nalwa '93



Nalwa '93

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Where on the sphere is

- (a) p=0, q=0
- (b) p=-5, q=0
- (c) p=0, q=-1
- (d) p=-0.707, q=-0.707

- How bright?
- E=
- E=
- E=
  - E=



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Where on the sphere is

- (a) p=0, q=0
- (b) p=-5, q=0?
- (c) p=0, q=-1?
- (d) p=-0.707, q=-0.707?

How bright?  $E=n^{\Lambda}.s=(0,0.1)(0,0,1)=1$ 





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Where on the sphere is

- (a) p=0, q=0
- (b) p=-5, q=0
- (c) p=0, q=-1?
- (d) p=-0.707, q=-0.707?

How bright?  $E = \stackrel{\Lambda}{n.s} = (0,0.1)(0,0,1) = 1$  $E = \stackrel{\Lambda}{n.s} = (5,0,1)/sqrt(25+1).(0,0,1) = 0.2$ 





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Nalwa '93

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How bright?  $E = \stackrel{\Lambda}{n.s} = (0,0.1)(0,0,1) = 1$   $E = \stackrel{\Lambda}{n.s} = (5,0,1)/sqrt(25+1).(0,0,1) = 0.2$   $E = \stackrel{\Lambda}{n.s} = (0,1,1)/sqrt(1+1).(0,0,1) \sim 0.707$  $E = \stackrel{\Lambda}{n.s} = (1/sqrt(2),1/sqrt(2),1)/sqrt(2*0.5+1).s = 0.707$ 



Nalwa '93

Where on the sphere is

- (a) p=0, q=0
- (b) p=-5, q=0
- (c) p=0, q=-1
- (d) p=-1, q=-1

How bright? E= 1 E= 0.2 E= 0.707 E= 0.707



Where is the brightest spot on the sphere?



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#### **s**=(-1,-0.5,1)<sup>⊤</sup>

Where is the brightest spot on the sphere?  $E=1=\hat{n}\cdot\hat{s}=(p^2+q^2+1^2)/(p^2+q^2+1^2)$ 

> <mark>39</mark> © Betke



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#### **s**=(-1,-0.5,1)<sup>⊤</sup>

Where are the points on the sphere with brightness E=0.707?





Nalwa '93

Where are the points on the sphere with brightness E=0.707?  $E=\stackrel{\wedge}{\mathbf{n}}\stackrel{\circ}{\mathbf{s}}=(-p,-q,1)/sqrt(p^2+q^2+1^2).\mathbf{s}=cos(45^\circ)$ 





Where are the points on the sphere with brightness E=0.707?  $E=\stackrel{\wedge}{\mathbf{n}}\stackrel{\circ}{\mathbf{s}}=(-p,-q,1)/sqrt(p^2+q^2+1^2).\mathbf{s}=cos(45^\circ)$ 

## R(p,q) of Lambertian Surface



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## **Reflectance Maps**



Computer Science

- How to obtain reflectance maps? Library or own experiment.
- One reflectance map per light source direction.
- For teaching purposes, we used a sphere in the previous slides. The goal is not to reconstruct the surface of a sphere but the unknown surface of a planet or face etc. The reflectance maps are valid for any object with Lambertian surface reflectance properties.
- Algorithms use the reflectance maps by looking up p and q. Input: brightness E, Output: n=(-p,-q,1)<sup>T</sup>

## **Photometric Stereo**

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Goal: Given images  $E_1$  and  $E_2$  under 2 lighting conditions (p<sub>1</sub>,q<sub>1</sub>) and (p<sub>2</sub>,q<sub>2</sub>), find surface orientation  $\mathbf{n} = (-p,-q,1)^T$ , i.e., find p & q.

#### 2 nonlinear equations: $E_1 = R_1(p,q)$ $E_2 = R_2(p,q)$

If  $(p_1,q_1) = (p_2,q_2)$ infinite number of solutions else 0, 1, or 2 solution(s)

Better, use *N* images & least-squares method



0.6

## **LSM for Photometric Stereo**



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Number of images = source directions = iGray value at a specific pixel in *i*th image:  $E_i$ 

$$\min_{\mathbf{n}} \sum_{i=1}^{n} = (\mathbf{n}.\mathbf{s}_i - E_i)^2$$

Take derivative with respect to unknown normal *n* of surface patch imaged at this pixel

$$2\sum_{i=1}^{n} (\mathbf{n}.\mathbf{s}_i - E_i) \frac{d(\mathbf{n}.\mathbf{s}_i - E_i)}{d\mathbf{n}} = 0$$

How do you take a derivative of a dot product with respect to a vector?

$$\frac{d}{d\mathbf{a}}(\mathbf{a}.\mathbf{b}) = \mathbf{b}$$

Solve this linear equation for surface normal *n*:

$$\sum_{i=1}^{n} (\mathbf{n} \cdot \mathbf{s}_{i} - E_{i}) \mathbf{s}_{i} = 0$$
$$\sum_{i=1}^{n} (\mathbf{n} \cdot \mathbf{s}_{i}) \mathbf{s}_{i} = \sum_{i=1}^{n} E_{i} \mathbf{s}_{i}$$
$$\sum_{i=1}^{n} (\mathbf{s}_{i} \mathbf{s}_{i}^{T}) \mathbf{n} = \sum_{i=1}^{n} E_{i} \mathbf{s}_{i}$$
$$S \mathbf{n} = \mathbf{E}$$
outer product

 $\mathbf{n} = S^{-1}\mathbf{E}$ 



#### Viking Lander I 1977









## **Shape from Shading**



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Find 3D shape in scene from a single 2D image

Horn's Algorithm, 1989

Least-squares method: Minimize sum of squared error

**Derivation of error function?** 



- The measured brightness E should not vary much from the brightness R(p,q) at each pixel (i,j).
- Values for p and q should lie on the isobrightness curve labeled with E in reflectance map R.





The surface orientation, defined by surface normal  $\mathbf{n}_{ij} = (-\mathbf{p}_{ij}, -\mathbf{q}_{ij}, 1)^{T}$ , at a pixel (i,j) should not vary much from the surface orientation at neighboring pixels (i+1,j), (i,j+1), (i-1,j), (i,j-1).

average

## **Combining Constraints**



**Computer Science** 

# Use compromise of isobrightness solution and average-of-neighbors solution.



## **Combining Constraints**



**Computer Science** 

Combine two error measures, departure from

brightness: Error<sub>brightness</sub>  $(E_{ij} - R(p_{ij},q_{ij}))^2$ smoothness: Error<sub>smoothness</sub>  $(p_{ij}-p_{ij})^2 + (q_{ij}-q_{ij})^2$ where <u>p</u>, <u>q</u> are local averages:  $\underline{p} = 1/4 (p_{i+1,j}+p_{i,j+1}+p_{i-1,j}+p_{i,j-1})$  $\underline{q} = 1/4 (q_{i+1,j}+q_{i,j+1}+q_{i-1,j}+q_{i,j-1})$ 

using regularization:

$$Error_{smoothness} + \lambda Error_{brightness}$$

## Horn's Shape-from-Shading Algorithm



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#### Minimization Problem:

 $\begin{array}{l} & \operatorname{Error}_{\text{smoothness}} + \lambda \operatorname{Error}_{\text{brightness}} \\ & \min \left\{ \sum \left( (p_{ij} - \underline{p}_{ij})^2 + (q_{ij} - \underline{q}_{ij})^2 \right) + \lambda \sum \left( \mathsf{E}_{ij} - \mathsf{R}(p_{ij}, q_{ij}) \right)^2 \right\} \\ & \mathsf{p}_{\mathsf{kl}}, q_{\mathsf{kl}} \end{array}$ 

Solution: Iterative Scheme

$$p_{ij}^{(n+1)} = \underline{p}_{ij}^{(n)} - \lambda \left( \mathsf{E}_{ij} - \mathsf{R}(\mathsf{p}_{ij}^{(n)}, \mathsf{q}_{ij}^{(n)}) \right) \left( \frac{\partial R}{\partial p_{ij}} \right)^{(n)}$$
$$q_{ij}^{(n+1)} = \underline{q}_{ij}^{(n)} - \lambda \left( \mathsf{E}_{ij} - \mathsf{R}(\mathsf{p}_{ij}^{(n)}, \mathsf{q}_{ij}^{(n)}) \right) \left( \frac{\partial R}{\partial q_{ij}} \right)^{(n)}$$





All non-boundary points:  $p_{ij}^{(0)} = 0$  and  $q_{ij}^{(0)} = 0$ 

Silhouette boundaries with smooth edges:

Surface orientation is perpendicular to viewer's line of sight (optical axis of camera) and to silhouette. All such surface orientations project onto circle in p-q-space.









#### Horn '86



Microscope image and needle diagram of reconstructed surface of droplet of flower of *Cannabis sativa* plant



#### Shape-from-Shading iterative algorithm:



Spherical cap on plane

Horn '90



Horn '90

Conical singularity



Horn '90

Too rapid reduction of penalty term in smoothness

Various programming errors

Propagation of instability at edge of image when penalty term set to zero

## Learning Objectives:



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- Shape from shading is a heavily underconstrained problem. Solutions involve iterative schemes & careful attention to implementation & testing methodology.
- Photometric Stereo is a problem that can be solved with 2 input images but the more the better (LSM). Ensure sufficiently different light source directions.
- Connection to Computer Graphics: The Lambertian surface reflectance model is convenient (but sometimes not applied properly).
- Tools learned: Modeling reflectance properties, with BRDF, E=cosθ<sub>i</sub>, reflectance map R(p,q), LSM, iterative method