## Search Strategies



Consider a path planning problem with a graph shown above. The starting point is node S and the end point is node T . The cost of each edge has been labeled in the figure. We want to find the shortest path from S to T . Suppose the estimated remaining cost from each node to T is given in the following table:

| Node | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| Estimated remaining cost | 12 | 10 | 10 | 6 |

Note that in the example we did in class, the estimated remaining costs of nodes next to the goal node were exactly the edge costs. In this example, however, we assume that the edge costs are not visible when the decision on which path has minimal costs is made.

Questions:

1. Run the "Greedy Search with Pruning" algorithm on the graph. What is the shortest path and its corresponding cost?

The procedure of the algorithm can be shown in the following table. One can extract the optimal path from the

| Step | Optimal Cost [Prev. Node] |  |  |  |  |  | Unvisited Nodes | Chosen Node |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | S | A | B | C | D | T |  |  |
| 0 | 0 | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | S, A, B, C, D, T | S |
| 1 | 0 | $4[\mathrm{~S}]$ | $\infty$ | $2[\mathrm{~S}]$ | $\infty$ | $\infty$ | $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}, \mathrm{T}$ | C |
| 2 | 0 | $4[\mathrm{~S}]$ | $6[\mathrm{C}]$ | $2[\mathrm{~S}]$ | $8[\mathrm{C}]$ | $\infty$ | $\mathrm{A}, \mathrm{B}, \mathrm{D}, \mathrm{T}$ | A |
| 3 | 0 | $4[\mathrm{~S}]$ | $6[\mathrm{C}]$ | $2[\mathrm{~S}]$ | $6[\mathrm{~A}]$ | $\infty$ | B, D, T | B |
| 4 | 0 | $4[\mathrm{~S}]$ | $6[\mathrm{C}]$ | $2[\mathrm{~S}]$ | $6[\mathrm{~A}]$ | $12[\mathrm{~B}]$ | $\mathrm{D}, \mathrm{T}$ | D |
| 5 | 0 | $4[\mathrm{~S}]$ | $6[\mathrm{C}]$ | $2[\mathrm{~S}]$ | $6[\mathrm{~A}]$ | $12[\mathrm{~B}]$ | T | T |
| 6 | 0 | $4[\mathrm{~S}]$ | $6[\mathrm{C}]$ | $2[\mathrm{~S}]$ | $6[\mathrm{~A}]$ | $12[\mathrm{~B}]$ |  |  |

last row: $S \rightarrow C \rightarrow B \rightarrow T$ and the corresponding cost is 12 .
2. Run the $\mathrm{A}^{*}$ algorithm on the graph. What is the shortest path and its corresponding cost? The procedure of $\mathrm{A}^{*}$ algorithm can be shown in the following table.

| Step | Optimal Cost [Prev. Node] |  |  |  |  |  | Unvisited Nodes | Chosen Node |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | S | A | B | C | D | T |  |  |
| 0 | 0 | $\infty$ | $\infty$ | $\infty$ | $\infty$ | $\infty$ | S, A, B, C, D, T | S |
| 1 | 0 | $4[\mathrm{~S}]$ | $\infty$ | $2[\mathrm{~S}]$ | $\infty$ | $\infty$ | A, B, C, D, T | C |
| 2 | 0 | $4[\mathrm{~S}]$ | $6[\mathrm{C}]$ | $2[\mathrm{~S}]$ | $8[\mathrm{C}]$ | $\infty$ | A, B, D, T | D |
| 3 | 0 | $4[\mathrm{~S}]$ | $6[\mathrm{C}]$ | $2[\mathrm{~S}]$ | $8[\mathrm{C}]$ | $15[\mathrm{D}]$ | A, B, T | T |
| 4 | 0 | $4[\mathrm{~S}]$ | $6[\mathrm{C}]$ | $2[\mathrm{~S}]$ | $8[\mathrm{C}]$ | $15[\mathrm{D}]$ | A, B |  |

One can extract the optimal path from the last row: $S \rightarrow C \rightarrow D \rightarrow T$ and the corresponding cost is 15 .
3. Change the estimated remaining cost of node $B$ from 10 to 5 and run the $A^{*}$ algorithm again. What's the shortest path and its corresponding cost? The new procedure of $\mathrm{A}^{*}$ algorithm can be shown in the following table.

\left.| Step | Optimal Cost [Prev. Node] |  |  |  |  |  | Unvisited Nodes | Chosen Node |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | S | A | B | C | D | T |  |  |$\right]$ S

One can extract the optimal path from the last row: $S \rightarrow C \rightarrow B \rightarrow T$ and the corresponding cost is 12 .

## Rule-Based Systems

Consider a system with the following rules.

| Index | Antecedents | Consequents |
| :---: | :--- | :--- |
| R1 | X has fictional characters | X is a novel |
| R2 | X is an invented prose narrative | X is a novel |
| R3 | X has fictional characters <br> X is very long <br> X contains intimate human experience | X is a novel |
| R4 | X is a novel <br> X has historical plots | X is a historical novel |
| R5 | X is a novel <br> X is imported <br> X describes the June Rebellion in Paris | X is a historical novel |
| R6 | X is a novel <br> X is imported <br> X elaborates upon the history of France | X is a historical novel |
| R7 | X was authored by Victor Hugo <br> X was published in Belgium in the 19th century | X is one of the greatest works at the time |
| R8 | X has doodles <br> X has tea stains | X is a historical novel lightly worn out <br> X is one of the greatest works at the time <br> X has hardcover <br> X is lightly worn out |

Suppose the following assertions about a book B are in the system's initial working memory:

- B contains intimate human experience
- B is very long
- B has fictional characters
- B is imported
- B describes the June Rebellion in Paris
- B was authored by Victor Hugo
- B was published in Belgium in the 19th century
- B has hardcover
- B is lightly worn out

The expert system evaluates the rules as follows:

- Conflicts are resolved based on rule ordering and antecedent ordering. In other words, when examining rules, the system always go from R1 to R9 sequentially; when examining antecedents in any rule, the system always follow the order as appeared in the table.
- A rule is never selected unless it adds a new assertion to the working memory.


## Questions:

1. Simulate how the expert system interprets the working memory by making a table with the relevant rule indices (e.g., R4) and by indicating any new assertions added to the working memory. An example of your answer is given as follows.

| Step | Rules ready to be instantiated | Rule selected | New assertion |
| :---: | :---: | :---: | :---: |
| 1 | R1, R3, R7 | R1 | B is a novel |
| 2 | R1, R3, R5, R7 | R5 | B is a historical novel |
| 3 | R1, R3, R5, R7 | R7 | B is one of the greatest works at the time |
| 4 | R1, R3, R5, R7, R9 | R9 | B is Les Misérables |

2. The expert system uses backward chaining and starts with the hypothesis that book B is Les Misérables but no initial assertions. The system asks the user questions to build the working memory and instantiate rules. The user answers "yes" to any question corresponding to an assertion on the list given in the initial working memory and "no" otherwise. In addition, the system can read a previously stored answer to a repeated question, so it never asks the same question twice.
It can be helpful to draw a diagram as in 1 .
(a) If the system follows in a depth-first way, list the questions in the order it asks. There should be 12 questions in total.
i. B is Les Misérables?
ii. B is a historical novel?
iii. B is a novel?
iv. B has fictional characters?
v. B has historical plots?
vi. B is imported?
vii. B describes the June Rebellion in Paris?
viii. B is one of the greatest works at the time?
ix. B was authored by Victor Hugo?
x. B was published in Belgium in the 19th century?
xi. B has a hardcover?
xii. B is lightly worn out?
(b) If the system follows in a breadth-first way, list the questions in the order it asks. There should be 13 questions in total.
i. B is Les Misérables?
ii. B is a historical novel?
iii. B is one of the greatest works at the time?
iv. B has a hardcover?
v. B is lightly worn out?
vi. B is a novel?
vii. B has historical plots?
viii. B is imported?
ix. B describes the June Rebellion in Paris?
x. B elaborates upon the history of France?
xi. B was authored by Victor Hugo?
xii. B was published in Belgium in the 19th century?
xiii. B has fictional characters?

Please be careful as an error in the middle of your answer means the portion after that is also incorrect.
3. Which of the two backward chaining versions of the expert system would be preferred? Choose each of the following that support your view.
(a) Depth-first, because questions tend to stay relevant to particular subproblems, rather than jumping around.
(b) Depth-first, because the answer will be produced more reliably.
(c) Breadth-first, because answers will be produced faster.
(d) Breadth-first, because questions tend to stay relevant to particular subproblems, rather than jumping around.


Figure 1: Backward chaining diagram

## Propositional Logic and Inference

Given the following axioms:

$$
\begin{gather*}
E_{1} \vee \neg E_{2}  \tag{1}\\
E_{2} \vee \neg E_{3}  \tag{2}\\
\neg E_{1}  \tag{3}\\
E_{2} \vee E_{3} \vee E_{4} \tag{4}
\end{gather*}
$$

Prove the following theorem by refutation:

$$
E_{4}
$$

Use indices to refer to the axioms or new statements from resolution to carry out the proof.

1. Negate the theorem:

$$
\begin{equation*}
\neg E_{4} \tag{5}
\end{equation*}
$$

2. Resolve (4) and (5):

$$
\begin{equation*}
E_{2} \vee E_{3} \tag{6}
\end{equation*}
$$

3. Resolve (2) and (6):

$$
\begin{equation*}
E_{2} \tag{7}
\end{equation*}
$$

4. Resolve (1) and (7):

$$
\begin{equation*}
E_{1} \tag{8}
\end{equation*}
$$

5. Resolve (3) and (8):

NIL

Therefore, $E_{4}$ is true.

