CS 332 - Homework 2

Due: Thursday, February 9 at midnight

Please turn in Homework 2 on gradescope, not emailed or submitted directly on-line.

Reading:

Chapter 3, Pages 165-187 and Chapter 4, section 1, pages 193-210.

If you get a chance, take a look at the 1936 article by Alan Turing that is about computation. A pointer to that article can be found in the extra reading section of the course homepage.

PROBLEMS: Answer the following questions. You should (briefly) justify all your answers if they are not obvious or if the problems says “no justification necessary”. This also holds for all future HW’s, tests and quizzes

1. Consider the Turing machine $R_1$ which is defined by $\Sigma = \{a,b\}$, $\Gamma = \{a,b,c,B\}$ and states $Q = \{q_0, q_1, q_a, q_r\}$ and whose program is the following five tuples:

   $(q_0, a, q_0, c, R)$ $(q_1, a, q_0, b, L)$

   $(q_0, c, q_0, a, R)$ $(q_1, b, q_r, c, R)$

   $(q_0, b, q_1, b, R)$ $(q_1, c, q_0, a, R)$

   $(q_0, B, q_r, a, R)$ $(q_1, B, q_a, b, R)$

   (i). Carry out the computation of $R_1$ on the string $aba$. Show your work as the tape head travels over the tape, changing states and symbols. State what string is left on the tape when this computation halts, and say whether the TM accepts $aba$ or rejects it.

   (ii). Do the same as you did in part (i) except for the computation of $R_1$ on the string $bbaba$.

   (iii). Do the same as you did in part (i) except for the computation of $R_1$ on the string $abaab$.

2. Write a TM $R_2$ which accepts the set of binary strings which, when interpreted as positive decimal integers, are evenly divisible by 4 (note this means the input string ends in 100), and rejects all other binary strings. $R_2$ should have a $\Sigma = \{0,1\}$, and a $\Gamma = \{0,1,B\}$. State the set of states $Q$ you choose. Your TM can have either 3, 4, or 5 states.

   Write the full TM program of your TM and give a small example of how your TM works on two different input strings, one which accepts the input and another string which rejects it.
3. Write a TM $R_3$ which decides the set of binary strings which contains a 101 as a substring of its input. $R_3$ should have the same $\Sigma$ and $\Gamma$ as machine $R_2$ in problem 2. Give the full TM program of the $R_3$ machine and give a small example of how your TM works on two different strings $x$ and $y$, one which accepts the input $x$ and one which rejects $y$. Briefly, in a few sentences, explain how $R_3$ works.

4. Consider a TM $R_4$ which accepts any input string of the form #W#W\# where W is a string of capital letters and #’s and where $W^r$ is the string W but written in reverse (from the rightmost symbol of W to the leftmost). So for example $(JHKG)^r = GKHJ$.

This time you should not write out the program for $R_4$ but rather you should describe what steps the TM $R_4$ takes when it does a computation. Essentially this requires writing a description of the steps that such a TM would take in order to compute whether an input string for $R_4$ is accepted or rejected by this Turing machine.

For an example of this type of description you should look at the examples 3.11 and 3.12 on pages 174 and 175 of our textbook. Said another way your job is to write the description of how $R_4$ works in English and as a step by step algorithm. No state diagram of a TM or full TM program is needed.