CS 332
Spring 2023

CS 332 - Homework 4

Due: FRiday, March 29

Reading: Chapter 3, pages 178-180, Chapter 7, pages 275-289 and pages 292-298

PROBLEMS:

1. (i). Prove that the halting problem is recognizable.
Explain your reasoning.

(ii). Prove that if J is set of binary strings and (\{0, 1\}^* - J) \leq_m J then J is decidable. Explain by saying how you would decide J once you know that (\{0, 1\}^* - J) \leq_m J.

2. Give an example of languages A and B where A, B, and (A - B) are all infinite languages, and A-B is decidable, but both A and B are not decidable. Explain.
You can use the fact (proved in class) that \( A_{TM} \) is an undecidable language, and make A and B be languages which are defined using \( A_{TM} \)
Hint: Once you think about what the languages A and B are, then the facts above about A, B and A-B are pretty easy to see.

3. Let I be the set of all even length binary strings. (So I is decidable.)
Give an example of a language \( K \subseteq I \) with K undecidable.
Here you need to define the set K and state your reasoning why the K you define is undecidable.

4. Show that it is decidable to determine whether, given a TM M and an input w of M, the computation of M(w) goes into a loop without ever reaching the first blank symbol B past w on its tape.

HINT: One way to do this is to compute a number of steps which M(w) could take before it has to repeat one of the configurations of M during its computation on w.
(The configuration of a TM is define in Chapter 3, pages 168-169.)

Note that if a M(w) ever reaches the same configuration twice then it is in a loop.