PROBLEMS:

1. A vertex cover (VC) of a graph G is a set of vertices of G which, covers every edge in G. (Covering an edge e mean that the VC contains at least one vertex of edge e.)

   The vertex cover optimization problem is, given a graph G, find a VC which is a smallest possible vertex cover of G.

   (i). Give an example of a graph with at least 7 vertices whose smallest vertex cover is size 4.

   (ii). Write (DVC), a version of the VC problem which is a decision problem.

   (iii). Now show that the decision vertex cover problem has an efficient verifier and so is in NP. You should state what the certificate is for your verifier algorithm, state the algorithm \( V_{VC} \), and say why it is in P (polynomial time).

2. The hitting set problem is: You are given a non-empty finite set \( S \) of integers and \( F \), a collection of non-empty subsets of \( S \).

   A hitting set \( H \) is a subset of \( S \) which intersects every set in \( F \) at least once. The hitting set problem is to find a smallest sized hitting set of \( S \).

   Prove that the decision version of the hitting set problem is in NP.

3. Show that the max-cut problem is in NP.

   The max-cut problem is defined in problem 7.27 on page 325 (page 297 in the second edition of the textbook) of Sipser’s book, is in NP.

4. Assume that a function \( f \) is in polynomial time and can be computed in time \( O(n^3) \) and that \( g \) is in polynomial time and can be computed in time \( O(n^4) \).

   Prove that the function \( f \) composed with \( g = h \) defined by \( h(x) = f(g(x)) \), can be computed in time \( O(n^{12}) \).

5. The 3-clique language \( L = \{ G \mid G \text{ is a graph and } G \text{ contains a 3-clique } \} \). Define an infinite subset of \( L \) which is in P.
6. Give a straightforward algorithm which runs on polynomial time and which decides whether a given graph G contains a 4-clique.

Explain why your algorithm is in polynomial time. (I’m not asking for a formal proof, just a justification.)