CS 332
Spring 2023

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A few Notes on Undecidability (so far)

1. A standard Turing Machine (TM) has an input alphabet Σ and has input strings x which are finite strings of symbols from Σ.

M(x) = the result of TM M on input x = either "accept" or "reject" or "loop". (Here loop means "never halt" and the TM M may either actually loop on a finite part of its tape or run forever moving farther and farther along on its tape.)

2. The TM M recognizes a language L(M) defined by, L = $\{x|x$ is an input string of M and M(x) accepts $\}$.
Given a TM M, the language L(M) is a subset of Σ^* and is unique.

3. The language L is decidable if there is some TM M which recognizes L and which halts on every legal input string x for M.
In this case we say M accepts x if x ϵ L (i.e. "x is in L"), and otherwise it rejects x.

4. A language L is enumerable if there is a Turing machine with a printer which enumerates (lists) the elements of L.
L is enumerable if and only if it is recognizable (Theorem 3.21).

5. A language is undecidable if there is no TM which decides it.
A language is unrecognizable if there is no TM which recognizes it.

6. Recall that a TM that decides a language L also recognizes L. It follows from this that if L is unrecognizable then L is undecidable.

7. The language $A_{TM} = \{ <M, w> |$M is a TM that accepts the input string w $\}$.
A_{TM} is is a language which is recognizable but A_{TM} is not decidable. In fact, A_{TM} is recognized by a UTM, a universal Turing Machine. UTM is not unique.

From this it follows that the complement of $A_{TM} = \{ <M, w> |$M is a TM and w is a legal input string for M and M does not accept the input string w $\}$ in not even recognizable.